Mortality risk in Mexico: a comparison of the general population mortality and life tables for insured lives

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Abstract

The aim of this study is to model the mortality behavior in the Mexican general population using data from 1990 to 2009 and to compare it to the mortality assumed in the life tables used in Mexico for insured lives. We describe the process of mortality rates calculation for each sex and age, a Lee-Carter model, a Renshaw-Haberman model and an Age-Period-Cohort model are fitted. The data used come from the INEGI and CONAPO. For each 5-year period, the number of deaths and the number of living people by age and sex are available. We also adjust a Brass-type model is used to compare the gap between the general population mortality rate estimates and the mortality rates estimates for the insured population that is being used by the National Insurance and Finance Commission in Mexico. Since, life tables for insured lives are not different for male and female, we assume possible scenarios of gender proportion in the insured lives mortality tables. We compare our results with the ones obtained for the population in Switzerland and we find similar results, for β_0 specially if we assume that the proportion of women among insured people is very small. We discuss the results and emphasize the limitations of the mortality tables for insured lives currently use din Mexico and, in general, the incurred bias when using unisex mortality tables.

Keywords: Mortality rates, Lee Carter, Longevity dynamics, Insured population.

1 Introduction

During the twentieth century the human population experienced a substantial decrease in mortality. This was due mainly to reduced mortality of infants from infectious diseases. Particularly in Mexico, life expectancy at birth increased from 57.04 years in 1960 to 76.47 years in 2009, but in the early nineties the yearly increase only improved 0.46 for men and 0.33 for women. Henceforth improving life expectancy for women has undergone very few changes but always counts down, the last change was 0.22 yearly increase from 2008 to 2009. The behavior is different for men, remaining constant throughout the years, improved 0.23 in 2002 and 0.24 in 2009. (`http://www.worldbank.org`)

The slowdown in the growth of life expectancy is associated with causes of death, such as deaths from accidents and violence, and the increase of chronic conditions such as heart-related diseases and hypertension as well as cancer. For the male population, changes can also be attributed to emigration, for instance to the United States, that continuously modify the composition and structure of the population (Camposortega,1997). It is a fact that mortality has declined in the early childhood and it has moved to a later age, although this is an achievement in the demographic aspect, in the actuarial area this behavior give rise to rethink how the mortality behavior has been modeled, that in order to make a good risk management, making calculations of reserves, annuities, pension plans and to calculate premiums for life products.

2 Metodology

Through the years, various models have been explored to describe mortality. In the nineteenth century the literature already mentioned aspects related to mortality, annuities and adverse selection risk. The first mortality table that consider alteration in mortality was created in the United Kingdom, in order to protect insurers against loss. These tables were constructed with data for insurers in the period 1900 to 1920 [10]. Recently, parametric models have been used to determine mortality behavior, better known as the laws of mortality: Heligman-Pollard, Gompertz, Makeham, other approach are: Poisson models, survival models like Cox proportional hazards model. As regards non-parametric models have been used generalized additive models (GAM). Among the models that take into account the cohort effect and age effect are: the British CMI (1990), the Lee-Carter (1992), Currie, Durban and Eilers (2004), the Renshaw-Haberman (2006), the Currie (2006), CBD (2006).

In order to obtain a better fit we adjusted some of the models mentioned above.

2.1 Lee Carter model

Lee and Carter (1992) model combines a demographic time series model to adjust rates that decrease exponentially, mortality represents $m(x,t)$ in a single index which can be modeled as a time series to obtain predictions and confidence intervals.

$$\log[m(x,t)] = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t + \epsilon_{x,t} \tag{1}$$

where κ_t is the index that varies in time and represents the variations in mortality by year . $\dfrac{\partial log(m(x,t))}{\partial t} = \beta_x^{(2)}\dfrac{\partial k}{\partial t}$. Negative $\beta_x^{(2)}$ is expected in certain ages indicating that for these ages mortality increases.

The solution of this system of equations 1 is not unique, therefore it is necessary to impose constraints to guarantee identifiability:

$$\sum_x \beta_x^{(2)} = 1 \qquad \text{and} \qquad \sum_t \kappa_t = 0 \tag{2}$$

$\beta_x^{(2)}$ and κ_t are estimated by using singular value decomposition applied to the matrix of logarithms of the mortality rates, once it has been subtracted the time average, $log[m(x,t)] - \hat{\beta}_x^{(1)}$, where $\hat{\beta}_x^{(1)}$ is the time average for $log[m(x,t)]$. As soon as κ_t has been estimated it is recalculated in order to minimize errors, since the estimated values do not generally reflect the true behavior of the number of deaths by age. So that κ_t is modeled as an ARIMA time series by using theory Box-Jenkins methodology.

2.2 Renshaw and Haberman model

The model proposed by Renshaw and Haberman in 2006 seeks to represent the cohort effect, creating mortality models such that generational changes are highlighted, cohort is defined as $c = t - x$. Then the term $\beta_x^{(3)}\gamma_{t-x}^{(3)}$ is added to the Lee-Carter model:

$$\log[m(x,t)] = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_{t-x}^{(3)} + \epsilon_{x,t}. \tag{3}$$

as well as Lee-Carter model, this new model also presents identifiability problems, thereby another restriction are added to the previous equations (2):

$$\sum_x \beta_x^{(3)} = 1 \qquad \text{and} \qquad \sum_{x,t} \gamma_{t-x}^{(3)} = 0 \tag{4}$$

The parameters are estimated using an iterative process with an initial estimates then adjust according to $log[m(x,t)] = \dfrac{l(x,t)}{d(x,t)}$.

2.3 Age-Period-Cohort model

This model was proposed by Currie in 2006:

$$\log[m(x,t)] = \beta_x^{(1)} + \frac{1}{n_a}\kappa_t^{(2)} + \frac{1}{n_a}\gamma_{t-x}^{(3)} + \epsilon_{x,t}. \tag{5}$$

n_a is the total number of ages. Currie used P-splines to estimate the parameters, although in this case no smoothing conditions are imposed , because the model described is the one presented in Cairns, A.J.G. et al. (2009). The following constraints are required:

$$\sum_t \kappa_t^{(2)} = 0 \qquad \text{and} \qquad \sum_{x,t} \gamma_{t-x}^{(3)} = 0 \tag{6}$$

2.4 Interpreting the estimated parameters

In all the models described before:

- $\beta_x^{(1)}$ is the average of $\ln m(x,t)$ in the calendar years t, thus $\beta_x^{(1)}$ is the general form of the mortality.

- $\beta_x^{(2)}$ describes the variations of the logarithm of mortality at age x when κ_t varies over time. The parameters $\beta_x^{(i)}$ show the effects of age.
- $\kappa_t^{(i)}$ is the index that varies in time and represents the variations in mortality by year, reflects the effects of age and year of death, ie the period. $\frac{\partial log(m(x,t))}{\partial t} = \beta_x^{(2)} \frac{\partial k}{\partial t}$.
- $\gamma_{tx}^{(i)}$ reflects the effects of age and year of birth, ie, describes the cohort.
- The error term $\epsilon_x(t)$ reflects the historical influence of each particular age which is not captured by the model. It is considered with mean 0 and variance σ_ϵ^2

2.5 Relational Brass-type model

Once we estimated mortality rates in Mexico we proceeded with the estimation of the Brass type model parameters. This model associates the mortality of policyholders to that or a reference population. In actuarial terms we are interested in measuring the impact of adverse selection.

$$f(m(x,t)) = \beta_0 + \beta_1 f([m(x,t)_{ref}]) + \epsilon_{x,t}. \tag{7}$$

We considered as reference mortality rates those estimated for the Mexican population with the Lee-Carter model, Renshaw-Haberman and Age-Period-Cohort ($m(x,t)_{ref}$). The main idea is that the behavior of mortality in the insurance sector is not the same to that in the general population and put it aside would be reflected in a bias estimation. This model can be extended to incorporate interaction between mortality rates and time t, as Gatzert and Wesker(2011) have proposed.

In our case we focalized in to explore the influence of gender composition on mortality giving weights w that reflect the proportion of men and women in the general population, so that we recalculated the mortality rates as follows $m_w(x,t) = wm_{female}(x,t) + (1-w)m_{male}(x,t)$. We propose the following model:

$$\log[m_{aseg}(x,t)] = \beta_0 + \beta_1 \log[m_w(x,t)] + \epsilon_{x,t} \tag{8}$$

which can be extended when including the interaction with age.

2.6 Model Selection Criteria

With the purpose of selecting the model that better fit standardized residuals were plotted and the Bayes Information Criterion was computed.

Bayes Information Criterion penalizes models that are over parameterized.

$$BIC_k = \hat{l}_x - \frac{1}{2} n_k \log N$$

where n_k is the number of parameters, $\hat{l}_x$ maximum of the likelihood function and N the number of observations.

Standardized residuals,

$$Z(t,x) = \frac{d(t,x) - l(t,x)\hat{m}(t,t;\phi)}{\sqrt{l(t,x)\hat{m}(t,t;\phi)}},$$

is assumed to be distributed identically and independently as $N(0,1)$.

R version 2.13 were used to perform the analysis.

3 Data description

Data for this study were obtained by consulting the website of the National Institute of Statistics, Geography and Informatics (INEGI) and The National Population Council (CONAPO). Mortality rates were computed as the ratio of between the number of people alive and the number of registered deaths with age x in a year t. The first ones were extracted directly from censuses and population counts conducted by the INEGI. The second ones were extracted from the CONAPO website. The data analyzed reflect the demographic behavior of the 32 states of Mexico from 1990 to 2009, we have separated by males and females with age between 0 and 100 years.

In order to avoid biased information, data without description of sex and/or age were excluded, this because the omission of such information could be caused by deaths at a young age so they are not reported gender, or in the case of old age the exact death-age is not known. Then it is assumed that the error is not random and if we make a proportional distribution of unspecified, it can alter the distribution of the population that stated both age and sex. The percentage of unclassified data is tiny so it is not losing a lot of information. The average percentage of those who did not report age is 0.6% for males, 0.34% for females and 23.29% for those who also did not report sex. The highest percentage of non reported sex were found in children under one year old, on average 0.26%. For other ages the average percentage is between 0.02% and 0.06%.

Mortality tables currently in use in the Mexican insurance sector were published in the official journal of the federation by CNSF on March 2000: individual and group Mortality Rates (CNSF 2000-I and CNSF 2000-G). These life tables were constructed by Manuel Mendoza Ramirez, Ana Maria Madrigal Martnez Gmez and Evangelina Torres, following a Bayesian logistic regression model whose description is found in Mendoza et al. (1999 and 2000). The data used for its construction were reported mortality experience for all Mexican companies that operate the life insurance in the SESA between 1991 to 1998, without differentiating by gender.

Once established the data source we will describe the treatment performed before proceeding to apply models described in paragraphs above.

As mentioned above, we only have information about alive people for the years where census was held, so it is necessary to complete information in the years which census was not performed so that linear interpolation have been done, according to presented by Delwarde and Denuit (2003):

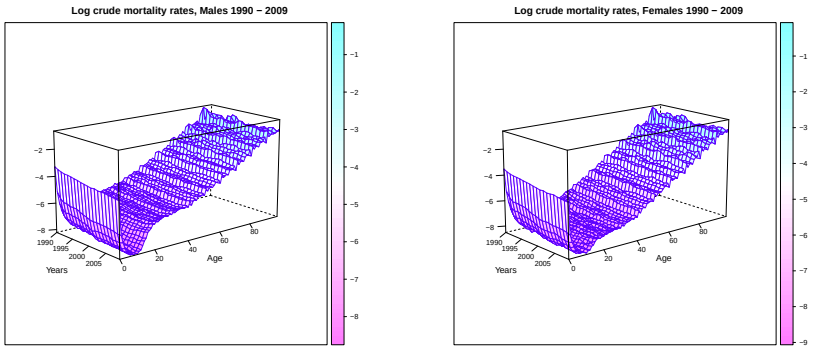
From the disaggregated information by sex and age for each age x from 0 to 100 years or more the missing data for the period t in 1992 to 1994, 1996 to 1999, 2001 to 2004 and from 2006 to 2009 were obtained. The first step is calculate the proportion of alive at age x in t year, on total live births in that year:

$$r_x(t) = \frac{l(x,t)}{\sum_{x=0}^{n} l(x,t)},$$

Once you have this ratio, you can perform linear interpolation between two years t_1 and t_2 for each age:

$$r_x(t) = r_x(t_1) + \left(\frac{t - t_1}{t_2 - t_1}\right)\Big(r_x(t_1) - r_x(t_2)\Big)$$

where t is the year for which you want the missing data.



(a) Males (b) Females

Figure 1: Log crude mortality rates for Mexican population from 1990 to 2009: (a) Males, (b) Females

Therefore we could calculate the log crude rate of mortality $m(x,t)$ as:

$$\log m(x,t) = \log \frac{l(x,t)}{d(x,t)}$$

where $l(x,t)$ is the number of persons living and $d(x,t)$ the number of deaths for each year t and age x.

The next step was setting the Lee Carter model and its extensions. The estimation was performed separately for men and women aged 0 to 99 and for the years 1990 to 2009. We selected the estimates obtained for ages 12 to 99 years, whereas 12 years is the minimum age of recruitment to some insurance companies. To make the estimate has been used Lifemetrics software, with implementation in R version 2.13.

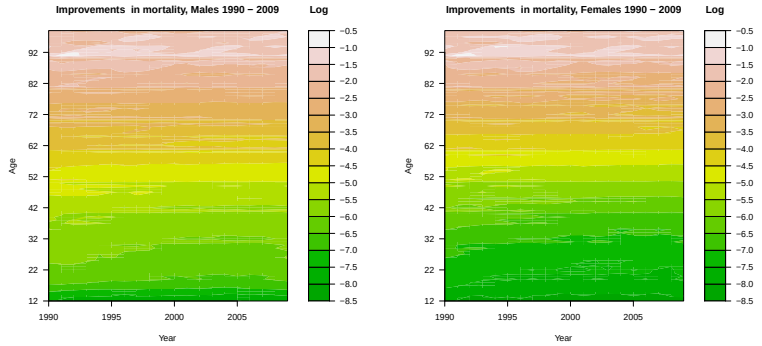
4 Results

We performed an exploratory analysis of the behavior of mortality over the study years t and at different ages x, separated by sex. Figure 1 shows that the curves of log mortality in men and women are very similar in childhood and adulthood, the largest differences are in the youth that is when men are more likely to have accidents and die, behavior that has not changed over the two decades of study.

Figure 2 presents improved mortality rates. The bottom of the figure, where green is more prominent, it is for women and note that this graph indicates that mortality in this age group is lower. The horizontal lines present in virtually all ages tell us that there has been no improvement in mortality in the years of study, the age for which there are more changes for over 80 years, fulfilling for both sexes.

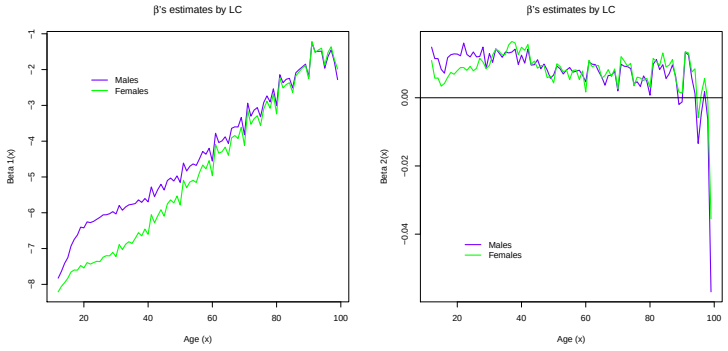
4.1 Lee-Carter model

We now present the parameter estimates for the Lee-Carter model. The number of estimated parameters is 88 for each β and 20 for κ, resulting in a total of 196 parameters.

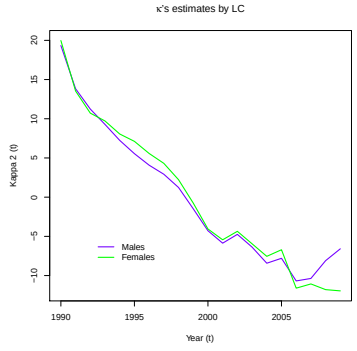


(a) Males (b) Females

Figure 2: Improvements in mortality for Mexican population, from 1990 to 2009: (a) Males, (b) Females



(a) $\beta_x^{(1)}$ (b) $\beta_x^{(2)}$



(c) $\kappa_t^{(2)}$

Figure 3: Estimated parameters for Lee-Carter model. Male and Female aged 12 to 99 from 1990 to 2009. Mexican population.

Notice that $\hat{\beta}_x^{(1)}$ estimates for women are lower than those of men (Figure 3), except for ages above 80 years. Applying the exponential function yields the crude mortality

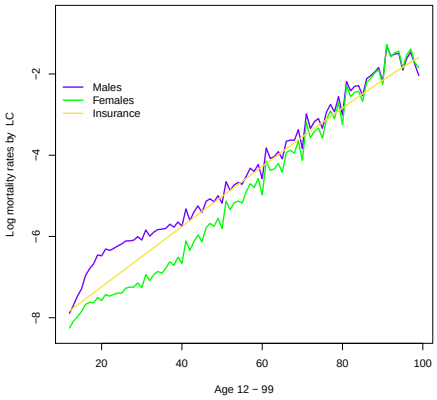


Figure 4: Log estimated mortality rates for 2000 LC, Men and Women for Mexican population. Log insured mortality rates (CNSF) Mexico.

rates, which in the case of women have a median of 0.007 (IQR $0.001, 0.045$) and for men have a median of 0.010 (IQR $0.003, 0.054$).

Another good observation to remark is that $\hat{\beta}_x^{(2)}$ for both sexes was maintained with a positive sign from age 12 to age 78. Changes in the logarithm of mortality ages are minimal because the β 's estimated are around 0.01. For women there is a median of 0.009 (IQR $0.006, 0.011$) and for men 0.009 (IQR $0.006, 0.012$). More abrupt changes between men and women are given the ages between 12 and 40 years for females where the logarithm of mortality rates grew steadily. After 40 years the curves do not differ by sex.

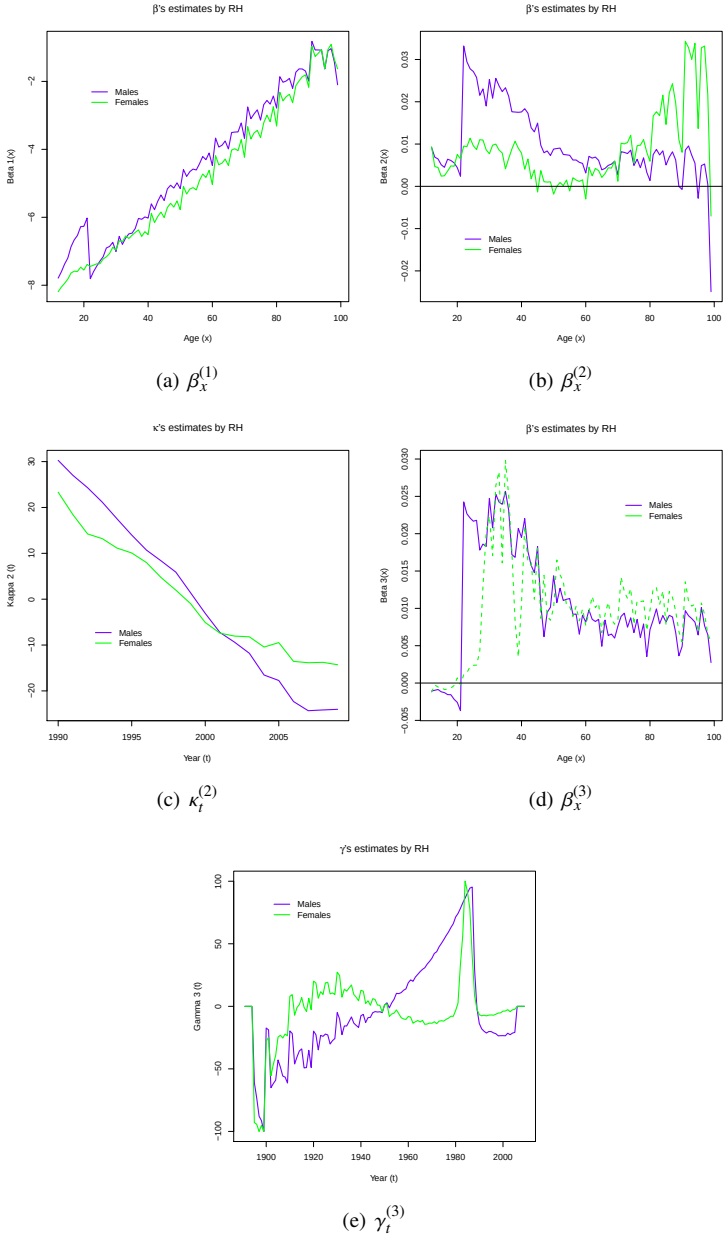
About the values of $\hat{\kappa}$'s they are steadily decreasing for both sexes. By differentiating the values of $\hat{\kappa}$ by gender, we note that from 1995 to 2005, women have a slight increase with an approach to male behavior in 2000. However, after 2005, the opposite happens, the $\hat{\kappa}$'s for men are higher than those of women.

We recall that we have obtained the mortality rate for the insured population, which allows us to draw a graph comparing the logarithm of these and the logarithm of the values set by the model Lee-Carter. We present this separately for men and women, this graph is shown in figure 4. One of the important facts to note from this chart is that the values currently used mortality corresponds to an average of the values sex-differentiated, but fails to reflect the actual behavior of men and women separately. Furthermore after age 70, it is maintained below the expected mortality for both sexes. This can create problems in the creation of reserves and in projecting the claims as they are always affected by the gender composition, so if there are more men in the insurance portfolio mortality would be expected to be less real. Later we will address this problem by studying in depth the Brass-type model described in equation 8.

4.2 Renshaw and Haberman model

Parameters were obtained for the Renshaw and Haberman model estimated using singular value decomposition. Figure 5 shows the values for both the male and the female population aged 12 to 99 years.The number of estimated parameters is 88 for every β (3×88), 20 for κ and 119 for γ, resulting in a total of 403 estimated parameters.

The estimated values by sex are closer to those estimated by LC, there is no perceived difference between ages 22 and 40, the most notable differences are just before age 22. Mortality rates are higher in males.



(a) $\beta_x^{(1)}$ (b) $\beta_x^{(2)}$

(c) $\kappa_t^{(2)}$ (d) $\beta_x^{(3)}$

(e) $\gamma_t^{(3)}$

Figure 5: Estimated Parameters Renshaw-Haberman. Men and women aged 12 to 99 from 1990 to 2009. Mexican population.

About the values of $\hat{\beta}_x^{(2)}$ for women has a median 0.008 (IQR $0.003, 0.011$) and for men 0.007 (IQR $0.005, 0.014$). For men are above zero, an increase at age 23 up to

0.033, then dropped gradually to age 60 where from this age and to 90 the behavior is maintained with ups and downs but no longer so marked. After 90 crosses zero to take negative values, the most dramatic being the value taken at age 99.

In women the estimates are more stable from age 12 to 40, where a slight decline starts to become negative, to continue oscillating between positive and negative values, but very close to zero between ages 45 and 60. From 61 onwards begins to rise steadily to peak at age 91, after this age have similar values except for age 95 and 99 with the latter taking negative values.

The slopes biggest indicate more abrupt increases in the logarithm of the mortality rate and therefore the same mortality. We found abrupt increases in the logarithm of the mortality rate, jumping in men age 23 reaffirmed the fact that mortality in youth grows to this genre, which does not happen with women. Unlike the Lee Carter Model, the Renshaw and Haberman model makes the difference between sex for older ages, this is clearly seen in women, whose mortality was increasing in recent years.

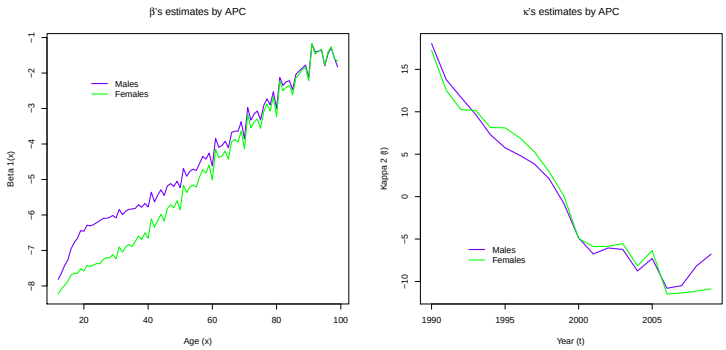
We know that the parameter $\hat{\beta}_x^{(3)}$ means the exchange rate for period mortality rates. The estimated values with this model tells us that women have a median 0.010 (IQR $0.007, 0.013$) and for men 0.009 (IQR $0.007, 0.016$). In males, there is still a parameter value increases, this is consistent with the increase already observed in the rest of the parameters. At age 22 occurs a large increase in the estimated value, and from this age began a temporary decline at age 27, to rise again into staying up until age 36, followed by decreases until age 47, where after lowering a continuous increase but at a lower speed than the former slope. For women, the behavior is similar, the most notable differences is the fall of the estimated value at age 39 and generally after 50 years are in excess of those of men.

$\hat{\kappa}$'s represents the overall level of dead in year t, and in the men's $\hat{\kappa}$'s in males is almost linear behavior up to 2005, where he begins a long curve to the last year of study, always decreasing. For women, the slope is less, but maintaining the negative sign, ie, the decrease in male mortality was faster than in women. In general it has a curvature greater than that of men, beginning in 2000, which exceeds the values for the other sex, indicating that mortality in women was lower than the male gender to that year.

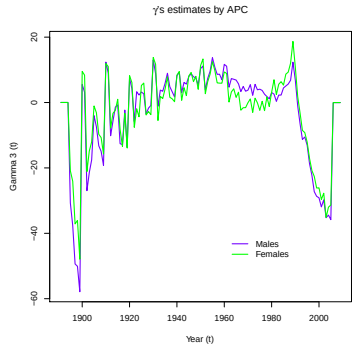
Focusing on the year of birth, and treating it as a series with non-constant variance, we note that gender behavior is different for those born between the years 1902 to 1950 and 1980 onwards, for this last set of years, the $\hat{\gamma}$'s are higher for women. The number of men is more linear, mostly rising trend, while for women it has a convex and concave before 1950 for subsequent years. Between 1984 and 1987 peaks are reached, where a possible explanation for this increase would be a strong earthquake in 1985, which affected several states in the country.

4.2.1 Age-Period-Cohort model

This is a special case of the previous model with $\hat{\beta}_x^{(1)}$ and $\hat{\beta}_x^{(2)}$ equal to one. The number of estimated parameters is 88 for β, 20 for κ and 119 for γ, resulting in a total of 227 estimated parameters. Like the Lee-Carter model, this model emphasizes the differences between genders for the deaths in youth, and slows to a later age is recognized that the actual behavior of the logarithm of mortality rates. Figure (6)



(a) $\beta_x^{(1)}$ (b) $\kappa_t^{(2)}$



(c) $\gamma_t^{(2)}$

Figure 6: Estimated Parameters Age-Period-Cohort. Male and Female aged 12 to 99 from 1990 to 2009. Mexican population.

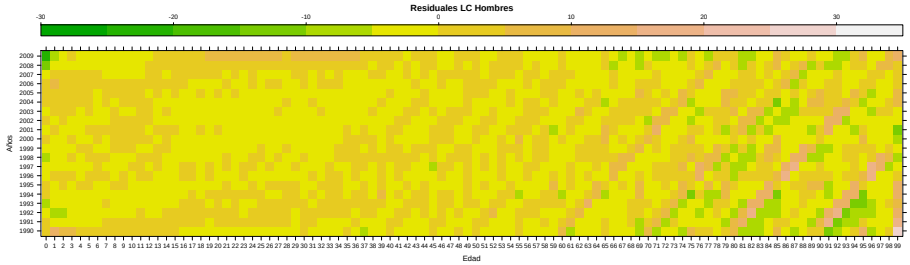
The values of the parameters $\hat{\kappa}$ were similar to those obtained by the Lee-Carter model especially in men. From 2000 onwards to previous years this model predicts lower values and marks a tonnage between 1995 and 1999. For women, there are continuous changes leave little pieces with linear behavior. When we study the year of birth, we observed no differences between sexes, the behavior is practically the same.

4.3 Model Selection

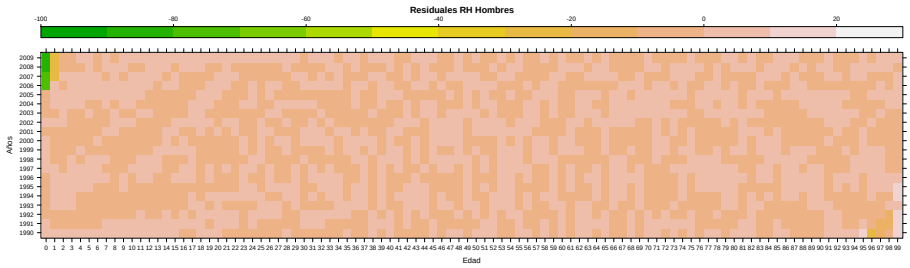
We made plots with the aim of locate patterns with the help of color gradients, guided by patterns. Axis x show age x and axis y show year t. Residual would be expected to be distributed randomly, ie it should be independent identically distributed.

Lee Carter residuals show cohort effect by diagonal lines through the years, mainly after 60 years for both sexes. Residuals has a median of -0.35 IQR $(-1.96, 1.60)$ in men and -0.3 IQR $(-1.88, 1.52)$ for women. The biggest residuals correspond to the estimated mortality for ages larger.

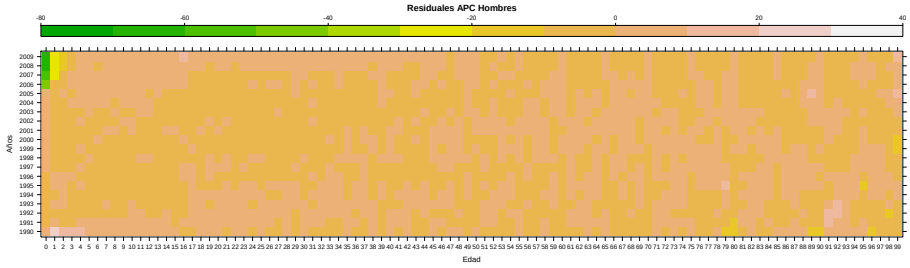
While Renshaw and Haberman residuals do not show pattern since cohort effects were included having a median of -0.06 IQR $(-1.13, 1.13)$ in men and a medina of -0.02 IQR $(-1.03, 1.02)$ for women. The residuals are larger at ages 96 to 99 years, mainly in the early nineties, which estimated values greater than those observed. Residuals are larger in women than in men, most values are estimated lower than real, but



(a) Lee Carter

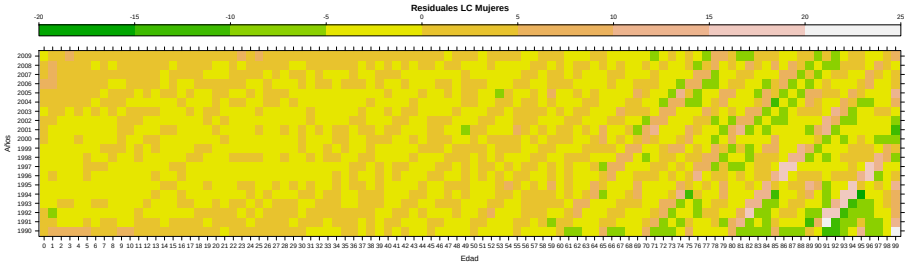


(b) Renshaw and Haberman

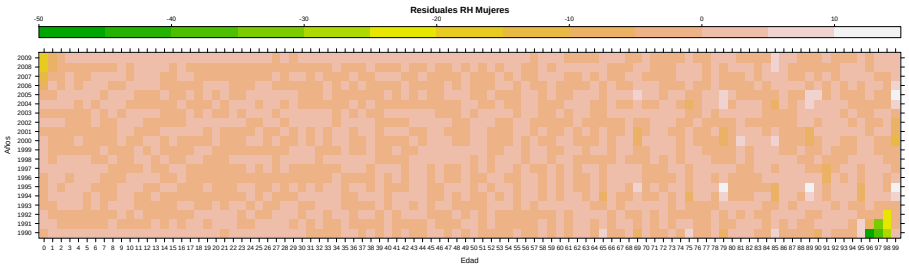


(c) Age-Period-Cohort

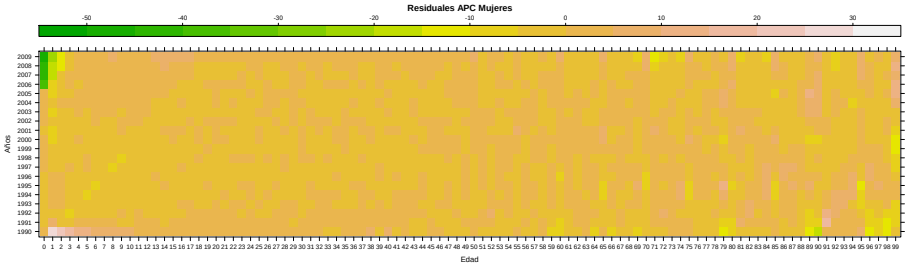
Figure 7: Standardized Residuals $Z(x,t)$ LC, RH y APC models for males. Biggest residuals appear in dark colors $Z(x,t) < 0$, or soft colors $Z(x,t) > 0$



(a) Lee Carter



(b) Renshaw and Haberman

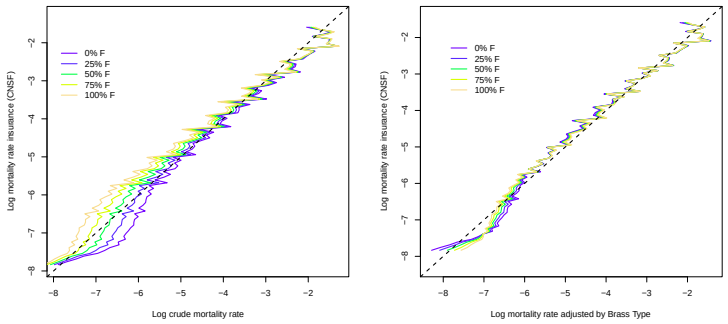


(c) Age-Period-Cohort

Figure 8: Standardized Residuals $Z(x,t)$ LC, RH y APC models for females. Biggest residuals appear in dark colors $Z(x,t) < 0$, or soft colors $Z(x,t) > 0$

Modelo	Hombres	Mujeres
LC	$-23,105.68$	$-23,559.63$
RH	$-14,963.81$	$-14,302.79$
APC	$-19,058.61$	$-18,273.52$

Table 1: Bayes Information Criterion



(a) (b)

Figure 9: (a)Log Mortality Rate with weight w vs. Log Mortality rate insurance population, (b): Adjusted Log Mortality rate by Brass Tyte vs. Log Mortality rate insurance population

close to zero.

APC residuals have a median of -0.22 IQR $(-1.61, 1.58)$ in men and -0.06 IQR$(-1.43, 1.4)$ for women. The biggest differences are not centralized in any particular age or year, it is possible found them for ages over 90 years as in younger ages. (figura 7 and 8). Like in the Renshaw and Haberman model, the diagonals in residuals pattern are no longer appreciate, but there are more differences in the estimated values.

According to the Bayes Information Criterion (BIC) Renshaw and Haberman is the model that better fits, indeed add cohort effects estimated better the mortality, achieving a better distribution of the residuals, so that they are independent and identically distributed.

4.4 Relational Brass type model

Now we proceed to set the type Brass type relational model. Linear regression was carried out to explain the mortality reported in the insurance industry with mortality in the general population. The parameters estimated and the BIC of the model (LC, RH and APC) are shown in Table 2, the first column indicates the percentage of women considered weight w in the linear regression.

The results indicate that the bigger the amount females the largest the difference between insurance mortality and general population mortality. In the case of most women, the logarithm of the death of the insured is greater than the general population and widening gaps in the younger ages, ie the mortality rate is higher for the insured. In 9 shows the figure of adjusted rates against those observed by varying the percentage of women w of 0% to 100%, the set values vary as the number of women.

% Mujeres (w)	LC			RH			APC		
	$\hat{\beta}_0$	$\hat{\beta}_1$	BIC	$\hat{\beta}_0$	$\hat{\beta}_1$	BIC	$\hat{\beta}_0$	$\hat{\beta}_1$	BIC
0%	−0.06	1.04*	33.36	−0.04	1.05*	22.06	−0.01	1.05*	24.17
25%	−0.14*	1.00*	17.88	−0.12	1.00*	3.48	−0.10	1.00*	7.79
50%	−0.22*	0.95*	8.70	−0.20*	0.95*	−7.86	−0.19*	0.96*	−1.18
75%	−0.30*	0.91*	6.00	−0.27*	0.91*	−11.18	−0.27*	0.91*	−2.64
100%	−0.38*	0.87*	8.77	−0.35*	0.87*	−7.58	−0.35*	0.87*	1.96

Table 2: $\hat{\beta}$'s by Brass-Type: $\log[m_{aseg}(x,t)] = \hat{\beta}_0 + \hat{\beta}_1 \log[m_w(x,t)]$, with different weight w according to the female proportion. * significance $\alpha = 0.05$

For the Swiss population, according to Gatzert and Wesker (2011), $\beta_0 = -0.3197$ and $\beta_1 = 1.0747$, comparing with the estimated parameters for the Mexican population with the Brass type model, β_0 is similar to that estimated by the model type brass when there is a primarily female population and β_1 in contrast to the previous parameter, the estimate is more like when there is low numbers of women.

Next step is to use the parameter estimates obtained in the working paper [9] data with mortality in Mexico but with $t = 0$ since there is a single table of mortality tables for insured. Then logarithms of the mortality rates are using in the model:

$$\log[\hat{m}(x)]_{Swiss} = -0.03197 + 1.0747 \log \hat{m}_{LC}(x) \tag{9}$$

Figure 10 shows two graphs, in the left graph the dotted line is the logarithm of mortality rates currently used for insurances companies , while the solid lines were obtained by replacing the logarithm of mortality rates estimated by the Lee-Carter model in the regression line data obtained with the general population and insured in Switzerland, equation 9. The top line corresponds to have only men and the bottom line corresponds to have only women, both are below that observed log rates in the insurance companies, so that the Swiss model estimated lower mortality than observed in Mexico. The opposite happens at ages 19, 21 and 81 where the differences are minimal and only for males and at ages 91, 92 and 93 is true for both sexes, with the largest difference occurs at age 91.

The chart at right compares the two linear regressions performed (equations 9 y 8). The y axis take values from the linear regression with parameters of the Switzerland population and the x axis represents the linear regression estimated purely Mexican data, since this last was made assuming different proportions w of men and women there are several lines estimates. The degraded of the lines to the right indicates more women, we have the following relations:

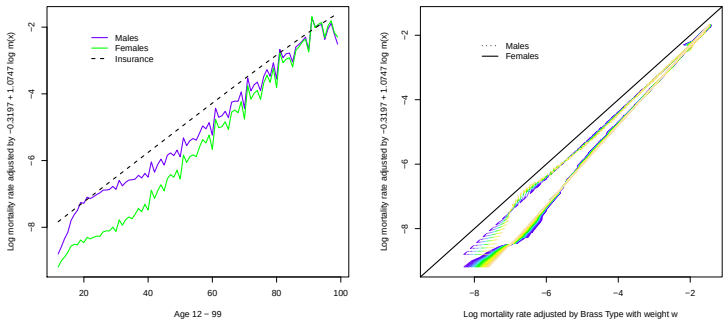
$$\text{Dot Lines} : \log[\hat{m}_H(x)]_{Swiss} \sim \log[m_{insurance}(x,t)] = \hat{\beta}_0 + \hat{\beta}_1 \log[m_w(x,t)]$$

$$\text{Solid Lines} : \log[\hat{m}_M(x)]_{Swiss} \sim \log[m_{insurance}(x,t)] = \hat{\beta}_0 + \hat{\beta}_1 \log[m_w(x,t)]$$

Once again we see that the behavior of mortality in Mexico and Switzerland are not very similar, although it was slightly similar when comparing women and when they are older.

5 Conclusions

The best estimate according to the BIC is Renshaw and Haberman model.



(a) (b)

Figure 10: Log-adjusted death rates obtained with the parameters for the Swiss population 9, (b)Comparison of log mortality rates obtained for the Mexican population parameters vs Switzerland. Log mortality rates obtained with the model equation Brass Type.

When we study the year of birth, we observed no differences between sexes, the behavior is practically the same. For those born in the late nineteenth and those born in the early twentieth century, there are more significant differences, this was expected. Overall for the other cohorts the mortality behavior is similar, with values hovering around zero.

The rates currently used in the insurance industry do not reflect the true behavior of mortality by modifying the proportion of men and women insured.

The rates from Swiss are lower than mexican rates. Comparing the estimates from Swiss with the estimated parameters for the Mexican population with the Brass type model β_0 is similar to that estimated by the model type brass when there is a primarily female population and β_1 in contrast to the previous parameter, the estimate is more like when there is low numbers of women.

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