Mortality risk in Mexico: a comparison of the general population mortality and life tables for insured lives

Arelly Ornelas and Montserrat Guillén

IAALS, October 2012

Motivation

- Is there a different behavior for policyholders and general population? is the same gap for men and women?
- The council directive 2004/113/EC
- Are the unisex tables the best choice?
- How does Gender proportion affect a life insurance portfolio?
- Insurance company should use their own experience. Can we trust SESA information?

Background

- Graduation models, which have been used to smooth crude mortality.
- Laws of mortality: Heligman and Pollard, Gompertz, Makeham, among others
- Poisson models, which fit the number of deaths given the number of exposed to risk
- The Cox proportional hazards model
- British Continuous Mortality Investigation (1990), the Lee and Carter model (1992), Currie, Durban and Eilers model (2004), the Renshaw and Haberman model (2006), the Currie model (2006) and the Cairns, Blake and Dowdmodel (2006) and its generalizations

In Pitacco et al. (2009) there is an interesting review of the historical background on early mortality tables.

Outline

Modeling the mortality behavior in the Mexican general population

- Lee-Carter model (1992)
- Renshaw-Haberman model (2006)
- Age-Period-Cohort model (2006)

Adjusted mortality rates were compared with life tables from CNSF used in Mexico for insured lives

- Using a Brass-type model
- Changing the gender proportion in the mortality for general population
- Results were compared with the ones obtained for the population in Switzerland

Data

Exposure to risk of death. $L(x,t)$

- Census and count population between 1990 and 2010:
  1990 - 1995 - 2000 - 2005 - 2010.
  INEGI web page.

Number of Deaths. $d(x,t)$

- From 1990 to 2009
  CONAPO web page

Log crude death rate

$L(x,t)$ interpolate within the available data at every year

Definition

$$\log[m(x,t)] = \log \frac{d(x,t)}{L(x,t)}$$

Separated by sex and age from 0 to 100+. t from 1990 to 2009.
Data without description of sex and/or age were not considered

Policyholders

The life table actually used in Mexico by the insurance companies: CNSF - 2000I

- It is used to calculate mathematical provision
- It is unisex table
- From age 12 to age 99
- There is just one table that summarize the mortality behavior in one decade
- It was built twelve years ago

Modeling Mortality

Lee-Carter model (LC)

$$\log[m(x,t)] = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t + \epsilon_{x,t}$$

Renshaw-Haberman model (RH)

$$\log[m(x,t)] = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_{t-x}^{(3)} + \epsilon_{x,t}.$$

A cohort is a group in the population that was born at the same period. It is defined as $c = t - x$.

Age-Period-Cohort model (APC)

$$\log[m(x,t)] = \beta_x^{(1)} + \kappa_t^{(2)} + \gamma_{t-x}^{(3)} + \epsilon_{x,t}.$$

Describes $\log[m(x,t)]$ by a sum of age- period- and cohort-effects.

Modeling Mortality

Parameter interpretation

- $\hat{\beta}_x^{(1)}$: Arithmetic mean of the $\log[m(x,t)]$ over time
- $\beta_x^{(2)}$: Reflect variations of the logarithm of mortality at age x when κ_t varies over time.
- $\beta_x^{(3)}$: Reflect variations variations of the logarithm of mortality at age x when $\gamma_{tx}^{(3)}$ varies over time.
- κ_t: Represents mortality variations by year
- $\gamma_{tx}^{(3)}$ Reflects birth year effects.
- The error term $\epsilon_x(t)$ reflects particular age-specic historical influence not captured in the model.

The advantage about modeling cohort mortality is that individuals in the same cohort evolve together and every year, they celebrate the same birthday and are exposed to the same external phenomena, such as epidemics for instance

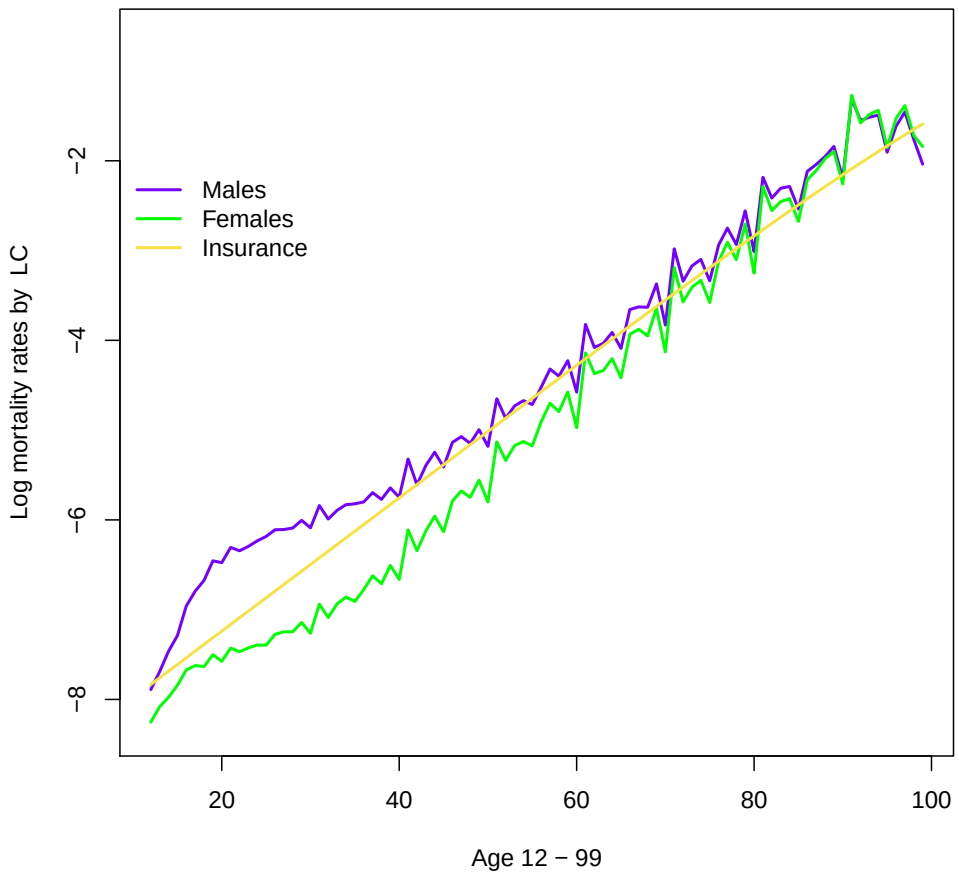
Result

Selection model was done plotting standardized residuals and computing Bayes Information Criterion.

- Residuals of RH are higher than those of LC or ACP, only for the older ages
- In other ages residuals are quite similar, and they are randomly distributed
- Models with cohorts effects improves the fit
- RH model minimized the BIC

Comparing rates

Mortality rates: Policyholders vs. Lee-Carter



Comparing rates

Brass-type model

It provides a link between the mortality rates of a given group to benchmark mortality rates.

In the actuarial context this model can also be used to measure the size of adverse selection.

In our application we consider as reference mortality rates those estimated for the Mexican population with the LC, RH and APC models.

Comparing rates

$$\log[m_{ph}(x,t)] = \beta_0 + \beta_1 \log[m_w(x,t)] + \epsilon_{x,t}$$

- $m_{ph}(x,t)$ represents the unisex death rate of Mexicans policyholders.
- $x \in [12, 99]$ and t is a constant
- Gender proportion was explored

Weighted death rate:

$$m_w(x,t) = w \cdot m_{female}(x,t) + (1-w) \cdot m_{male}(x,t)$$

w represents the proportion of females. We assume that the proportion is stable and constant for all adult age groups

Result

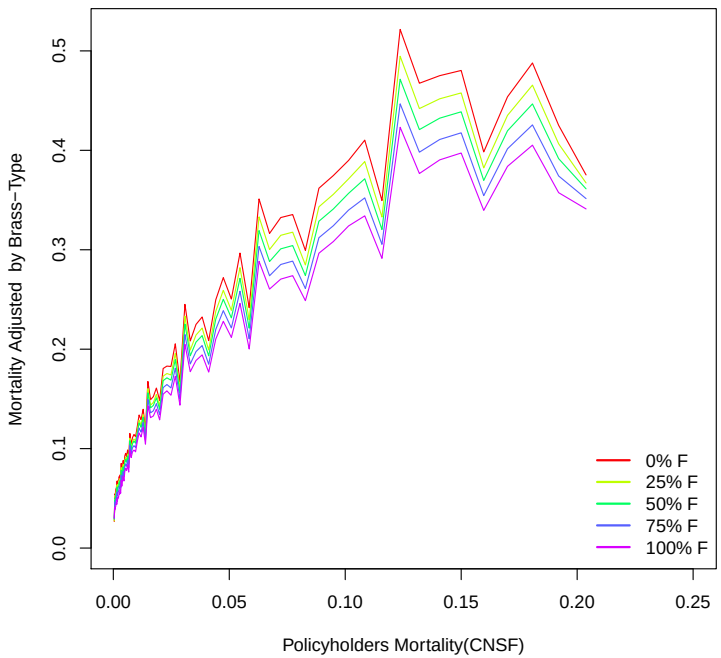
β estimated by Brass-type model

% Females (w)	LC		RH		APC	
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_0$	$\hat{\beta}_1$
0%	−0.06	1.04	−0.04	1.05	−0.01	1.05
25%	−0.14	1.00	−0.12	1.00	−0.10	1.00
50%	−0.22	0.95	−0.20	0.95	−0.19	0.96
75%	−0.30	0.91	−0.27	0.91	−0.27	0.91
100%	−0.38	0.87	−0.35	0.87	−0.35	0.87

$$\log[m_{ph}(x,t)] = -\beta_0 + \beta_1 \log[m_w(x,t)]$$

Result

$m_{ph}(x,t)$ vs. $\widehat{m_{ph}(x,t)}$



Model estimated for Switzerland

β estimated for Switzerland population

$$\log[\hat{m}_{ph}(x,t)] = -0.032 + 1.07 \log \hat{m}_{pop}(x,t) - 0.0004 \log \hat{m}_{pop}(x,t) \cdot t$$

Gatzert and Wesker (2011)

- $\hat{\beta}_0$ are similar between models when the percentage of women is higher than 75%
- $\hat{\beta}_1$ are similar between models when the percentage of women is lower than 25%

Final Comments

- The rates currently used in the insurance industry do not reflect the true behavior of the general population mortality
- Life tables for different purposes. Longevity and mortality risk.
- Unisex tariffs should be based on reasonable assumptions on the proportion of women in the portfolio, and on the gap between the respective general population mortality and the insured mortality
- It is important to have consistent databases. Data quality.
- Introduce lifestyle information in actuarial modeling. The main problem when replacing gender information by lifestyle data is moral hazard and the cost of verifying that a policyholder has not provided fraudulent information.
- The experience of other countries could be useful meanwhile the insurance companies collect their own resources.

Some References

- Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A., and Balevich, I. (2009) "A Quantitative Comparison of Stochastic Mortality Models Using Data from England and Wales and the United States," *North American Actuarial Journal*, 2009, Vol.13, No. 1, 1-35.
- Currie, I.D. (2006) "Smoothing and forecasting mortality rates with P-splines," Talk given at the Institute of Actuaries, URL:`http://www.ma.hw.ac.uk/~iain/research/talks/`
- Delwarde, A. and Denuit, M. (2003) "Importance de la période d'observation et des âges considérés dans la projection de la mortalité selon la méthode de Lee-Carter," *Belgian Actuarial Bulleti*, Vol. 3, No. 1, pag. 1-21.
- Lee, R.D. and Carter, L.R. (1992) "Modeling and Forecasting U. S. Mortality," *Journal of the American Statistical Association*, Vol. 87, No. 419, pag. 659-671.
- Gatzert, N. and Wesker, H. (2011) "Mortality Risk and its Effect on Shortfall and Risk Management in Life Insurance," *Working paper 2011/04, Friedrich-Alexander-University of Erlangen-Nürnberg*.
- Renshaw, a and Haberman, S. (2006) "A cohort - based extension to the Lee - Carter model for mortality reduction factors," *Insurance: Mathematics and Economics*, Vol. 38, No. 3, pag. 556-570. URL:`http://linkinghub.elsevier.com/retrieve/pii/S0167668705001678`.