Real World Economic Scenario Generators

David Wilkie

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Input:

Real world mathematical model

Engine:

Economic scenario generator programme

Output:

N (= 10,000) simulated future scenarios

Real world models

Risk neutral models

Different purposes, often different time scale

May be the same, but with different parameters

May be different

What outputs?

Wilkie model has no yield curves (yet)

Share prices and share dividends
or total return share index

Index-linked

Foreign currencies

Price inflation

Wages/earnings

Property (real estate)

House prices

What frequency?

Wilkie model uses annual steps

but can use stochastic interpolation

i.e. Brownian or O-U bridges

Others monthly or daily steps

A good short term model is not always a good long term one

State space

set of variables at each step

Single paths or branching

Need initial conditions (state space time 0)

Neutral (roughly medians)

or Market on some date

or Arbitrary

Neutralising parameters

e.g. UK on 31 Dec 1999

Median inflation (QMU) was historic 0.04

actual inflation was 0.0175 (logged)

so put QMU = 0.0175

Median long term bond yield 7.5%

= 4.0% inflation + 3.5% (= CMU)

actual yield was 4.89%

so put CMU = 4.89% − 1.75% = 3.14%

Median share dividend yield (YMU) was 4.0%
actual dividend yield was 2.36%, so put YMU = 2.36%

Median share price total return
= inflation + dividend yield + dividend growth
= 4.0% + 4.0% + 1.6% (DMU) = 9.6%

This is 2.1% more than long-term bond yield of 7.5%

Actual bond yield was 4.89%; adding 2.1% gives 6.99%

To get this we put neutralised DMU = 2.88% (0.0128)
to give 1.75% + 2.36% + 2.88% = 6.99%

Short term forecasts can use exogenous data.

So these forecasts can be better than model's.

"Select period":

Adjust first few years as you wish,

bias mean, alter standard deviations.

The let model take over.

Uncertainty about parameter values.

Allow by using “hypermodel”.

Assume parameters for each simulation are random and drawn from some multivariate distribution,

e.g. multivariate normal.

New parameters for each simulation.

Perhaps adjust parameters, perhaps use limits.

Based on covariance matrix of standard errors from maximum likelihood estimation.

Or otherwise (e.g. Bayesian, MCMC).

Residuals often not normal, but fat-tailed.

Use some other distribution for innovations.

Empirically useful:

$X_1 \sim \text{lognormal}(\mu_1, \sigma_1^2)$

$X_2 \sim \text{lognormal}(\mu_2, \sigma_2^2)$

$Y = X_1 - X_2$

$Z = (Y - E[Y])/\sigma(Y)$, so normalised (0,1)

Instead of lognormal use Pareto, Gamma, …

Good fit in many cases is Burr:

$$F(x) = 1 - \{\lambda^\tau / (\lambda^\tau + x^\tau)\}^\alpha$$

Can't fit Z properly by MLE, but fit positive and negative residuals separately.

This gives empirically a good fit.

Wilkie model for exchange rates (1995):

Based on purchasing power parity

$XR_{ij}(t)$ is number of units of j for one unit of i,

e.g. \$1.65 = £1.

$Q_i(t)$, $Q_j(t)$ are price indices in i and j.

$$X_{ij}(t) = \ln(XR_{ij}(t)) + \ln(Q_i(t)) - \ln(Q_j(t))$$

$$X_{ij}(t) = XMU_j + XA_j \times \{X_{ij}(t-1) - XMU_j\} + XE_j(t)$$

standard AR(1) model

OK for country i alone

Cross-rates messy:

$XR_{jk}(t) = XR_{ik}(t) / XR_{ij}(t)$

$X_{jk}(t)$ is not AR(1)

but is difference between two AR(1)s

unless $XA_j = XA_k$

New model:

Assume hypothetical or hidden series for each country, $HR_i(t)$, representing "relative strength".

Put $S_i(t) = Q_i(t) / HR_i(t)$

Then $XR_{ij}(t) = S_j(t)/S_i(t) = Q_j(t)/Q_i(t) \times HR_i(t)/HR_j(t)$

Then

$$\ln(XR_{ij}(t))$$
$$= \ln(Q_j(t)) - \ln(Q_i(t)) + \ln(HR_i(t)) - \ln(HR_j(t))$$

Put $H_j(t) = \ln(HR_j(t))$

Data: Exchange rates and CPIs :

Monthly: September 1972 to December 2010,

460 months (month-end values)

Twelve countries:

Australia, Canada, Denmark, Germany/Euro

Japan, New Zealand, Norway, South Africa

Sweden, Switzerland, USA, UK

Take exchange rates w.r.t. UK

11 exchange rates, 12 unknown Hs for each date

Fit Hs for each date by least squares

Take $\ln(Q_i(t))$, deduct mean to get $q_i(t)$, all i

Take $\ln(XR_{ij}(t))$, deduct mean to get $x_j(t)$, all j,
fixed i = UK

Then we get:

$$x_j(t) – q_j(t) + q_i(t) = h_i(t) – h_j(t)$$

All x_i's and q_i's have zero mean over time, so also could h_i's.

Minimise $\mathrm{Sum}(t) = \sum_j h_j(t)^2$

$$= h_i(t)^2 + \sum_{j\neq i} \{h_i(t) – x_j(t) + q_j(t) – q_i(t)\}^2$$

Solution is $h_i(t) = \sum_{j\neq I} \{x_j(t) – q_j(t) + q_i(t)\} / n$

h_j's for each day have zero mean, and for each j

Model each h_j as AR(1):

$h_j(t) = \alpha_j . h_j(t-1) + \sigma_j . z_j(t)$

$z_j(t) \sim (0,1)$, perhaps normally

On an annual scale, also AR(1)

$h_j(t) = \alpha_{(12)j} . h_j(t-12) + \sigma_{(12)j} . z_{(12)j}(t)$

$\alpha_{(12)j} = \alpha_j^{12}$

$\sigma_{(12)j}^2 = \sigma_j^2 \times (1 - \alpha_{(12)j}^2) / (1 - \alpha^2)$

Monthly α_j's range:

0.9231 (Norway) to 0.9863 (Japan)

Annual α_j^{12}'s range from 0.3829 to 0.8471

Monthly σ_j's range:

0.0163 (Canada) to 0.0407 (South Africa)

Annual σ_j^{12}'s range:

0.0405 (Norway) to 0.1287 (South Africa)

Large simultaneous cross-correlations of z's

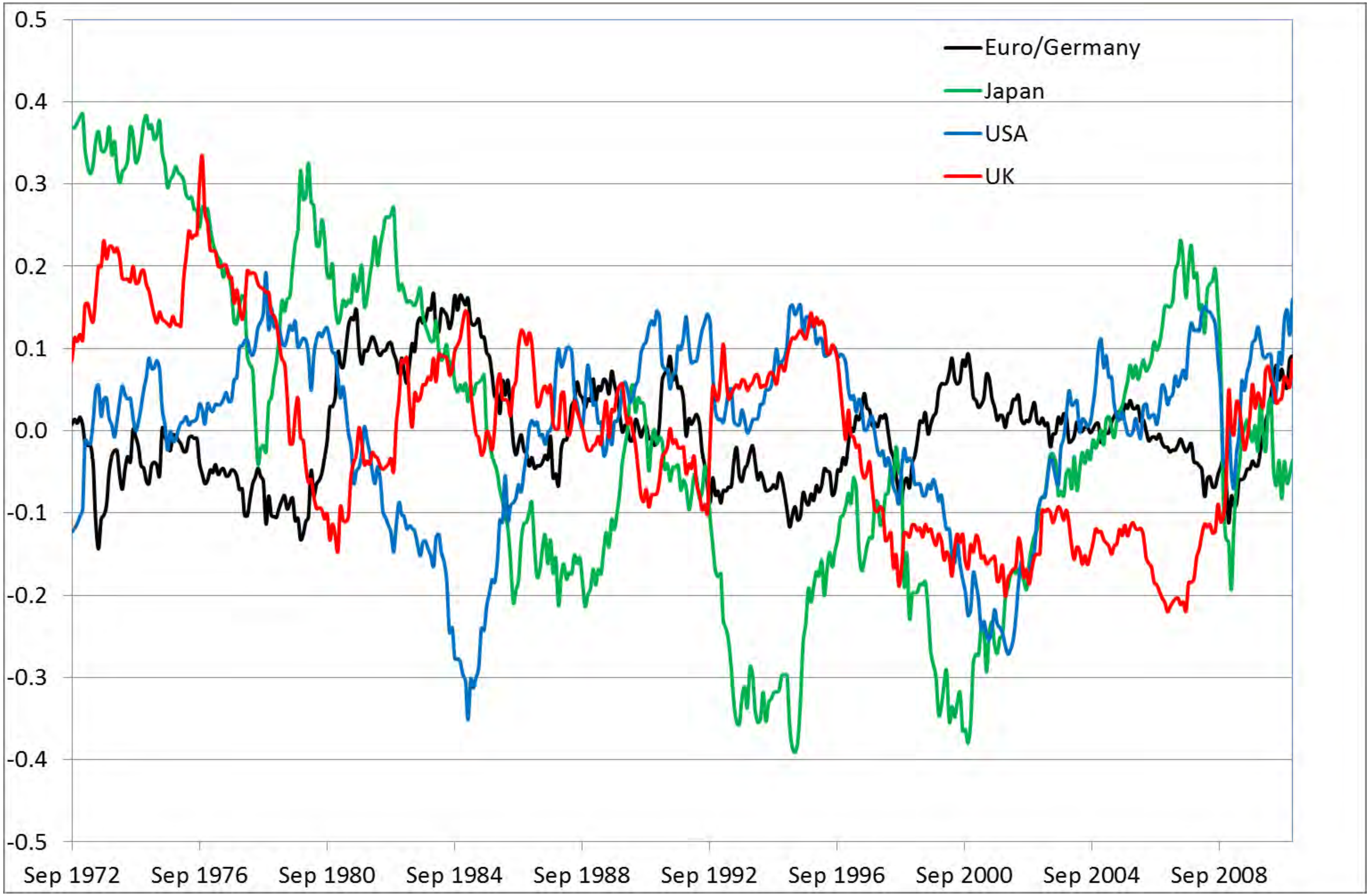
Euro/Denmark	0.84
Euro/Switzerland	0.65
Denmark/Switzerland	0.58
Canada/USA	0.62
Australia/New Zealand	0.44
…	
Canada/Euro	−0.53
Canada/Switzerland	−0.52
Euro/USA	−0.47

Quite small lagged auto and cross-correlations

Very little evidence for AR(2) model monthly, none for AR(2) annually.

Values of h as at 31 December 2010

Sweden	0.1917	South Africa	0.0055
USA	0.1594	Canada	−0.0138
Euro	0.0912	Japan	−0.0367
UK	0.0860	Switzerland	−0.1321
Norway	0.0076	New Zealand	−0.1615
Denmark	0.0069	Australia	−0.2041



0.5
0.4
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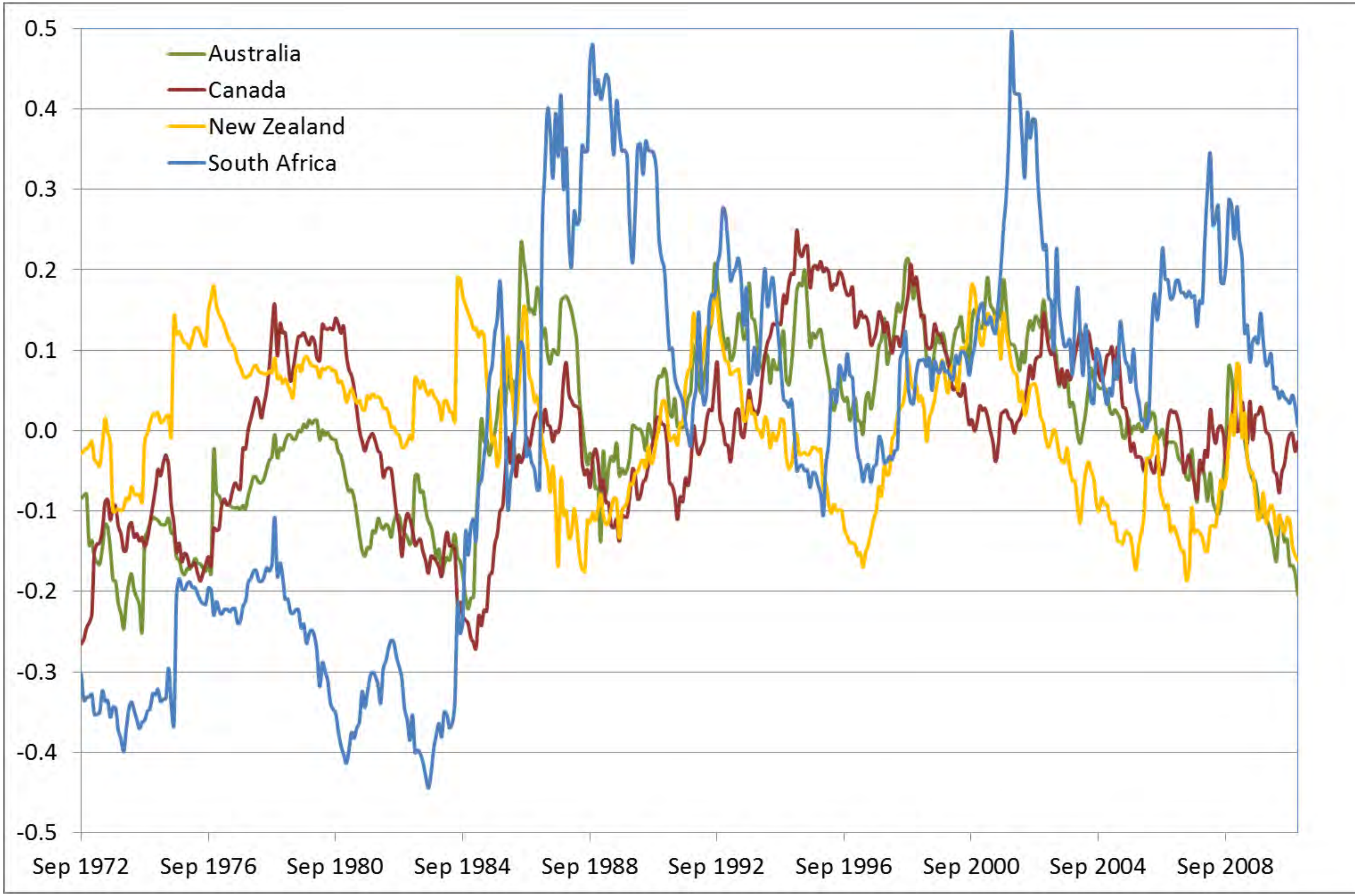
Euro/Germany
Japan
USA
UK
Sep 1972
Sep 1976
Sep 1980
Sep 1984
Sep 1988
Sep 1992
Sep 1996
Sep 2000
Sep 2004
Sep 2008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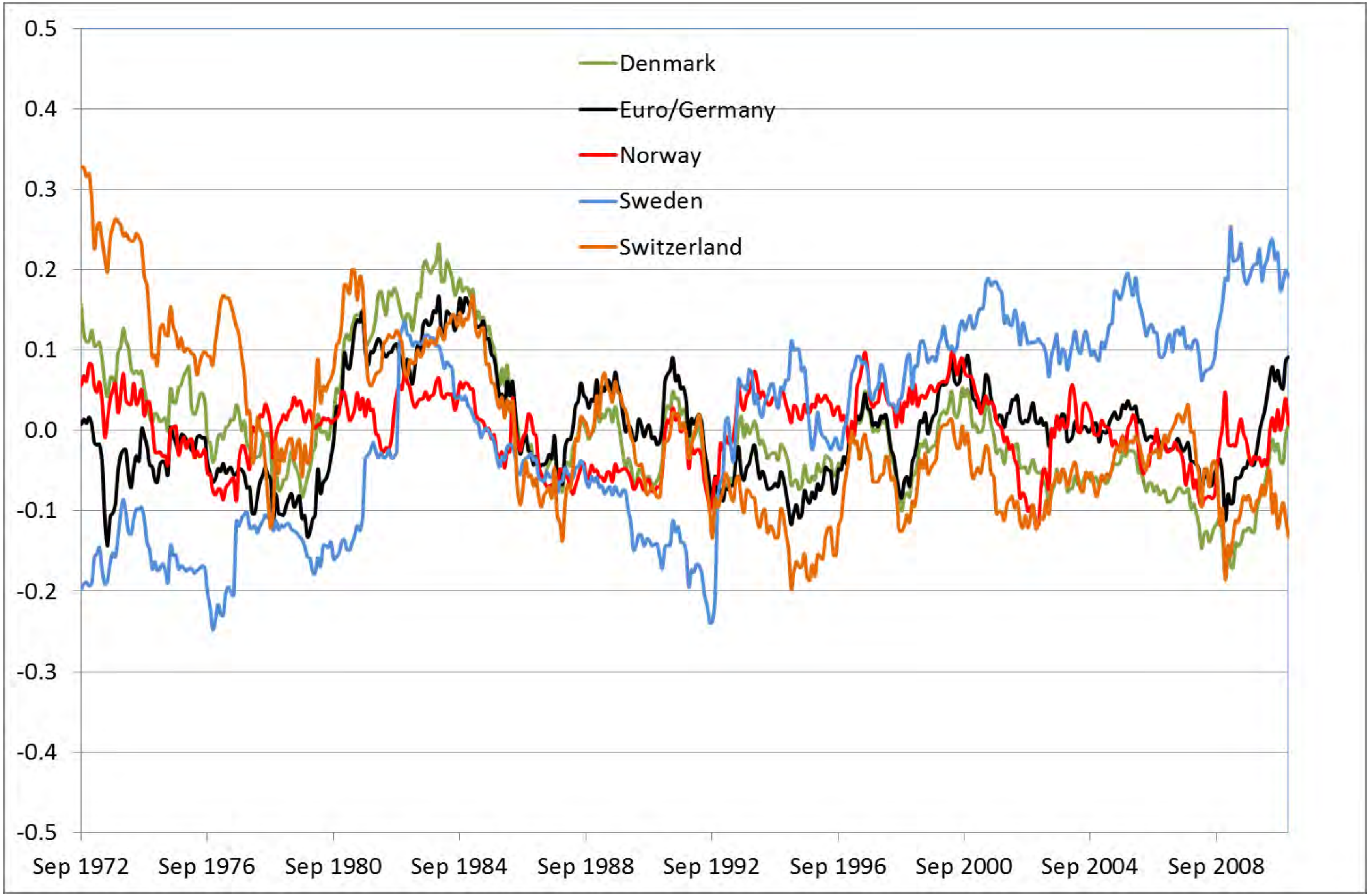

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Very high kurtosis of residuals:

Sweden	13.00
New Zealand	12.26
South Africa	12.03
…	
Switzerland	4.11
USA	4.05
Canada	3.42

To be investigated:

Distribution of residuals

Different periods

Different currencies

The h’s are the same for any base currency

If a currency is omitted or added the other h’s are all altered by a constant,

but keep their relative positions:

but the AR coefficients would change.

A. D. Wilkie & P. J. Lee (2000)

"A comparison of stochastic asset models"

Proceedings of the 10th International AFIR Colloquium, Tromsø, Norway, June 2000, 447-465.

A. D. Wilkie, Şule Şahin, A. J. G. Cairns and Torsten Kleinow (2011)

"Yet More on a Stochastic Economic Model: Part 1: Updating and Refitting, 1995 to 2009"

Annals of Actuarial Science, Volume 5, Part 1, pp. 53–99

and further Parts