

A Model Study about the Applicability of the Chain Ladder Method

By

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Abstract

Risk orientated modelling of claim processes is increasing in importance as the application of Solvency II approaches in insurance practice. “Internal models” are of great importance for those insurance companies that find their business not adequately modelled by the standard model. We have developed a three dimensional (3D) stochastic model for claim reserving [9]. This model integrates a realistic risk claim occurrence, claim reporting and run-off behaviour of a given claim portfolio. The model is therefore especially useful for lines of business with long reporting and / or run-off times.

In this paper, we investigate the ability of the Chain Ladder method to forecast the right reserves in the framework of the 3D model. We find conditions under which the CL model is adequate and others where it fails. We perform Monte Carlo Simulations to apply our results to several concrete examples. The theoretical and the numeric results coincide.

Keywords:

Solvency II, Risk Management, Claim reserving, IBNR reserve, Chain Ladder method, collective model of risk theory, Monte Carlo simulation, stochastic model.

1. Introduction

The Solvency II framework forces the insurance industry to implement efficient methods of risk capital allocation. One of the most important liability positions for P&C insurance companies is the claim reserve. It may determine up to 70% of the balance sum. Claim reserves are necessary to cover the liabilities arising from insurance contracts written in the present and the past. Therefore, besides Solvency II, claim reserving is decisive for regulatory and accounting issues as well as for product pricing and reporting. Within Solvency II internal models are possible and even favourable, if the risk situation is described in a better way. We compare in this paper the performance of an internal stochastic reserving model with one of the most popular classical reserving methods, the Chain Ladder (CL).

The claim reserves are calculated for homogeneous portfolios of insurance contracts via actuarial methods. The bases of these classic reserving methods are two dimensional matrices, the run-off triangles. They are generated via accumulation of claim data. An overview on claim reserving and its classical methods can be found in literature, see for instance [2, 6, 10, 11, 14]. The idea of using the information on claim occurrence, reporting- and payment-delay for reserving methods can be found in the literature for instance in [5, 13].

We developed a stochastic model (the “3D model”) that takes into account the claim process from first principles (see [9]). We did this primarily to have a sound model that gives not only estimations of the expectation of the reserves but also their statistic distribution. This meets the requirements of Solvency II and modern risk management, where risk measures have to be calculated on the basis of distribution functions. This makes risk measures accessible as VaR or expected shortfall.

In this paper we want to analyse for which cases this 3D model is superior to the classical Chain Ladder (CL) method for claim reserving. We ask if it is worthwhile to calibrate the 3D model. This is more inconvenient than using the CL method. Further we examine, if there are systematic deviations in the reserve forecast between the 3D model and the CL method.

This article is organised as follows: In section 2, we give a short summary of the 3D model and show the connections to classical actuarial methods, especially the CL. In section 3, we find the deviations between the 3D model and the CL method from the conditions on the validity of the CL model. In section 4, we validate the theoretical findings of section 3 with the help of several, realistic, numerical examples. In section 4, we investigate the expectations and compare them with the means of the 3D model. We also generate and evaluate the reserves’ distribution function and discuss the results.

2. The 3D Model and the connection to the classical 2D models

In this section, we introduce our stochastic 3D model; for details see [9, 7]. Our model describes the single claim process on a “microscopic” level: The stochastic process contains the claims’ occurrence, reporting and run-off. We perform this parameter controlled process in a Monte Carlo simulation, and in this way model a whole claim portfolio on the basis of a single claim process.

The classical reserving methods such as the Chain Ladder or the Bornhuetter - Ferguson [1, 6] operate on a cumulated 2D data structure and calculate the expectation of the reserve. This means that a complex stochastic process with many degrees of freedom is projected onto a two dimensional structure – the run-off matrix. It is expected to receive a convenient forecast

of the future in doing so. This is only true under very special assumptions, as we will show in section 3.

2.1. The 3D Model

The basis of our claim model is the collective model of risk theory. We model the number of active claims N_{ijk} which occurred in year i , reported j years after occurrence and still active k years after reporting as well as Z_{ijk} , the total amount paid for the N_{ijk} claims of occurrence year i with reporting delay j years in the k th year after reporting.

The ultimate number of claims in occurrence year i is given by:

$$N_i = \sum_j N_{ij0}$$

It is modelled as Poisson distributed with the (given) parameter \bar{N}_i .

$$N_i \propto \text{Poisson}(\bar{N}_i)$$

The claim numbers obey a multinomial distribution along the years of reporting delays with parameters

$$\lambda_j \quad j \in \{1, 2, \dots, I_{\max}\} \text{ with } \sum_j \lambda_j = 1.$$

I_{\max} is the number of occurrence years for which the claim reserve has to be calculated. J_{\max} is the maximum number of years between occurrence and reporting of the claim. For convenience we have chosen $I_{\max} = J_{\max}$. This is a suitable assumption if the claim history of the company is known over a certain period. Other conventions are possible.

Therefore the claim numbers for $k = 0$ are distributed according to:

$$N_{ij0} \propto \text{MultNom}[N_i, \lambda_1, \lambda_2, \dots, \lambda_j, \dots, \lambda_{I_{\max}}]$$

The closure of active claims along the years after reporting is described by the parameters

$$\eta_k \quad k \in \{1, 2, \dots, K_{\max}\} \text{ with } \eta_0 = 1 \text{ and } \eta_{k+1} < \eta_k.$$

K_{\max} is the maximum number of years a reported claim needs for the run-off to be completed. The fact that an open claim is not closed in year k can be modelled as a survival process resulting in a binomial distribution with survival probability η_k :

$$N_{ijk} \propto \text{BiNom}[N_{ijk-1}, \eta_k]$$

Therefore the expectation of the active number of claims is:

$$E[N_{ijk}] = \bar{N}_i \cdot \lambda_j \cdot \eta_k. \quad (2.1)$$

The claim payments are modelled according the collective model of risk theory:

$$Z_{ijk} = \sum_{l=1}^{v_{ijk}} X_l^{(ijk)}$$

with the binomially distributed number of single claim payments v_{ijk}

$$v_{ijk} \propto \text{BiNom}[N_{ijk}, p_{ik}]$$

The parameters of this distribution are N_{ijk} , the active number of claims, and p_{ik} , the probability that those active claims cause payments. The X_{ijk} are the payments for single claims. Applying the model to Monte Carlo simulations (section 4.2) we assume them to be Gamma distributed with parameters depending on i , j , and k :

$$X_{ijk} \propto \Gamma[EW_{ijk}, Var_{ijk}]$$

This completes the setting of our model. The choice of the distributions for claim numbers and payments is, of course, a matter of best fit to the data. We have made the assumptions here as characterised above, being not too far from real world claim data, in order to apply the model to a concrete simulation example.

The expectation of the payments in our model is:

$$E[Z_{ijk}] = \bar{N}_i \cdot \lambda_j \cdot \eta_k \cdot EW_{ijk} \cdot p_{ik} \quad (2.2)$$

2.2. The connection between the 2D and the 3D models

In this section, we give a short review of how our 3D model is connected to the classical 2D models. For Details we refer to [9].

As for all classical actuarial models the data matrix with the known data is the basis for the forecasting method. The difference is the dimension of the data set. Our model uses number of claim data N_{ijk} and payment data Z_{ijk} in a three dimensional form. They can be represented in a cube (see figure 2), analogously to the 2D data triangle in the classical case.

All relevant reserves can, of course, be found in this data cube: We visualise all relevant relations in figure 2.

The claims reported so far are found in the run-off triangle:

$$N_{ij0} \text{ with } i + j \leq I_{\max}$$

I_{\max} is given by the known history of claim data. The estimation of the ultimate claim number consists of the sum of already reported claims and the incurred but not reported (IBNR) claims (see figure 2a). The table of active claims in the present is given by the set (see figure 2b):

$$\{N_{ijk} \mid i + j + k = I_{\max}\}$$

The IBNR reserve can be written as:

$$\hat{R}_{IBNR} = \sum_{\{i,j,k|i+j+k>I_{\max} \vee i+j<I_{\max}\}} \hat{Z}_{ijk}.$$

The number of IBNR claims is given by:

$$\hat{N}_{IBNR} = \sum_{\{i,j|i+j>I_{\max}\}} \hat{N}_{ij0}$$

The total reserve evaluates as (sum of reserves for known and unknown claims):

$$\hat{R}_{total} = \hat{R}_{IBNR} + \sum_{\{i,j,k|i+j>I_{\max}\}} \hat{Z}_{ijk}.$$

The visualisation of these reserves can be seen in figure 2 on the right hand side: For the IBNR reserve see figure 2d), for the total reserve figure 2c) and d). In both cubes the plain separating the known from the unknown numbers is drawn with sketched lines.

By aggregating the components of the Z_{ijk} tensor (payments) in the following way the run-off triangles used by standard 2D reserving methods can be generated:

The run-off triangle “payments: occurrence versus run-off year”, well known in the 2D world, can be generated from the 3D according to:

$$S_{mn}^{(1)} = \sum_{\{j,k|j+k=n \wedge m+j+k \leq I_{\max}\}} Z_{mjk}$$

This triangle is used in the context of standard reserving methods for **total reserve estimation**.

The 2D run-off triangle “payment: reporting year versus run-off year” can be generated via:

$$S_{mn}^{(2)} = \sum_{\{i,j|i+j=m \wedge i+j+n \leq I_{\max}\}} Z_{ijn}$$

This triangle is used in the context of standard reserving methods for **reserve estimation of the known claims**. The difference between this reserve and the total reserve is the IBNR Reserve.

In other words: The kind of the data triangle used as input for a classical 2D method is responsible for the kind of estimated reserve: total, IBNR, IBNER. This fact is visualised in figure 2: The total reserve (the sum of reported claims’ and IBNR reserves; see left side of figure 2) or two separate results for both parts of the total reserves (see right side of figure 2). In practice one is often faced with the following problem: Applying both methods to the same data set delivers inconsistent results. This is well known and up to now this insufficiency was ascribed to properties of the estimators, particularly the bias (see for instance [12]). In the following section we give an explanation for this behaviour and show that the problem is the projection of a 3D to a 2D space where important information is lost.

3. The possibilities and limitations of the CL method

In this section, we investigate which attributes of the 3D model parameters are necessary to get the right CL results. After that, we will know what “behaviour” of the 3D single claim process is allowed or not allowed for using the CL method. This delivers the connection between the claim process and the applicability of the CL method. In the next section we will demonstrate the results of this section with the help of three numerical examples.

It is well known under which conditions the CL method gives the right results in reserve estimation [3, 4]. We apply these conditions to our 3D model, especially to the 2D data matrices generated as described in section 2.2:

For the bias free use of the CL method the cumulated Claims S have to meet the following condition; X denotes the non cumulated payments:

$$S_{ik} = \sum_{l=1}^k X_{il}$$

$$f_k \sim \frac{S_{ik+1}}{S_{ik}} = \frac{\sum_{l=1}^{k+1} X_{il}}{\sum_{l=1}^k X_{il}} = \frac{\sum_{l=1}^k X_{il} + X_{ik+1}}{\sum_{l=1}^k X_{il}} = 1 + \frac{X_{ik+1}}{\sum_{l=1}^k X_{il}}$$

The Chain Ladder factor f may **not depend** on the **vertical** dimension (rows) of the data matrix.

This has the following implication for the 3D model:

The **occurrence – run-off** matrix for the payments is defined as:

$$A_{iy} = \sum_{\{j,k|j+k=y\}} Z_{ijk}$$

The CL method has to be applied on the cumulated payments in the following way:

$$S_{in}^{(1)} = \sum_{y=1}^n A_{iy} = \sum_{y=1}^n \sum_{\{j,k|j+k=y\}} Z_{ijk}$$

Therefore the CL factor reads:

$$\frac{S_{in+1}^{(1)}}{S_{in}^{(1)}} = 1 + \frac{\sum_{\{j,k|j+k=n+1\}} Z_{ijk}}{\sum_{y=1}^n \sum_{\{j,k|j+k=y\}} Z_{ijk}}$$

The CL method will give the right estimator in the framework of the 3D model if, and only if, the second term of the right side of the above equation is independent of i, the occurrence year. This is our result for the forecast of the total reserve.

For the **reporting – run-off** year an analogy holds:

Claims (Payments)

$$B_{xk} = \sum_{\{i,j|i+j=x\}} Z_{ijk}$$

Cumulated Claims (Payments)

$$S_{xn}^{(2)} = \sum_{k=1}^n B_{xk} = \sum_{k=1}^n \sum_{\{i,j|i+j=x\}} Z_{ijk}$$

CL factor

$$\frac{S_{xn+1}^{(2)}}{S_{xn}^{(2)}} = 1 + \frac{\sum_{\{i,j|i+j=x\}} Z_{ijn+1}}{\sum_{k=1}^n \sum_{\{i,j|i+j=x\}} Z_{ijk}}$$

The CL method will give the right estimator in the framework of the 3D model if, and only if, the second term of the right side of the above equation is independent of x, the sum of occurrence year i and reporting delay j. This is our result for the forecast of the reported claims' reserve.

In this general case none of the conditions is met and the CL method will lead to a systematic error for both reserves (total and known claims), (see example 3 in section 4).

In the special case that the payments have a multiplicative structure in the three dimensions i, j, k, both conditions hold:

Let the payments be of the form: $Z_{ijk} = \alpha_i \beta_j \gamma_k$,

then the CL factor for the occurrence – run-off year reads:

$$\frac{S_{in+1}}{S_{in}} = 1 + \frac{\sum_{\{j,k|j+k=n+1\}} \alpha_i \beta_j \gamma_k}{\sum_{y=1}^n \sum_{\{j,k|j+k=y\}} \alpha_i \beta_j \gamma_k} = 1 + \frac{\sum_{\{j,k|j+k=n+1\}} \beta_j \gamma_k}{\sum_{y=1}^n \sum_{\{j,k|j+k=y\}} \beta_j \gamma_k}$$

and for the reporting – run-off year:

$$\frac{S_{xn+1}}{S_{xn}} = 1 + \frac{\sum_{\{i,j|i+j=x\}} \alpha_i \beta_j \gamma_{n+1}}{\sum_{k=1}^n \sum_{\{i,j|i+j=x\}} \alpha_i \beta_j \gamma_k} = 1 + \frac{\gamma_{n+1} \sum_{\{i,j|i+j=x\}} \alpha_i \beta_j}{\sum_{k=1}^n \gamma_k \sum_{\{i,j|i+j=x\}} \alpha_i \beta_j} = 1 + \frac{\gamma_{n+1}}{\sum_{k=1}^n \gamma_k}$$

In the first case the i-dependence is dropped, in the second case the x-dependence.

Therefore: in the special case of a multiplicative structure of the claim payments, the CL method is appropriate for reserve estimation (see example 1 in section 4).

Two other special cases can be found, where the CL Method is at least partially successful:

- 1) Let $Z_{ijk} = \alpha_i \mu_{jk}$, then we find for the occurrence vs. run-off year:

$$\frac{S_{in+1}}{S_{in}} = 1 + \frac{\sum_{\{j,k|j+k=n+1\}} \alpha_i \mu_{jk}}{\sum_{y=1}^n \sum_{\{j,k|j+k=y\}} \alpha_i \mu_{jk}} = 1 + \frac{\sum_{\{j,k|j+k=n+1\}} \mu_{jk}}{\sum_{y=1}^n \sum_{\{j,k|j+k=y\}} \mu_{jk}}$$

the i dependence vanishes in this case, but not the x dependence in the other run-off triangle. Therefore the CL method is appropriate only for the estimation of the total reserve, not for the known claims' reserve.

- 2) Let $Z_{ijk} = \mu_{ij} \gamma_k$, then we find for the reporting vs. run-off year:

$$\frac{S_{xn+1}}{S_{xn}} = 1 + \frac{\sum_{\{i,j|i+j=x\}} \mu_{ij} \gamma_{n+1}}{\sum_{k=1}^n \sum_{\{i,j|i+j=x\}} \mu_{ij} \gamma_k} = 1 + \frac{\gamma_{n+1} \sum_{\{i,j|i+j=x\}} \mu_{ij}}{\sum_{k=1}^n \gamma_k \sum_{\{i,j|i+j=x\}} \mu_{ij}} = 1 + \frac{\gamma_{n+1}}{\sum_{k=1}^n \gamma_k}$$

the x dependence vanishes in this case, but not the i dependence in the other run-off triangle. Therefore the CL method is appropriate only for the estimation of the known claims' reserve, not for the total reserve.

We give an example for this behaviour in our example 2 in section 4.

Therefore, we have shown that it is a matter of structure or symmetry of the claim payments if the CL method is an appropriate estimation method for the reserves.

In the following section 4, we will demonstrate our results on a concrete example. We will evaluate the expectation values of the 2D data triangles and the resulting reserves for all three cases analysed in section 3. Additionally, we will compare these mean values with their numerically generated distribution functions. We will use the Monte Carlo simulation method for the calculation of the distribution function.

4. Numerical Examples

To illustrate our results with a concrete example, we compare the expectation values of the reserves for a given claim process with their CL estimates. For the reserves' distribution function we repeat the same analysis. The systematic of our numerical work is depicted in figure 3. We will explain it and refer to it throughout the following two subsections. In the framework of our 3D model, defined in section 2.1, the claim reserves are modelled stochastically. It is assumed that the parameters of the “microscopic” claim process are known or can be measured [8]. This is quite a realistic assumption provided the insurance company is ready to collect claim data over a relevant time horizon.

4.1. Expectation Value

As we pointed out in section 2.1 the expectation values of the two components of the claim model, the number of claims and the claim payments are given by: $E[N_{ijk}] = \bar{N}_i \cdot \lambda_j \cdot \eta_k$ and $E[Z_{ijk}] = \bar{N}_i \cdot \lambda_j \cdot \eta_k \cdot EW_{ijk} \cdot p_{ik}$. They depend only on the model parameters.

We use realistic parameters for our three examples of the claim portfolios. The parameters for the claim numbers are given in table 1. They are the same for all three examples. The expectations of the total claim number for each occurrence year \bar{N}_i are given in table 1a). After an increase from 80 to 220 in the first four years it stays at a level of about 200 claims per year. The parameter λ_j (see tab. 1b) governs the reporting lag of the claims: In the occurrence year, 41% of the claims are known, the following year 33% and so on. Eight years after occurrence, the last 1% of the claims is reported. The parameters for the closing of the claims can be found in table 1c): In the reporting year all claims are open. One year after reporting 55% of the claims are open and this drops to 35% after two years. Eight years after reporting all claims are closed. Note that the claim numbers have a multiplicative structure in any case.

For the first example, we chose a constant value of 1000 for the expectation of a single payment and a constant probability of payment 30%. Therefore the expectation of the payment Z has a multiplicative structure, too. For this example the Chain Ladder method is expected to work quite well.

For the second example, we let $EW_{ijk} = 700 + 300j + 50k$. This means, that the basis value for a payment is 700 and it is increased by 300 for every year of reporting delay. A much smaller increase of 50 is modelled for every year the claim stays open. This is quite a realistic behaviour that can be experienced in many claim portfolios: The claims are more expensive the larger the reporting delay is and the longer the claims are open. The probability for a payment is left constant as in the first example. In this second example, we expect (according to section 3) the CL method to calculate the right total reserve, but to fail for reserve of known claims.

For the third example, we set: $EW_{ijk} = 700 + 300j + 50k + 20i$ and $p_{ik} = 0,2 + 0,02i$.

Therefore, in this example, we model additionally a slight dependence on the occurrence year i in the expectation of the single claim payment as well as in the probability of payment. This can be sometimes observed in real claim portfolios, for example if there is a trend in evaluating or detecting claims. In this third example, it is expected (according to section 3)

that the CL method fails in all points. The deviation from the right value will be small because the i -dependence of Z is small, too.

Given these model parameters, we can calculate the expectation of all cells in the 3D cubes, the claim numbers as well as the claim payments according to equations 2.1 and 2.2. Following the equations in section 2.2 we can calculate the expectation for the two run-off triangles and by applying the CL method, the total and reporting year reserve. We perform these calculations for all three examples and receive the CL estimates of the reserves on the basis of the expectation triangles. The results can be found in tables 2 – 7. This procedure is shown in the first line of figure 3, systematic of numerical calculation. Tables 2, 4 and 6 show the occurrence vs. run-off year tables for all three examples. The upper part (a) is the expectation of the payment according to the 3D model. The sum of the blue marked values gives the total reserve. The CL method is performed on the basis of a 10×10 matrix of cumulated payment, the estimates of the 3D model. This is a situation that is often found in practice. The run-off data are not known long enough for the total run-off, therefore the “tail” (run-off years > 10) has to be estimated with different methods (see [6, 11]) The lower triangle of figures 2b), 4b) and 6 b) gives the cumulated payments as a result of the CL method. From these, the values of the CL reserve estimates follow. We compare the estimation of the 3D model for the “short” table with the expectation of the CL method. The values are given on figures 2, 4 and 6 on the lower right side. As expected, they coincide for the first and second example, but not for the third one (see section 3).

We also calculate the 3D estimates of the reporting year tables for all three examples (tables 3, 5 and 7). We use the cumulated data as input for the CL method and calculate the CL estimate of the reported claim reserve. The resulting values of the reserves are given on the right side of figures 3, 5 and 7. The calculated reserve of the known claims is the difference of the total reserve and the IBNR reserve. The comparison between the estimates of the 3D model and the CL predictions shows coincidence in the first example and deviance in the second and third examples. This can be explained through the structure of the claim payments as was pointed out in section 3.

Therefore, we find quite realistic scenarios where the chain ladder estimate does not deliver the right prediction for the claim reserves. The conditions for those systematic deviations are derived in section 3. In our examples, all CL estimates are upward “biased”. Upward bias of CL estimates is a well known phenomenon and has been investigated on a 2D basis so far [12].

4.2. Monte Carlo (MC) Simulation

In this section we look at the distributions and stochastic properties of the 3D as well as the CL estimates. It is important to evaluate the CL induced systematic deviations of the reserve in comparison with its model induced volatility.

We model a set of claims, i.e. a claim portfolio, stochastically, including their projection into the future. Every element of the set (i.e. claim) obeys the stochastic process (3D model) of section 2.1. The parameters of the process are known from data analysis of claim databases containing single claim information. We use Monte Carlo methods for our simulations and generate 1000 scenarios of the claim portfolio, in order to get as a result statistical properties, for instance the reserves’ distributions.

The systematic of our numeric investigation is shown in figure 3: After simulating 1000 scenarios of the claim portfolios according to the 3D model we can calculate the MC

simulated reserves (and their empirical distributions) by adding the relevant cells (see section 2.2) on the one hand. On the other hand we generate the occurrence year and the reporting year triangle from the simulated claim data and calculate the chain ladder estimates for each of the 1000 MC claim portfolios. Then we compare the simulated 3D reserves and the CL estimates.

We have chosen the same values of model parameters as in section 4.1. For the single claim payment distribution, we have chosen a Log Normal with mean as given in 4.1 and variance 1.8 times the mean. Overdispersed payment distributions are very often found in third party liability claim size distributions. For the MC simulation we used the MATHEMATICA package.

In the following, we give the results of the MC simulation: The comparison of the 3D model results and the CL estimates.

First of all, we compare the mean values of the reserves (total and known claims). In table 8 we see the estimation values and the empirical means of the MC simulation for the reserves of all three examples. The values (estimates) of the first and the third row depend only on the parameters of the 3D model. They are already known from tables 2 – 7. The values in the second and the fourth row are the empirical means of the 1000 realisations of the MC simulation. The first two rows show the reserves according to the 3D model, the second two rows show the CL results. It is expected that the values in the first row coincide with that in the second and those in the third row with the values in the fourth, because due to the law of large numbers the empirical mean should meet the theoretical expectation for a large number of MC runs. We find an excellent coincidence with less than 1% deviation. The small deviations are due to the finite number (1000) of MC scenarios. The fact that the CL estimates deviate from the 3D model predictions in examples 2 and 3 as discussed in section 3 can be reproduced by the MC simulations quite well. This can also be seen in table 8.

We further compare the empirical distributions of the CL estimates and those of the 3D model for all three examples. The empirical distributions are generated from the 1000 MC realisations. Our results are depicted in figures 4 and 5. In all cases, the CL reserves' distribution is broader than the distribution of the 3D model reserves. This is the case because the CL method adds an error to the model – inherent fluctuation. Our simulation results make these two types of errors (stochastic model vs. estimation) clearly visible.

The deviance between the model and the CL distributions is smaller for the reserves of the known claims than for the total reserve. This can be explained by the proportion of payments that have to be estimated: For the total reserve, a much higher reserve has to be forecast by a relatively small volume of known payments compared to the reserves of the known claims. Therefore, the CL estimation of the known claims' reserve is statistically much more stable than that of the total reserves. This is reflected by the widths of the green distributions in comparison to the blue ones in figures 4 and 5. In example 1 (see figs. 4 and 5 upper part) both distributions' means fit the theoretically forecast values (red lines). Example 3 (fig. 5 lower part) shows a shift of both reserves' distributions to the right in coincidence with the results in section 4.1. This means that the CL method produces an overestimation of the reserves. The reasons for this behaviour have been explained in section 3. For example 2 (fig. 4 lower part) we find as expected (see above): The shift exists only for the reserve of known claims and not for the total reserve.

As a further statistical property we examine the correlations between the CL reserves and the 3D model reserves. The values can be found in table 9. They are all positively correlated. This means that the CL estimate tends to forecast a large (small) reserve when the reserve is really large (small). This is of course a very welcome property of the CL method, but on the other hand the correlation values are quite small – not higher than 10 – 20 %. The correlation values

are better for the case, where a statistically more stable estimation is possible, the known reserves (compare with the discussion of the width of the distribution above). The correlation also tends to be better for the cases where the CL “does a good job” – compare example 1 with 3.

5. Summary and Discussion

This paper discusses the ability and conditions under which the CL method can predict the claim reserves of a 3D type stochastic claim model. The 3D model describes a real world scenario because it is based on a stochastic single claim process that is aggregate to a claim portfolio. The 3D model is orientated on first principles, namely the claim occurrence, reporting and run-off [9]. It includes therefore much more specific information about the claim process than the accumulated run-off triangles which are the input data of the CL method and classical reserving methods. In this way, the possibilities and limitations of the CL method for practical use can be evaluated.

We derive theoretical conditions under which the CL method should give the right predictions for our 3D model (see section 3). This depends on the structure of the claim numbers and payments. The CL predictions are unbiased in the case of a multiplicative structure.

The CL method predicts the right total reserve if the payment is multiplicative in the occurrence year dimension (i). It predicts the right reserve of known claims if the payment is multiplicative in the “run-off after reporting” time dimension (k). Therefore it is a matter of structure or symmetry of the claim payments if the CL method is an appropriate estimation method for the reserves or not. These arguments concerning the structure of the claim payments are of course also valid for the claim numbers. Therefore CL estimates of claim numbers are biased, if the data structure does not have the above described form. This is of importance because in practice CL estimates of the ultimate claim numbers are sometimes used for IBNR calculation.

We are able to reproduce these theoretical results with numerical examples. Therefore we have shown that there are realistic scenarios where the CL method generates biased reserve predictions. While the expectations of the 3D reserves are analytical functions of the model parameters, the reserve distribution can be calculated by Monte Carlos simulation techniques. We perform these stochastic computations in order to investigate the statistical properties of the 3D model reserves in comparison to the CL predictions. We are especially interested in the correlation and in the width and shape of the reserves’ distribution.

In our examples the deviations due to the not appropriate CL method are smaller than the 3D reserves’ standard deviation, but of the same order of magnitude. Due to lay-out reasons we have chosen a relatively short run-off period of only 10 years. Longer run-off periods will increase this deviance. Additionally, it depends very much on the deviance of the claim data from the multiplicative structure. This is a further source that will enlarge the deviation effect of the CL estimates. Concerning the correlation between the 3D model reserve and the CL reserve we find small positive correlation in all cases, which is a very favourable attitude for a reserving method, although small in size.

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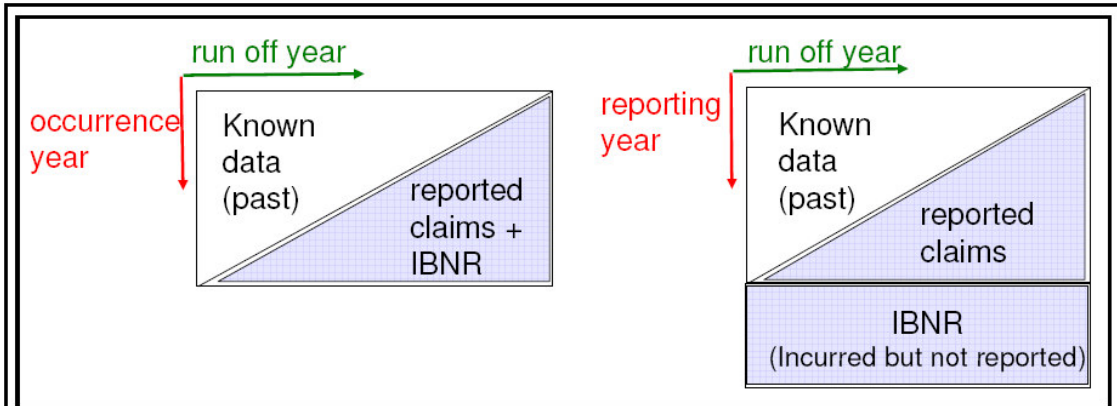


Fig. 1:
Typical data structures for reserve estimation.

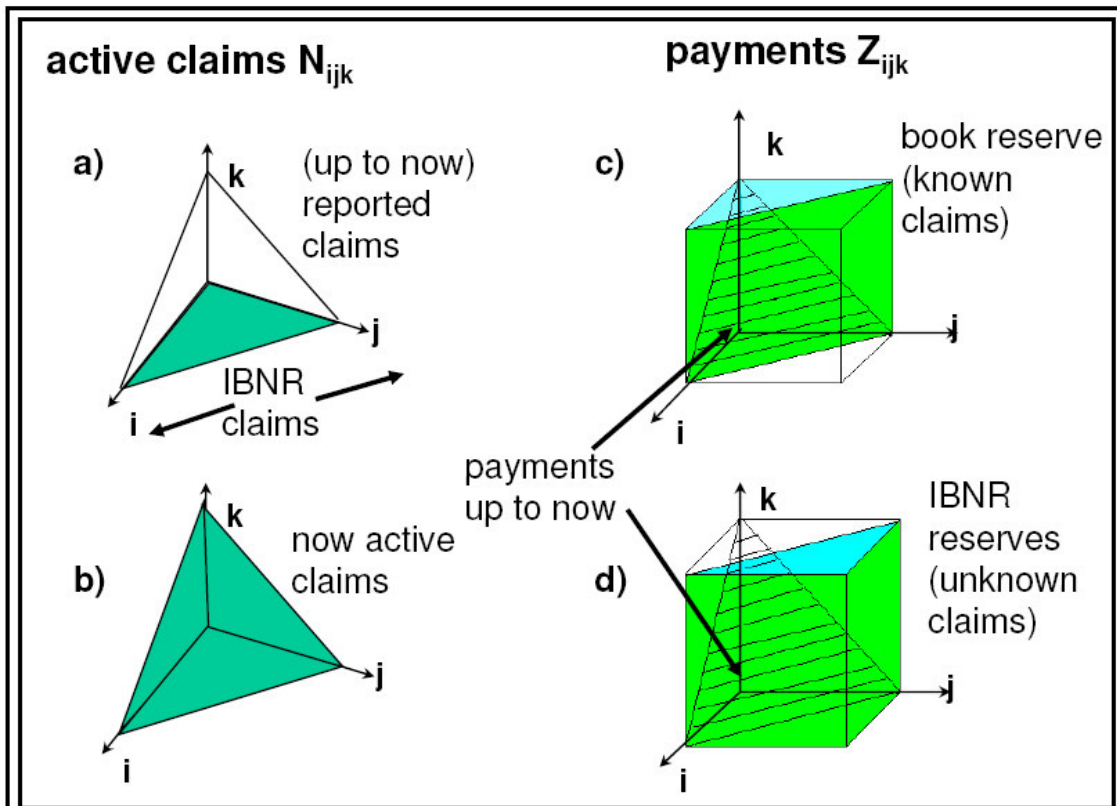
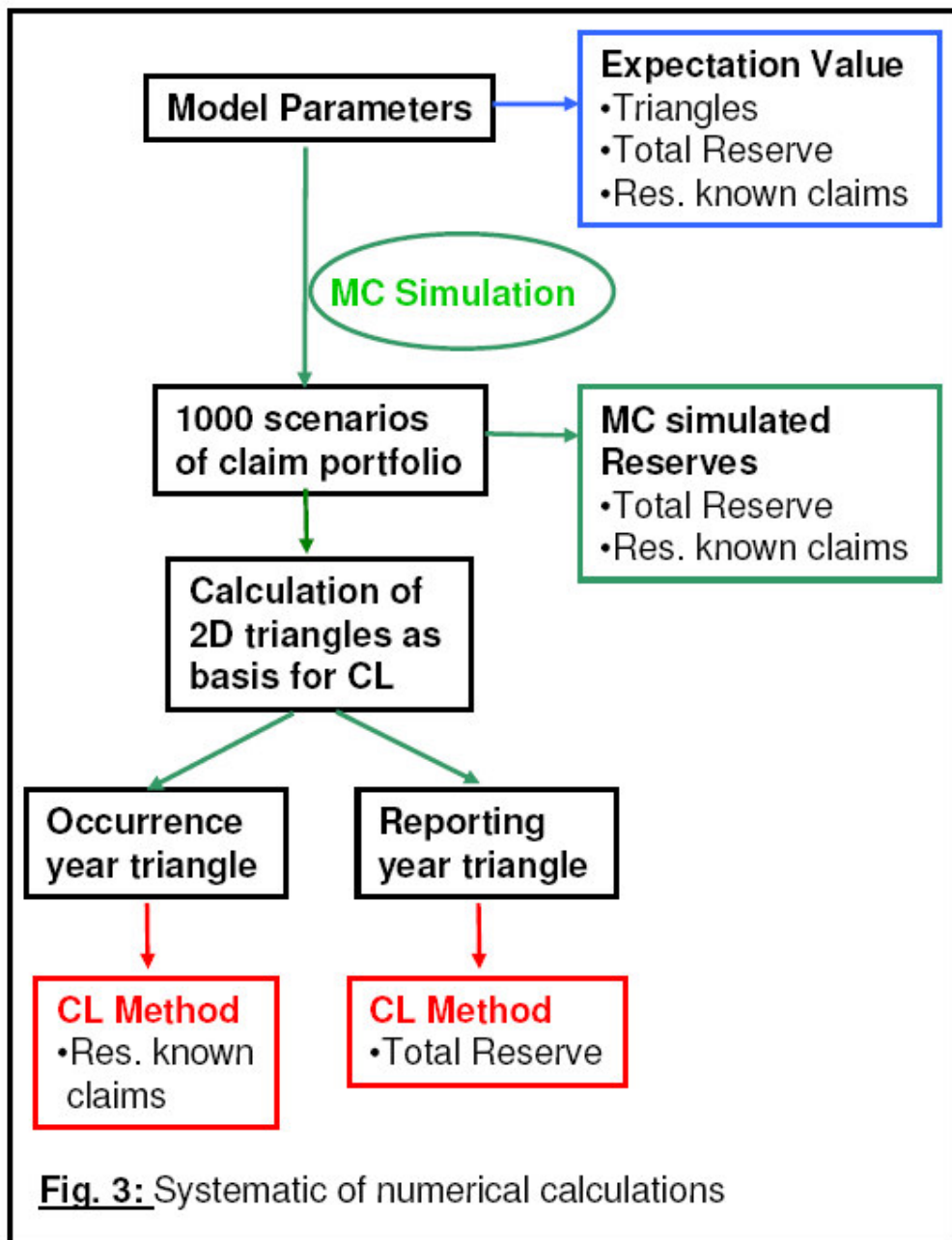


Fig. 2:
Visualisation of the 3D model. Left: The active claim numbers;
Right: Payments and reserves.



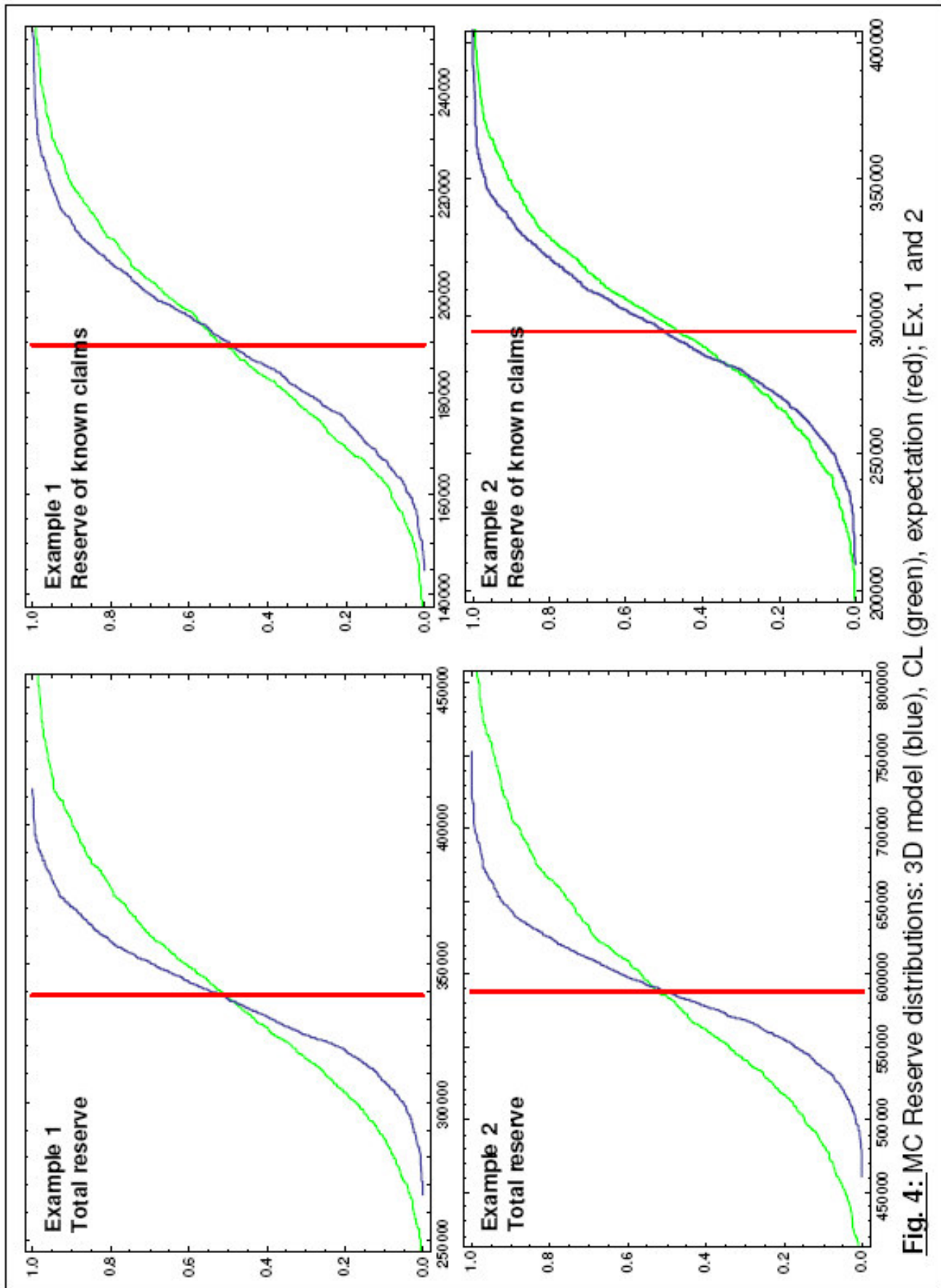


Fig. 4: MC Reserve distributions: 3D model (blue), CL (green), expectation (red); Ex. 1 and 2

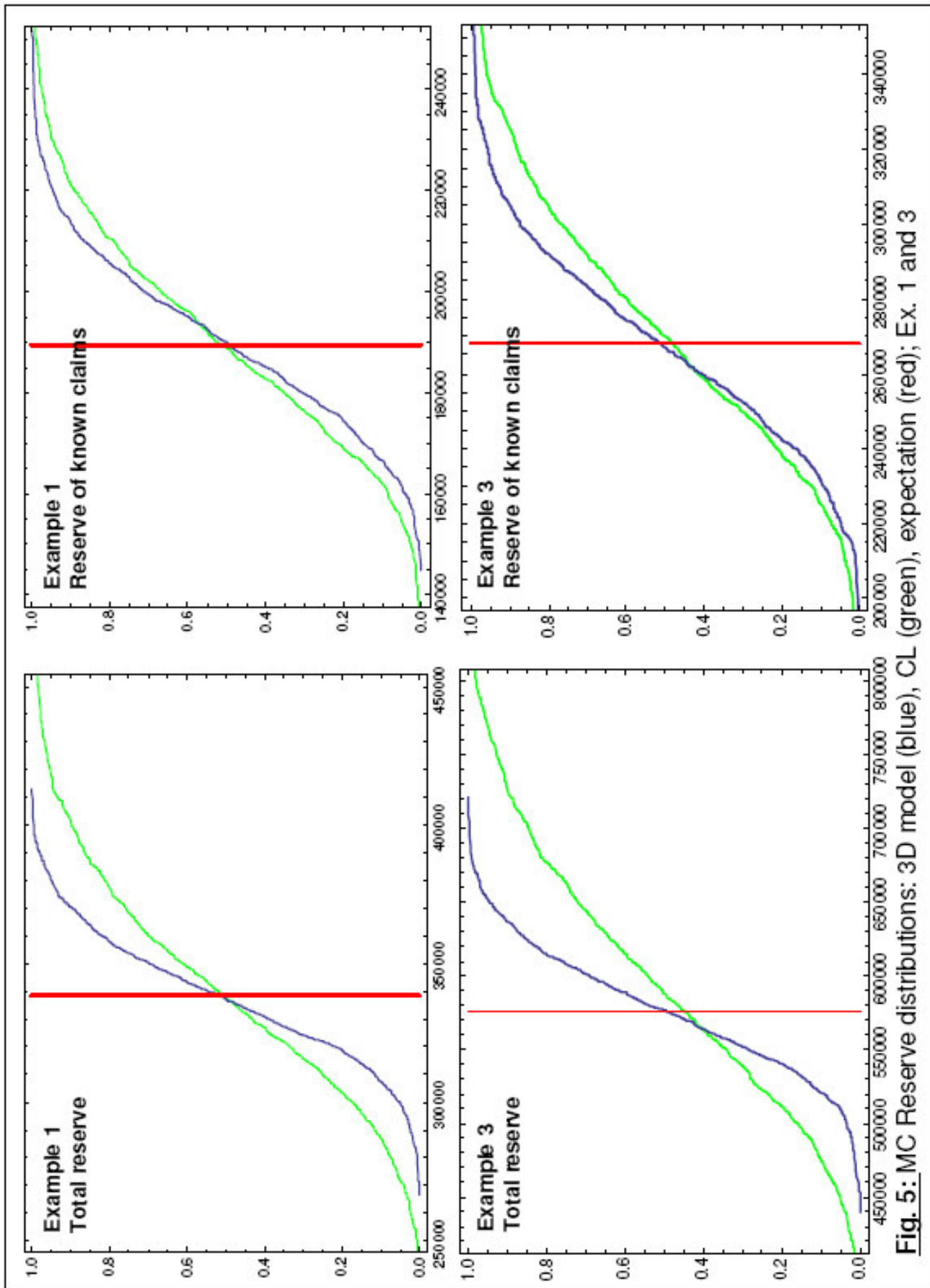


Fig. 5: MC Reserve distributions: 3D model (blue), CL (green), expectation (red); Ex. 1 and 3

a)

Occ. Year i	Number of Claims
1	80
2	140
3	200
4	220
5	216
6	190
7	180
8	184
9	210
10	196

b)

rep. Lag j	lambda
1	0,41
2	0,33
3	0,12
4	0,05
5	0,04
6	0,02
7	0,01
8	0,01
9	0,01

c)

k	open claims
1	1,00
2	0,55
3	0,35
4	0,23
5	0,15
6	0,10
7	0,06
8	0,03
9	0,01

Tab. 1: Claim number parameters for examples 1-3.

a) 3D model payment estimations: Occurrence year vs. run-off year table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	9.840	13.332	10.680	7.819	5.926	4.262	2.930	1.999	1.339	677	346	185	98	50	24	10	2
2	17.220	23.331	18.690	13.684	10.370	7.459	5.128	3.499	2.344	1.184	605	323	172	88	42	17	4
3	24.600	33.330	26.700	19.548	14.814	10.656	7.326	4.998	3.348	1.692	864	462	246	126	60	24	6
4	14.760	19.998	16.020	11.729	8.888	6.394	4.396	2.999	2.009	1.015	518	277	148	76	36	14	4
5	26.568	35.996	28.896	21.112	15.999	11.508	7.912	5.398	3.616	1.827	993	499	266	136	65	26	6
6	23.370	31.664	25.365	18.571	14.073	10.123	6.960	4.748	3.181	1.607	821	439	234	120	57	23	6
7	22.140	29.997	24.030	17.593	13.333	9.590	6.593	4.498	3.013	1.523	778	416	221	113	54	22	5
8	11.316	15.332	12.282	8.992	6.814	4.902	3.370	2.299	1.540	778	397	213	113	58	28	11	3
9	25.830	34.997	28.035	20.525	15.555	11.189	7.692	5.248	3.515	1.777	907	485	258	132	63	25	6
10	24.108	32.663	26.166	19.157	14.518	10.443	7.179	4.898	3.281	1.658	847	453	241	123	59	24	6

Total Reserve: 352.884

b) Short and cumulated version of a) (green) and CL estimates

	1	2	3	4	5	6	7	8	9	10
1	9.840	23.172	33.852	41.671	47.597	51.859	54.790	56.789	58.128	58.805
2	17.220	40.551	59.241	72.925	83.294	90.754	95.882	99.380	101.724	102.908
3	24.600	57.930	84.630	104.178	118.992	129.648	136.974	141.972	145.320	147.012
4	14.760	34.758	50.778	62.507	71.395	77.789	82.184	85.183	87.192	88.207
5	26.568	62.564	91.400	112.512	128.511	140.020	147.932	153.330	156.946	158.773
6	23.370	55.033	80.398	98.969	113.042	123.166	130.125	134.873	138.054	139.661
7	22.140	52.137	76.167	93.760	107.093	116.683	123.277	127.775	130.788	132.311
8	11.316	26.648	38.930	47.922	54.736	59.638	63.008	65.307	66.847	67.626
9	25.830	60.827	88.862	109.387	124.942	136.130	143.823	149.071	152.586	154.363
10	24.108	56.771	82.937	102.094	116.612	127.055	134.235	139.133	142.414	144.072

Total Reserve for
10 year triangle:

3D Model estimate
338.366

CL estimate
338.366

Tab. 2: Example 1: Occurrence year run-off tables .

a) 3D model payment estimations: Reporting year vs. run-off year table.

	1	2	3	4	5	6	7	8	9	10
1	9.840	5.412	3.444	2.263	1.476	984	590	295	98	0
2	25.140	13.827	8.799	5.782	3.771	2.514	1.508	754	251	0
3	41.340	22.737	14.469	9.508	6.201	4.134	2.480	1.240	413	0
4	40.800	22.440	14.280	9.384	6.120	4.080	2.448	1.224	408	0
5	48.708	26.789	17.048	11.203	7.306	4.871	2.922	1.461	487	0
6	54.234	29.829	18.982	12.474	8.135	5.423	3.254	1.627	542	0
7	54.006	29.703	18.902	12.421	8.101	5.401	3.240	1.620	540	0
8	42.516	23.384	14.881	9.779	6.377	4.252	2.551	1.275	425	0
9	48.840	26.862	17.094	11.233	7.326	4.884	2.930	1.465	488	0
10	55.866	30.726	19.553	12.849	8.380	5.587	3.352	1.676	559	0

Reserve of reported claims:

189.427

b) Cumulated version of a) (green) and CL estimates

	1	2	3	4	5	6	7	8	9	10
1	9.840	15.252	18.696	20.959	22.435	23.419	24.010	24.305	24.403	24.403
2	25.140	38.967	47.766	53.548	57.319	59.833	61.342	62.096	62.347	62.347
3	41.340	64.077	78.546	88.054	94.255	98.389	100.870	102.110	102.523	102.523
4	40.800	63.240	77.520	86.904	93.024	97.104	99.552	100.776	101.184	101.184
5	48.708	75.497	92.545	103.748	111.054	115.925	118.848	120.309	120.796	120.796
6	54.234	84.063	103.045	115.518	123.654	129.077	132.331	133.958	134.500	134.500
7	54.006	83.709	102.611	115.033	123.134	128.534	131.775	133.395	133.935	133.935
8	42.516	65.900	80.780	90.559	96.936	101.188	103.739	105.015	105.440	105.440
9	48.840	75.702	92.796	104.029	111.355	116.239	119.170	120.635	121.123	121.123
10	55.866	86.592	106.145	118.995	127.374	132.961	136.313	137.989	138.548	138.548

Reserve of reported claims:

CL estimate
189.472

Tab. 3: Example 1: Reporting year run-off tables .

a) 3D model payment estimations: Occurrence year vs. run-off year table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	10.332	16.645	14.811	11.768	9.821	7.612	5.603	4.245	3.297	1.812	1.013	579	326	176	87	36	9
2	18.081	29.129	25.919	20.594	17.187	13.322	9.805	7.430	5.769	3.171	1.773	1.013	570	307	152	63	16
3	25.830	41.613	37.028	29.420	24.554	19.031	14.008	10.614	8.242	4.530	2.533	1.447	814	439	218	90	23
4	15.498	24.968	22.217	17.652	14.732	11.418	8.405	6.368	4.945	2.718	1.520	868	488	264	131	54	14
5	27.896	44.942	39.990	31.774	26.518	20.553	15.128	11.463	8.901	4.892	2.736	1.563	879	474	235	97	25
6	24.538	39.532	35.176	27.949	23.326	18.079	13.307	10.083	7.830	4.304	2.407	1.375	773	417	207	85	22
7	23.247	37.452	33.325	26.478	22.098	17.128	12.607	9.552	7.417	4.077	2.280	1.302	733	395	196	81	21
8	11.882	19.142	17.033	13.533	11.295	8.754	6.443	4.882	3.791	2.084	1.165	666	374	202	100	41	11
9	27.122	43.694	38.879	30.891	25.781	19.982	14.708	11.144	8.654	4.757	2.660	1.520	855	461	228	94	24
10	25.313	40.781	36.287	28.832	24.062	18.650	13.727	10.401	8.077	4.439	2.483	1.418	798	430	213	88	23

Total Reserve: 632.854

b) Short and cumulated version of a) (green) and CL estimates

	1	2	3	4	5	6	7	8	9	10
1	10.332	26.977	41.788	53.556	63.378	70.990	76.593	80.838	84.135	85.947
2	18.081	47.210	73.129	93.723	110.911	124.232	134.038	141.467	147.236	150.407
3	25.830	67.443	104.471	133.891	158.444	177.475	191.483	202.096	210.338	214.868
4	15.498	40.466	62.682	80.334	95.066	106.485	114.890	121.258	126.203	128.921
5	27.896	72.838	112.828	144.602	171.120	191.673	206.801	218.264	227.165	232.057
6	24.538	64.071	99.247	127.196	150.522	168.501	181.908	191.991	199.821	204.124
7	23.247	60.699	94.023	120.502	142.600	159.727	172.334	181.887	189.304	193.381
8	11.882	31.024	48.056	61.590	72.884	81.638	88.082	92.964	96.755	98.839
9	27.122	70.815	109.694	140.585	166.366	186.349	201.057	212.201	220.855	225.611
10	25.313	66.094	102.381	131.213	155.275	173.925	187.653	198.054	206.131	210.570

Total Reserve for
10 year triangle:

3D Model estimate
587.676

CL estimate
587.676

Tab. 4: Example 2: Occurrence year run-off tables .

a) 3D model payment estimations: Reporting year vs. run-off year table.

	1	2	3	4	5	6	7	8	9	10
1	10.332	5.953	3.961	2.716	1.845	1.279	797	413	143	0
2	28.773	16.517	10.950	7.485	5.070	3.506	2.179	1.127	388	0
3	49.293	28.248	18.699	12.764	8.634	5.963	3.702	1.913	658	0
4	52.884	30.208	19.937	13.571	9.157	6.308	3.907	2.015	692	0
5	62.069	35.478	23.429	15.956	10.772	7.425	4.601	2.374	816	0
6	71.389	40.755	26.884	18.291	12.335	8.495	5.260	2.711	931	0
7	73.207	41.749	27.513	18.701	12.601	8.671	5.365	2.763	948	0
8	61.796	35.157	23.117	15.680	10.545	7.242	4.473	2.300	788	0
9	67.196	38.301	25.228	17.140	11.545	7.941	4.911	2.529	867	0
10	76.910	43.837	28.874	19.617	13.212	9.088	5.620	2.894	993	0

Reserve of reported claims:

294,221

b) Cumulated version of a) (green) and CL estimates

	1	2	3	4	5	6	7	8	9	10
1	10.332	16.285	20.246	22.962	24.807	26.066	26.883	27.296	27.439	27.439
2	28.773	45.290	56.240	63.725	68.795	72.301	74.480	75.607	75.995	75.995
3	49.293	77.541	96.240	109.004	117.638	123.601	127.303	129.216	129.882	129.882
4	52.884	83.092	103.030	116.801	125.757	132.066	135.973	138.026	138.739	138.739
5	62.069	97.547	120.976	136.932	147.704	155.129	159.767	162.179	163.016	163.016
6	71.389	112.144	139.029	157.319	169.654	178.223	183.551	186.323	187.285	187.285
7	73.207	114.956	142.468	161.169	173.874	182.656	188.117	190.958	191.943	191.943
8	61.796	96.953	120.069	135.911	146.624	154.030	158.635	161.031	161.862	161.862
9	67.196	105.497	130.812	148.071	159.743	167.811	172.828	175.438	176.344	176.344
10	76.910	120.830	149.825	169.592	182.961	192.202	197.948	200.938	201.975	201.975

Reserve of reported claims:

CL estimate
297,428

Tab. 5: Example 2: Reporting year run-off tables .

a) 3D model payment estimations: Occurrence year vs. run-off year table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	7.721	13.125	12.072	9.852	8.560	6.842	5.181	4.149	3.500	1.978	1.147	674	388	215	108	45	12
2	13.765	23.330	21.423	17.464	15.154	12.102	9.157	7.325	6.173	3.486	2.021	1.186	683	378	190	79	21
3	20.024	33.844	31.030	25.268	21.898	17.472	13.210	10.557	8.885	5.015	2.906	1.705	982	542	273	114	30
4	12.231	20.615	18.873	15.352	13.288	10.594	8.004	6.390	5.371	3.030	1.755	1.030	593	327	165	69	18
5	22.406	37.664	34.431	27.977	24.188	19.267	14.547	11.601	9.741	5.493	3.179	1.865	1,073	592	298	124	32
6	20.051	33.620	30.691	24.912	21.514	17.123	12.919	10.293	8.632	4.865	2.815	1,650	949	524	264	110	29
7	19.321	32.314	29.458	23.888	20.606	16.387	12.355	9.834	8.238	4.641	2.684	1,573	904	499	251	105	27
8	10.041	16.753	15.252	12.356	10.647	8.460	6.374	5.069	4.242	2.388	1.380	809	465	257	129	54	14
9	23.299	38.782	35.261	28.538	24.564	19.504	14.686	11.668	9.753	5.489	3.171	1,857	1,067	589	296	123	32
10	22.099	36.702	33.327	26.947	23.171	18.384	13.834	10.981	9.168	5.158	2.978	1,744	1,002	552	278	116	30

Total Reserve: 630.270

b) Short and cumulated version of a) (green) and CL estimates

	1	2	3	4	5	6	7	8	9	10
1	7.721	20.845	32.918	42.770	51.330	58.172	63.352	67.501	71.001	72.979
2	13.765	37.095	58.518	75.982	91.136	103.238	112.394	119.719	125.892	129.399
3	20.024	53.868	84.898	110.166	132.064	149.536	162.746	173.303	182.257	187.333
4	12.231	32.847	51.720	67.071	80.360	90.953	98.957	105.398	110.843	113.931
5	22.406	60.070	94.501	122.478	146.666	165.934	180.612	192.367	202.305	207.941
6	20.051	53.672	84.362	109.274	130.788	148.071	161.169	171.658	180.527	185.555
7	19.321	51.635	81.093	104.981	125.759	142.410	155.008	165.096	173.626	178.463
8	10.041	26.794	42.046	54.515	65.320	73.951	80.493	85.731	90.161	92.672
9	23.299	62.081	97.695	126.665	151.771	171.826	187.025	199.198	209.490	215.325
10	22.099	59.220	93.194	120.829	144.778	163.909	178.408	190.019	199.837	205.403

Total Reserve for
10 year triangle:

3D Model estimate
576.124

CL estimate
589.941

Tab. 6: Example 3: Occurrence year run-off tables .

a) 3D model payment estimations: Reporting year vs. run-off year table.

	1	2	3	4	5	6	7	8	9	10
1	7.721	4.445	2.955	2.025	1.375	953	593	307	106	0
2	22.445	12.866	8.519	5.817	3.936	2.719	1.688	872	300	0
3	39.605	22.652	14.969	10.200	6.889	4.751	2.946	1.520	523	0
4	44.154	25.169	16.579	11.265	7.588	5.219	3.228	1.662	570	0
5	52.746	30.060	19.796	13.448	9.056	6.228	3.851	1.983	680	0
6	62.125	35.351	23.248	15.772	10.608	7.287	4.501	2.315	793	0
7	65.291	37.100	24.366	16.510	11.092	7.611	4.696	2.413	826	0
8	56.959	32.290	21.160	14.308	9.594	6.571	4.047	2.076	710	0
9	62.612	35.519	23.292	15.759	10.572	7.245	4.465	2.292	784	0
10	72.438	41.079	26.929	18.214	12.216	8.369	5.157	2.646	904	0

Reserve of reported claims:

266.032

b) Cumulated version of a) (green) and CL estimates

	1	2	3	4	5	6	7	8	9	10
1	7.721	12.166	15.121	17.146	18.521	19.473	20.066	20.374	20.480	20.480
2	22.445	35.311	43.830	49.647	53.583	56.301	57.989	58.862	59.162	59.162
3	39.605	62.257	77.226	87.426	94.316	99.067	102.012	103.533	104.064	104.064
4	44.154	69.323	85.902	97.166	104.754	109.973	113.201	114.899	115.488	115.488
5	52.746	82.806	102.602	116.050	125.106	131.335	135.233	137.261	137.965	137.965
6	62.125	97.476	120.724	136.496	147.104	154.480	159.065	161.450	162.279	162.279
7	65.291	102.391	126.757	143.266	154.483	162.228	167.044	169.549	170.419	170.419
8	56.959	89.248	110.409	124.888	134.665	141.418	145.616	147.799	148.557	148.557
9	62.612	98.131	121.569	137.512	148.277	155.712	160.334	162.738	163.573	163.573
10	72.438	113.670	140.818	159.286	171.756	180.368	185.722	188.507	189.474	189.474

Reserve of reported claims:

CL estimate
272.400

Tab. 7: Example 3: Reporting year run-off tables .

	Only Model parameter dependent	MC simulated	Only Model parameter dependent	MC simulated
	3D Estimation	3D Mean	CL Estimation	CL Mean
Example 1 Total Reserve	338366	338103	338366	341029
Example 1 Reserve of known claims	189427	190158	189427	190390
Example 2 Total Reserve	587676	589595	587676	591713
Example 2 Reserve of known claims	294221	295583	297428	298571
Example 3 Total Reserve	576124	577304	589941	596173
Example 3 Reserve of known claims	268032	268004	272400	272347

Tab. 8: Comparison of estimation and (empirical) mean values.

	Example 1	Example 2	Example 3
Total Reserve	16 %	10 %	7 %
Reserve of known claims	22 %	27 %	19 %

Tab. 9: Correlations of MC reserves: CL vs. model.