Monitoring actuarial assumptions in life insurance

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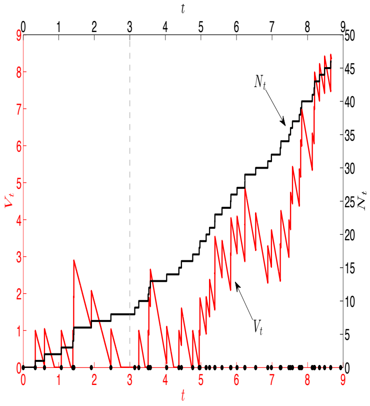
LIFE

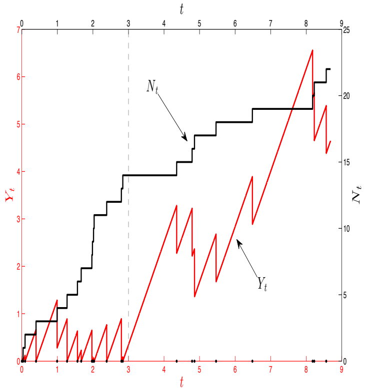
PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

LoLitA

Typical paths with change of regime at date 3



(a) Processes N and V_t

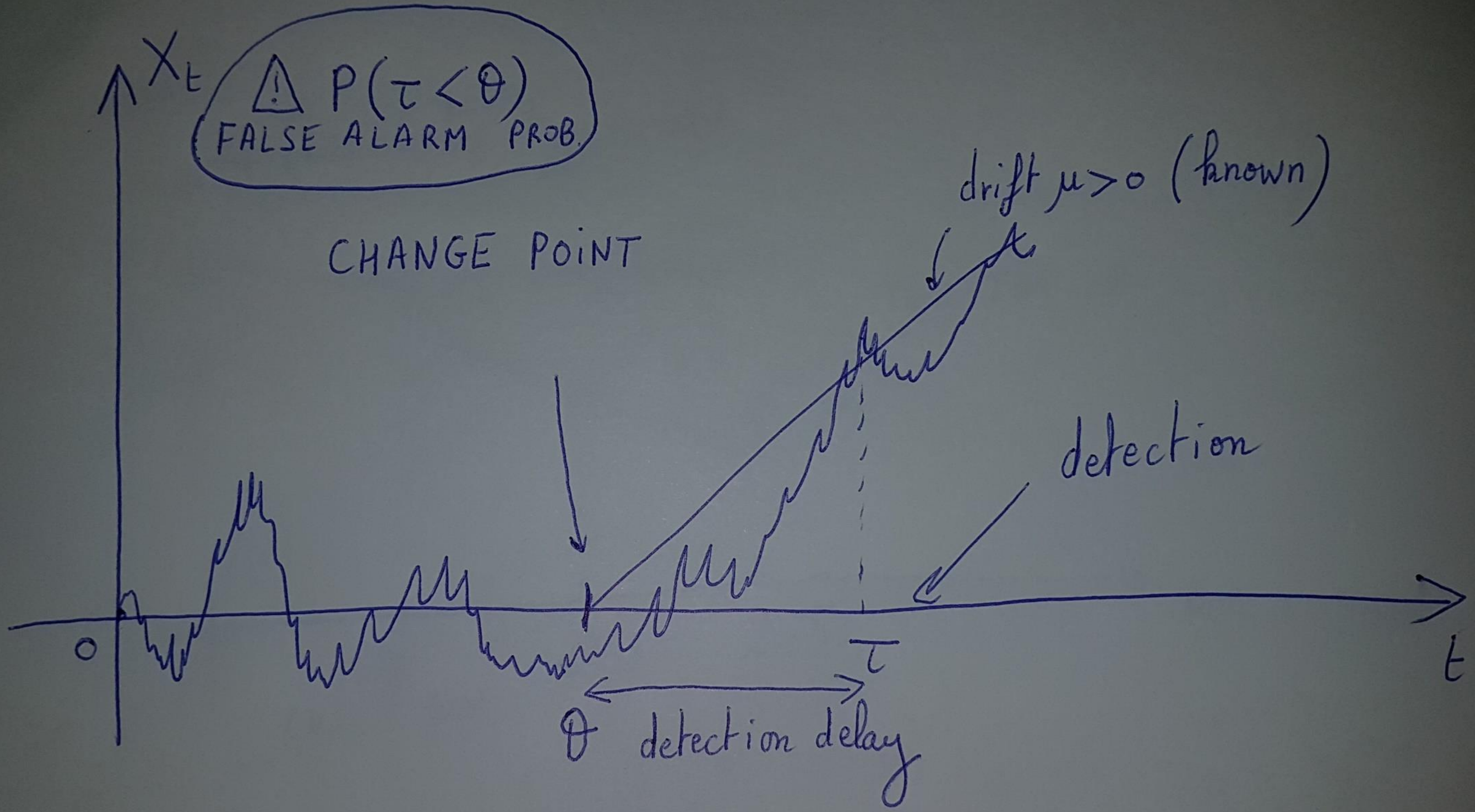


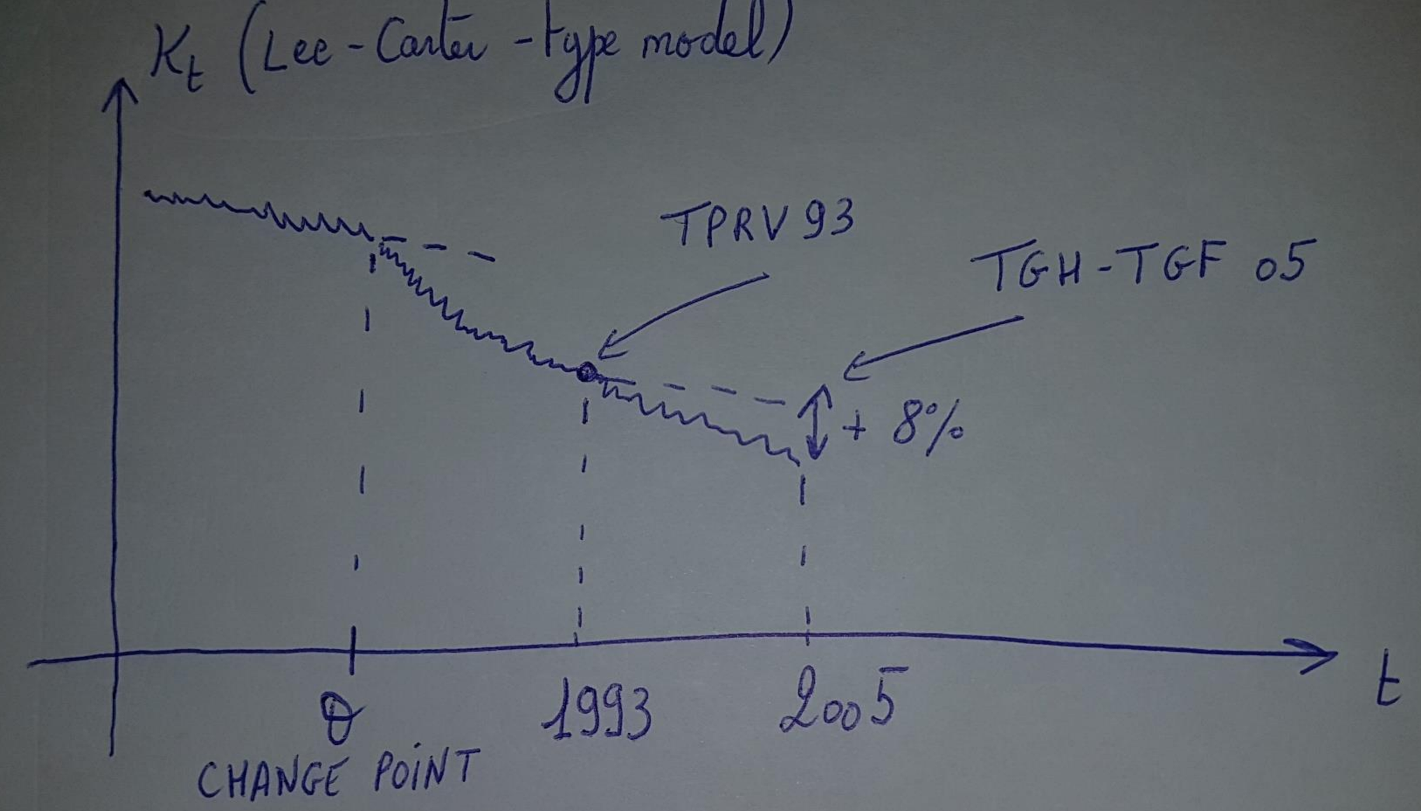
(b) Processes N and Y_t

Figure: Sample paths, for $\rho = 1.5$, of the cusum processes N, V^ρ (left) and N, Y_t^ρ for $\rho = 0.5$ (right).

Quick Outline

- Why is quick detection important in insurance?
- Quick version of quickest detection
- Monitoring populations in practice
- Monitoring other insurance portfolios (quickly if time permits)


X_t
⚠ $P(\tau < \theta)$
FALSE ALARM PROB.
drift $\mu > 0$ (known)
CHANGE POINT
detection
0
τ
t
θ
detection delay



K_t (Lee-Carter-type model)
TPRV 93
TGH-TGF 05
+ 8%
t
θ
1993
2005
CHANGE POINT

Why is quick detection important in insurance?
How to choose parameter rho?

	females				males			
	Doubled improvements		Mortality level at 80% of the expected		Doubled improvements		Mortality level at 80% of the expected	
	pension value	interest rate	pension value	interest rate	pension value	interest rate	pension value	interest rate
55	+5.4%	+32bp	+3.1%	+19bp	+6.7%	+42bp	+3.7%	+24bp
65	+5.76%	+43bp	+4.7%	+36bp	+7%	+57bp	+5.7%	+48bp
75	+5.2%	+55bp	+7.6%	+80bp	+6.3%	+74bp	+9.1%	+107bp
85	+3.6%	+60bp	+13.2%	+207bp	+4.3%	+84bp	+15.4%	+281bp

TABLE: TGH05/TGF05 with flat interest rate of 3%

Bayesian setup for random change-point

Brownian framework with abrupt change in the drift

- Based on the conditional distribution of the time of change,
- Formulated as an optimal stopping problem
- Page(1954), Shiryaev(1963), Roberts(1966), Beibel(1988), Moustakides (2004), and many others...

Poisson framework with abrupt change in intensity

- Based on the conditional distribution of the time of change, with exponential or geometric prior distribution
- More recent studies : Gal (1971), Gapeev (2005), Bayraktar (2005, 2006), Dayanik (2006) for compound Poisson, Peskir, Shyriaev(2009) and others

Mathematical settings

We consider a portfolio of insured population:

- Let $N = (N_t)_{t \geq 0}$ be a counting process indicating the deaths of policyholders and $\lambda = (\lambda_t)_{t \geq 0}$ its intensity.
- The counting process N_t, is available sequentially through the filtration $\mathcal{F}_t = \sigma\{N_s, 0 < s \leq t\}$.
- We suppose that the insurance company relies on a Cox-like model to project her own experienced mortality:

$$\lambda_t = \underline{\rho}\lambda_t^0,$$

  - λ_t^0 is a reference intensity and $\underline{\rho}$ is a positive parameter.
  - λ^0 is considered deterministic and may refer whether to a projection of national population/best estimate...

Model risk/parameter uncertainty: **Change-point**

$$\lambda_t = \mathbf{1}_{\{t<\boldsymbol{\theta}\}}\underline{\rho}\lambda_t^0 + \mathbf{1}_{\{t\geq\boldsymbol{\theta}\}}\overline{\rho}\lambda_t^0.$$

Without loss of generality we can assume that $\underline{\rho} = 1$ and let $\boldsymbol{\rho} = \overline{\rho} > 1$.

Probabilistic formulation

Let $\mathbb{P}_\theta$ (resp. $\mathbb{E}_\theta[\cdot]$) be the probability measure (resp. expectation) induced when the change takes place at time θ

Example

- For $\theta = 0$, the process is *out-of-control*
- For $\theta = \infty$, the process is *in-control*

Detect the change-point θ as quick as possible while avoiding false alarms

Optimality Criteria, Lorden (1971)-like

- The detection delay $\mathbb{E}_\theta\left[(N_\tau - N_\theta)^+ \middle| \mathcal{F}_\theta\right]$
- The frequency of false alarm $\mathbb{E}_\infty[N_\tau]$

Optimization Problem

Optimization Problem

$$\text{Find } \tau^* \text{ such that } C(\tau^*) = \inf_\tau \mathbf{sup}_{\theta\in[0,\infty]} \mathbf{ess\,sup} \mathbb{E}_\theta \left[(N_\tau - N_\theta)^+ \middle| \mathcal{F}_\theta \right]$$
$$\text{subject to } \mathbb{E}_\infty[N_\tau] = \omega.$$

Assumption

1. $\int_0^t \lambda_s ds < \infty, \quad \mathbb{P}_\infty, \mathbb{P}_0$-a.s.
2. $N_\infty = \infty \qquad \mathbb{P}_\infty, \mathbb{P}_0$-a.s.

Optimality of the Cusum procedure (1/7)

Let the Radon-Nikodym density of $\mathbb{P}_0$ with respect to $\mathbb{P}_\infty$ be defined as

$$\frac{d\mathbb{P}_0}{d\mathbb{P}_\infty}\bigg|_{\mathcal{F}_t} = \exp U_t,$$

where $U_t = \log(\rho)N_t + (1-\rho)\int_0^t \lambda_s^0 ds$ is the log-likelihood ratio.
Let $V(x)$ be the CUSUM process; with head-start $0 \leq x < m$; defined as

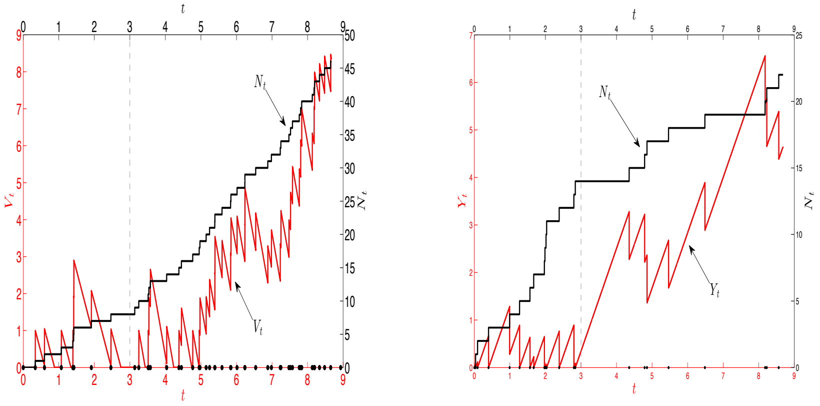
$$V_t(x) = U_t - (-x) \wedge \underline{U}_t \tag{1}$$

where $\underline{U}_t$ is the running infimum of U, i.e. $\underline{U}_t = \inf_{s \leq t} U_s$.
The process $V(x)$ measures the size of the drawup, comparing the present value of the process U to its historical infimum $\underline{U}$.
Let $\tau_m(x)$ be the fist hitting time of $V(x)$ of the barrier m, i.e.

$$\tau_m(x) = \inf\{t \geq 0, V_t(x) \geq m\}.$$

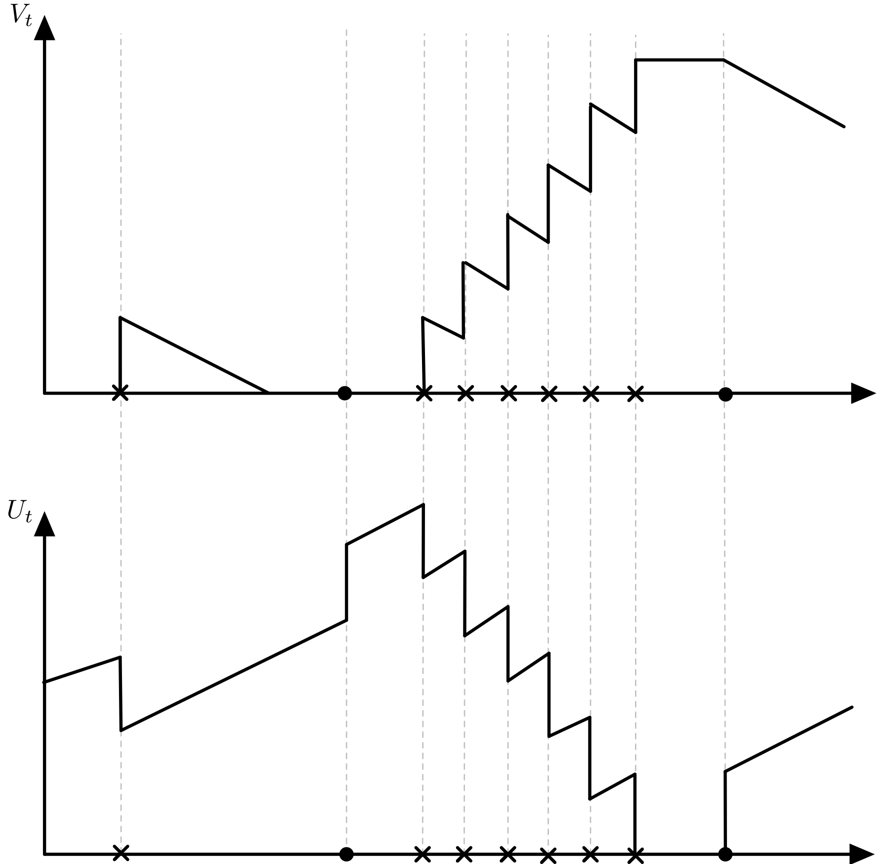
Theorem

If $\mathbb{E}_\infty[N_{\tau_m(0)}] = \omega$ *then* $\tau_m(0)$ *is optimal, i.e.* $\inf_\tau C(\tau) = C(\tau_m(0))$



(a) Processes N and V_t (b) Processes N and Y_t

Figure: Sample paths, for $\rho = 1.5$, of the cusum processes N, V^ρ (left) and N, Y_t^ρ for $\rho = 0.5$ (right).


V_t
U_t

Detection Procedure – Algorithm

Step 1: Fix the input parameters: The post-change intensity through the specification of ρ and the false alarm constraint ω.

Step 2: Determine the threshold m as the solution of the equation $\mathbb{E}_\infty[N_{\tau_m}] = \omega$.

Step 3: For each new observation at time t compute the value of the CUSUM process V given by the iterative relation $V_{t+1} = (V_{t-1} + U_t)^+$.

Step 4: Compare the current value of V to the threshold m and stop the procedure once $V_t \geq m$ and sound an alarm. Hence $\tau_m(0) = t$.

Detection Procedure – Real World (1/4)

We consider the Continuous Mortality Investigation assured lives dataset and England & Wales national population. We split data into two periods:

- We consider the period 1947-1969 as a training period.
- The Cox model is estimated over this period using the MLE.

Hence we monitor the sequentially the dataset over the period 1970-2005 and look for changes on the mortality of assured lives.

Detection Procedure – Real World (2/4)

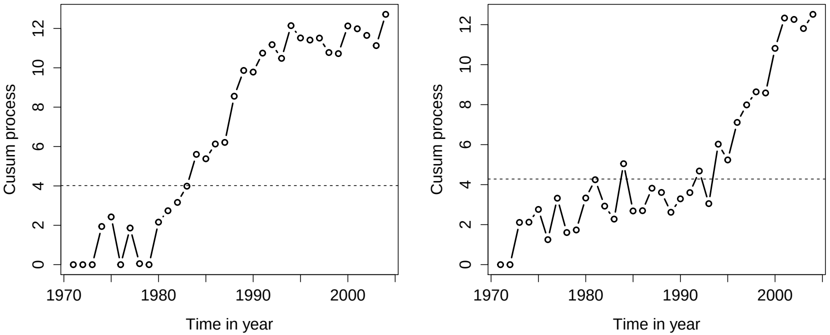


Figure: Detection scheme for age groups $50-59$ (right) and $80-89$ (left). The post-change is set to $\rho = 15\%$ and the false alarm constraint to $\omega = 100\bar{\lambda}$.

Detection Procedure – Real World (4/4)

	τ_m		
Age	$\rho = 1.50$	$\rho = 1.15$	Observed
$50 - 59$	1984	1978	1970
$60 - 69$	1991	1985	1974
$70 - 79$	1988	1984	1974
$80 - 89$	1983	1978	1973

Table: Detection of mortality change with a post-change ratio of $\rho = 1.15$ and an average run length (false alarm) constraint of 100. The right column reports the detected change-point using an off-line procedure.

Monitoring Mortality

Sounding an alarm for the change $\rho^{\text{Hyp}} \to \rho^{\text{Targer}}$

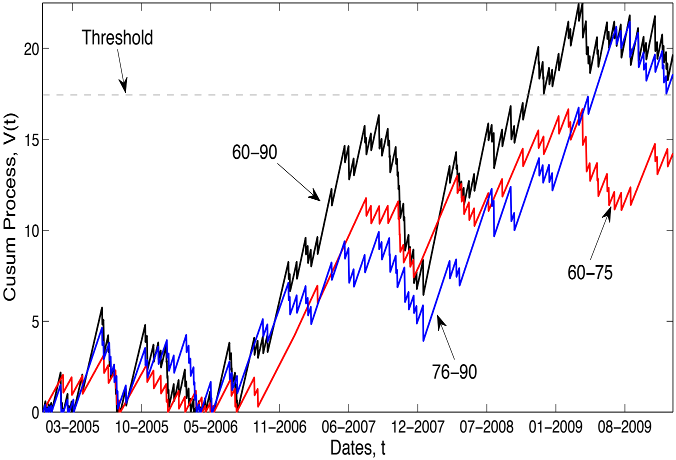
- We simulate deaths on the portfolio with different levels $\rho^{\text{Targer}} = 95\%, 90\%$ and 85% s.t.

$$D(x,t) \sim \text{Pois}(\rho^{\text{Targer}} \times L(x,t) \times \mu^{\text{ERM00}}(x,t))$$

- We suppose that *the actuary* made an assumption of $\rho^{\text{Hyp}} = 100\%$
- We set-up the monitoring/surveillance on the observed deaths and try to detect a change from $\rho^{\text{Hyp}} = 100\%$ to $\rho^{\text{Targer}} = 95\%, 80\%$ and 85% respectively.
- We test different sizes of the portfolio small sized $1000, 5000$ and a (relatively) large 10000 and compare the results

Monitoring Mortality

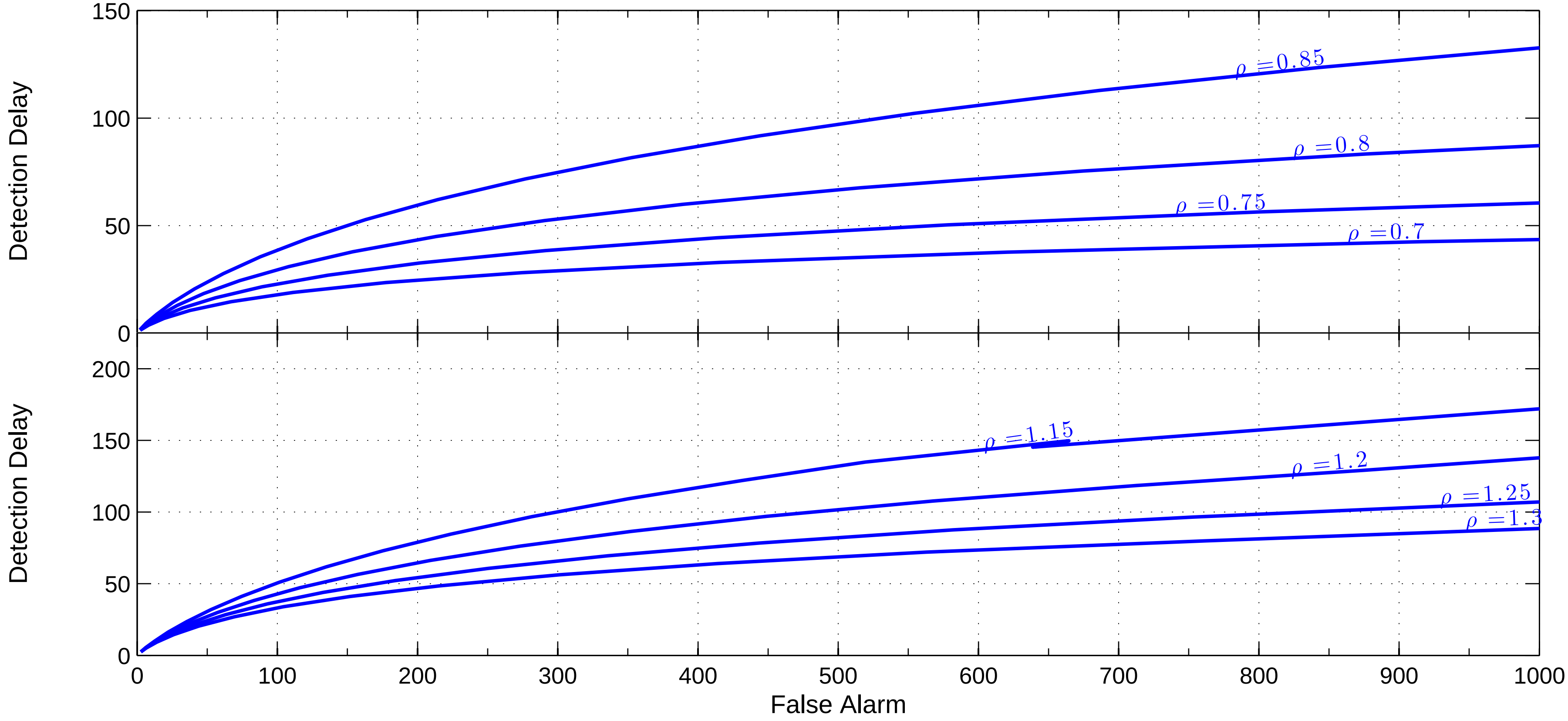
Sounding an alarm for the change $\rho^{\text{Hyp}} = 100\% \rightarrow \rho^{\text{Targer}} = 95\%$



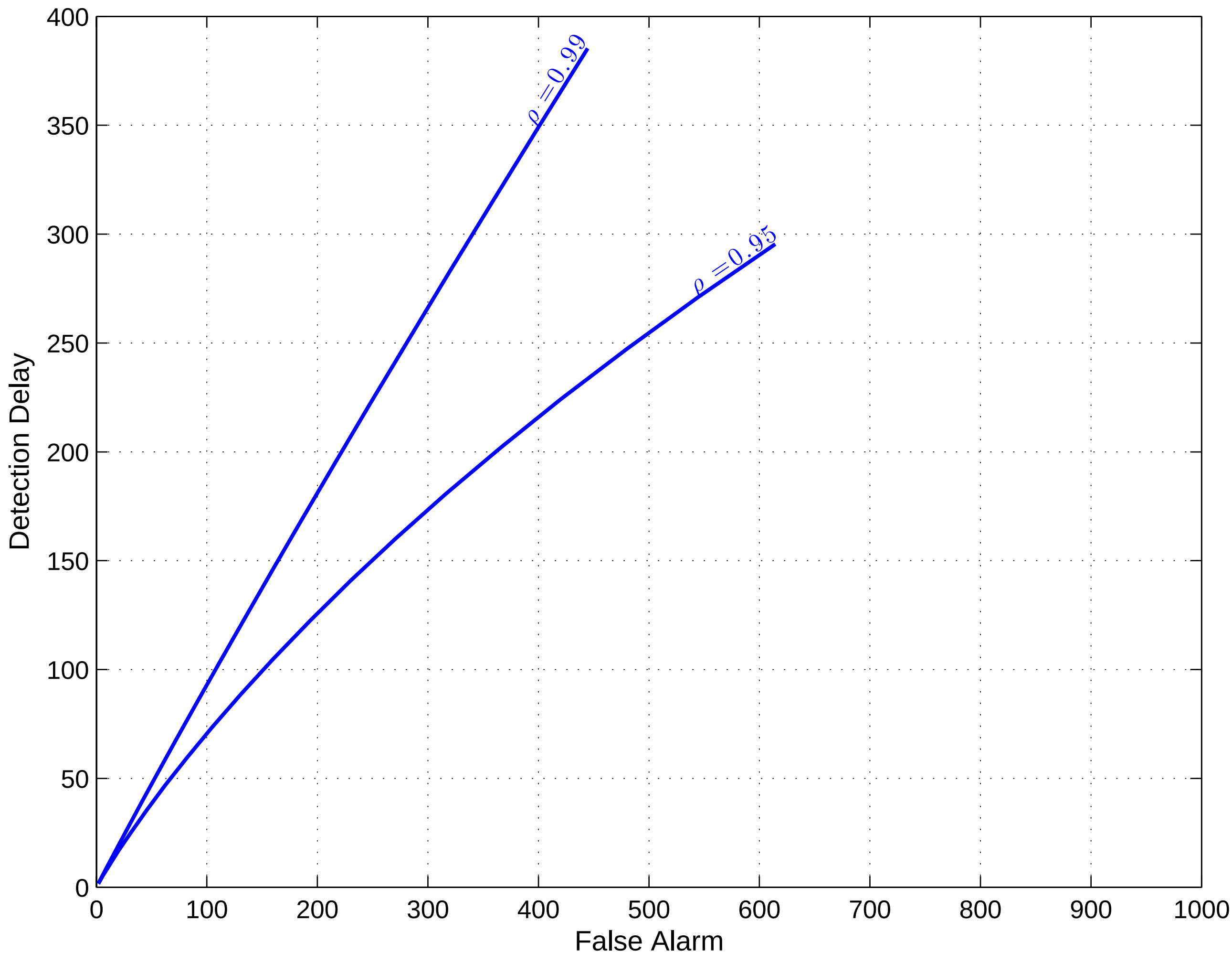
Detection Delay

Impact of Portfolio Size and Age Tranches

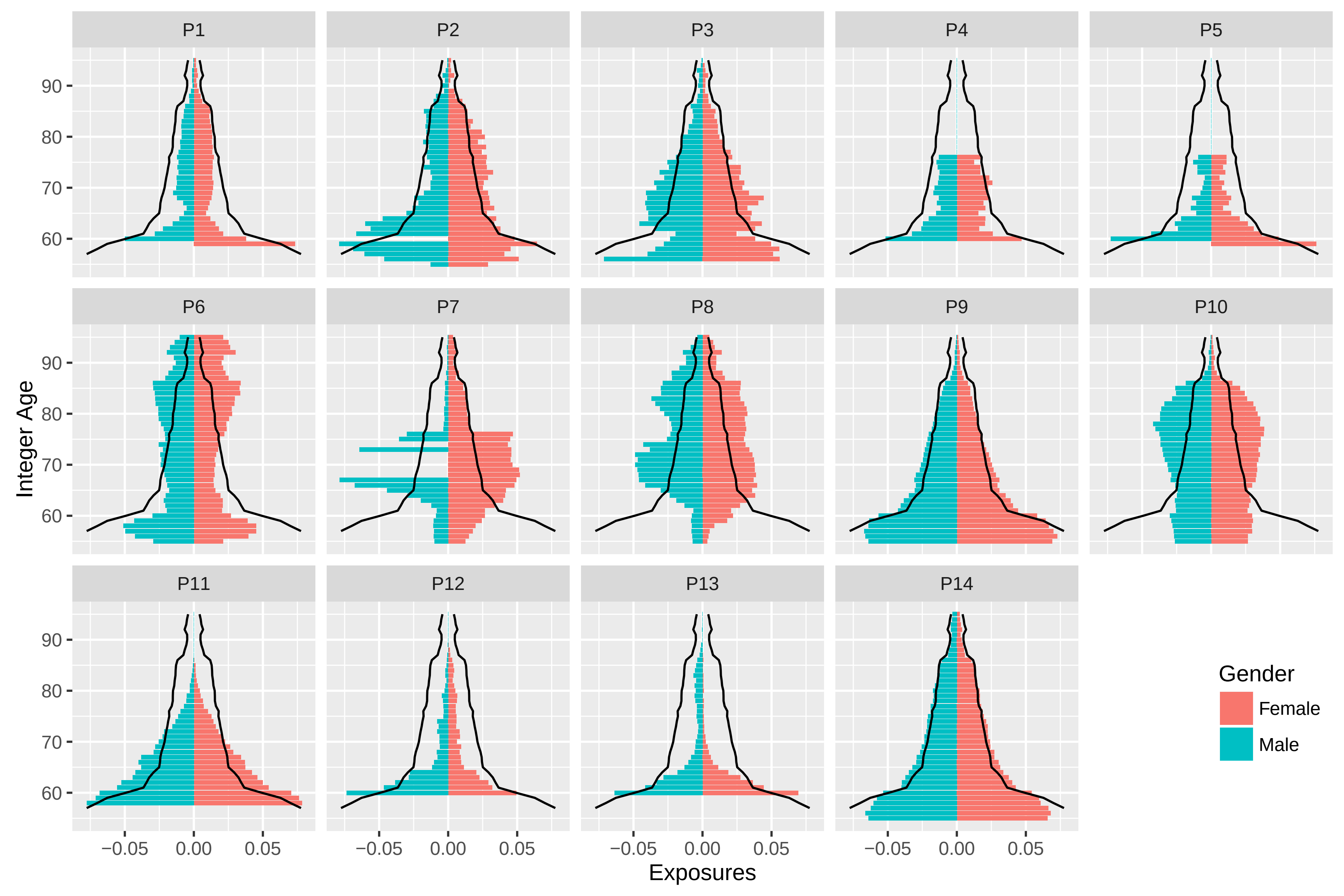
	Size	1000			5000			10000		
	Hyp. \ Ages	60-90	60-75	76-90	60-90	60-75	76-90	60-90	60-75	76-90
deaths	100% → 95%	596	710	498	246	99	107	240	99	106
deaths	100% → 90%	244	320	186	106	55	59	112	55	58
deaths	100% → 85%	92	122	100	58	35	36	61	34	36
time	100% → 95%	1086	1130	1120	576	617	422	308	327	212
time	100% → 90%	931	1124	947	276	373	241	151	192	127
time	100% → 85%	707	980	734	161	247	159	84	127	80

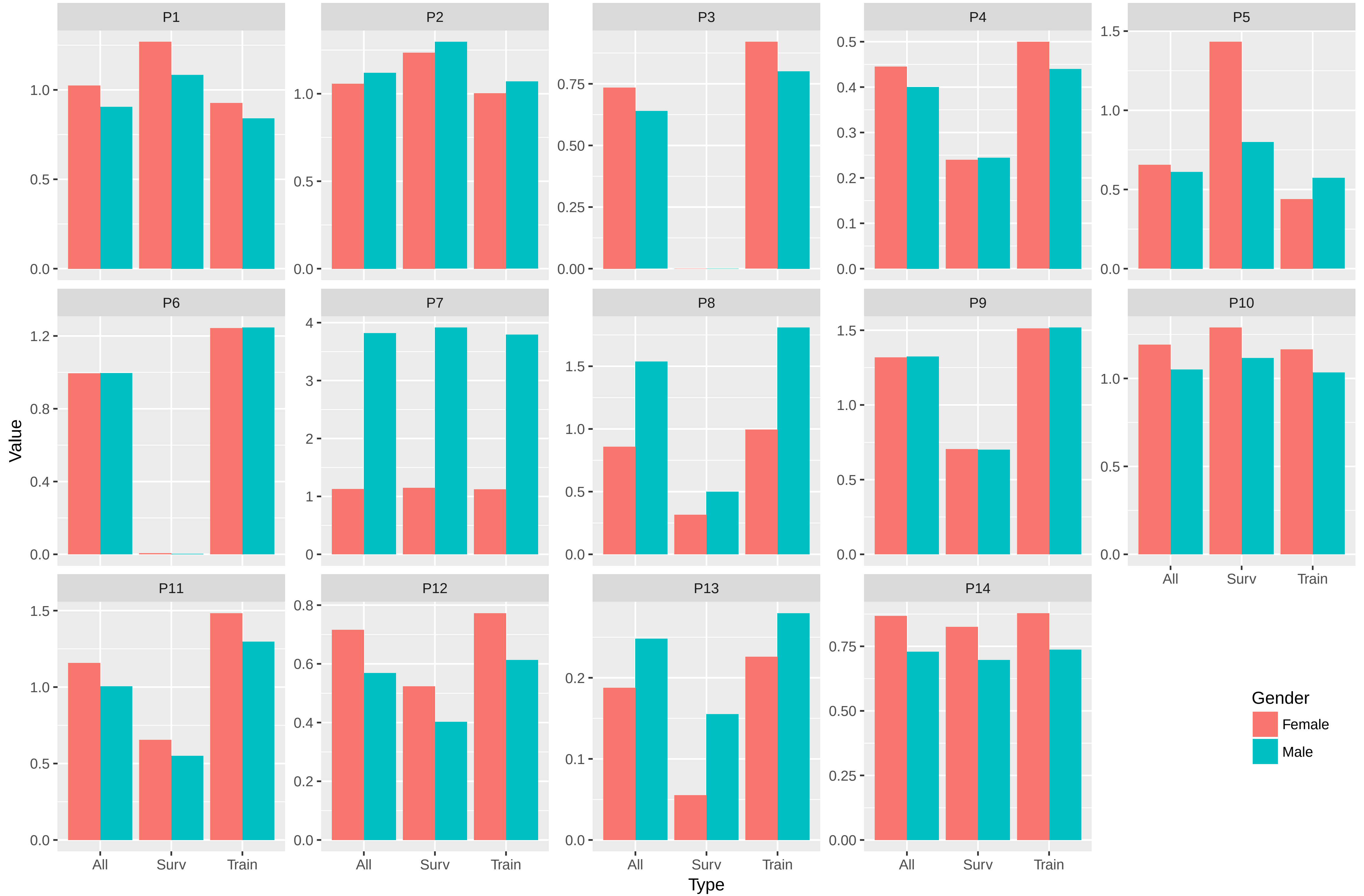


Detection Delay
Detection Delay
False Alarm
ρ =0.85
ρ =0.8
ρ =0.75
ρ =0.7
ρ =1.15
ρ =1.2
ρ =1.25
ρ =1.3

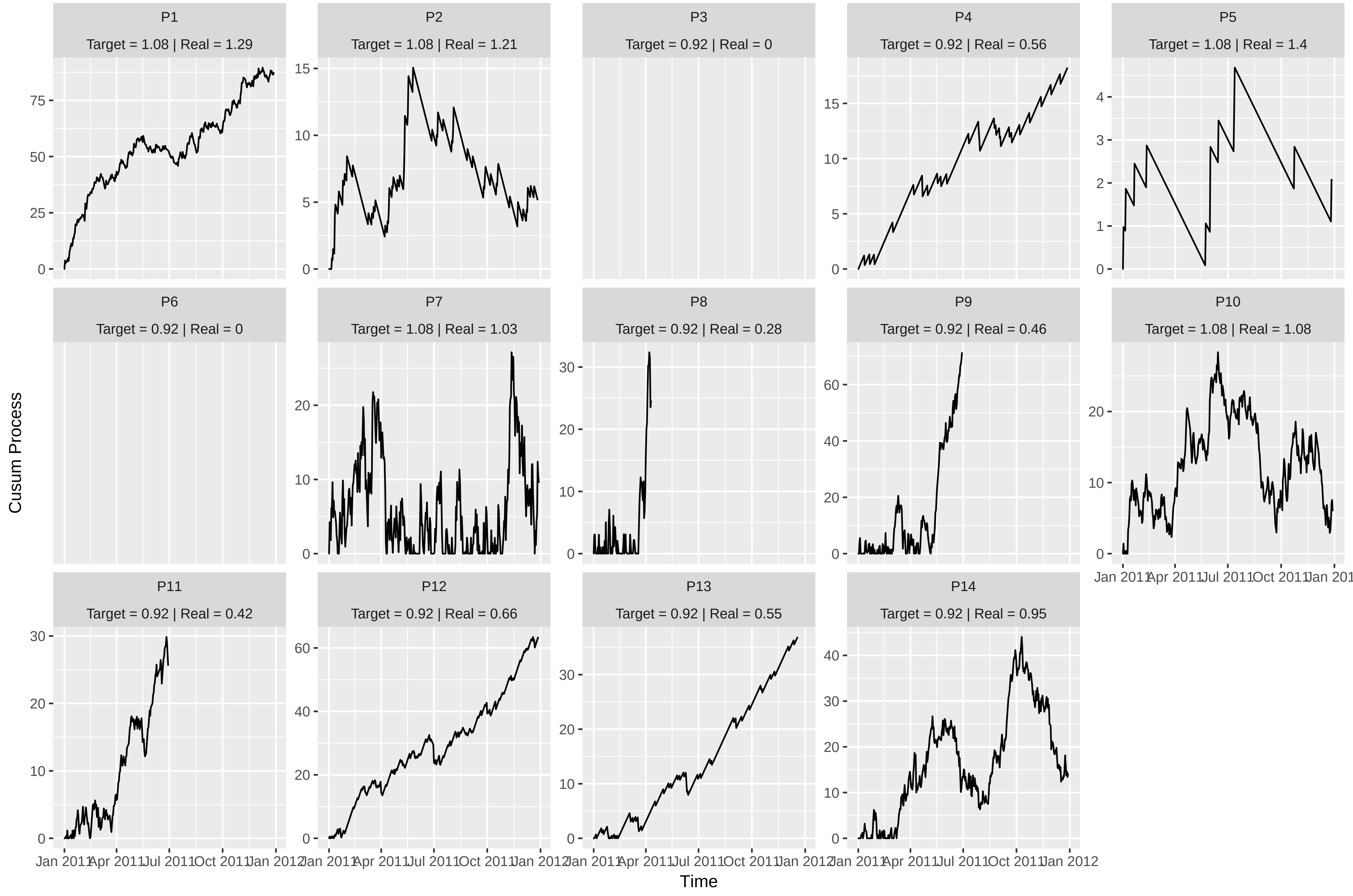


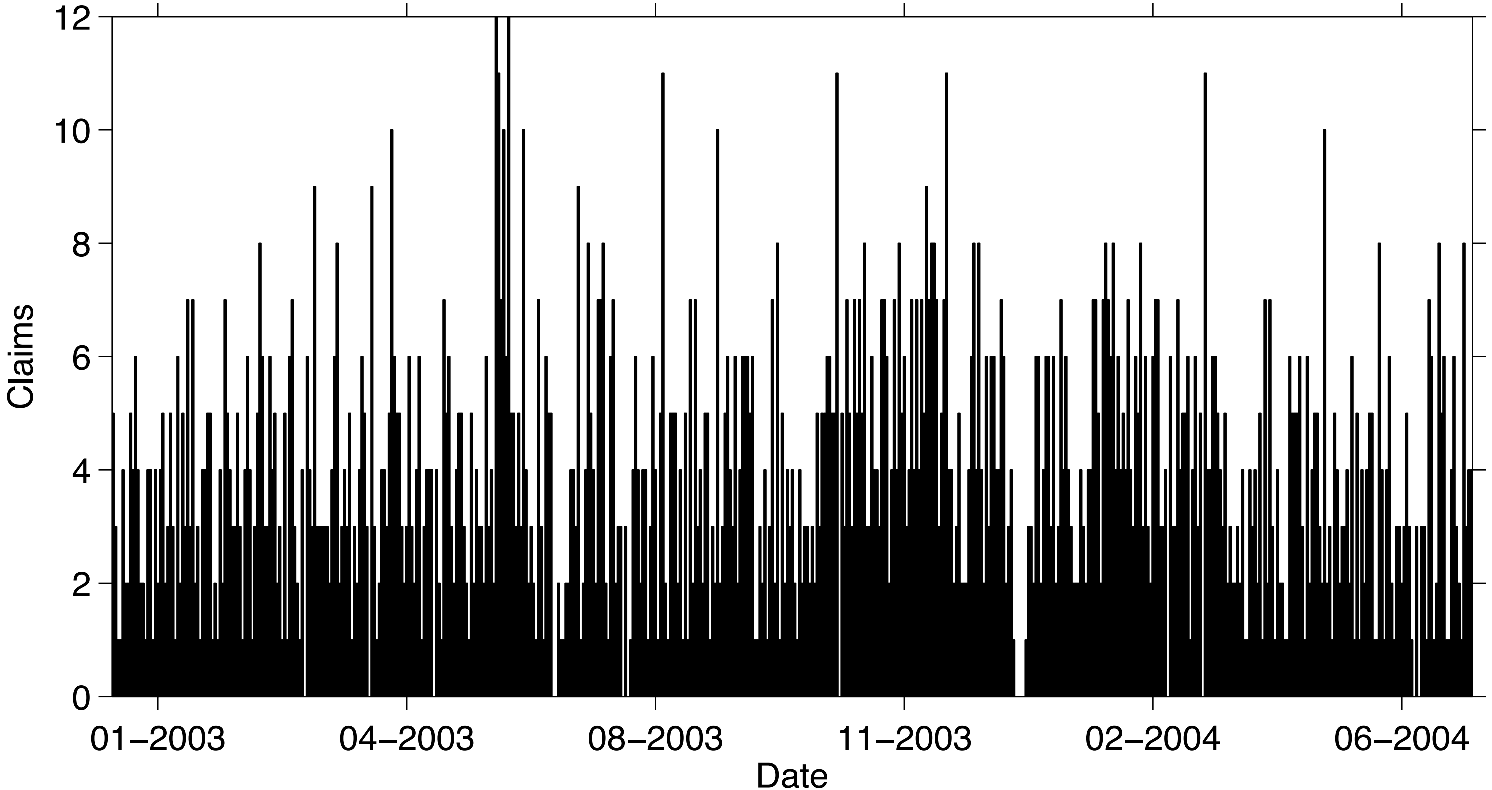
400
350
300
250
200
150
100
50
0
Detection Delay
0
100
200
300
400
500
600
700
800
900
1000
False Alarm
$\rho = 0.99$
$\rho = 0.95$

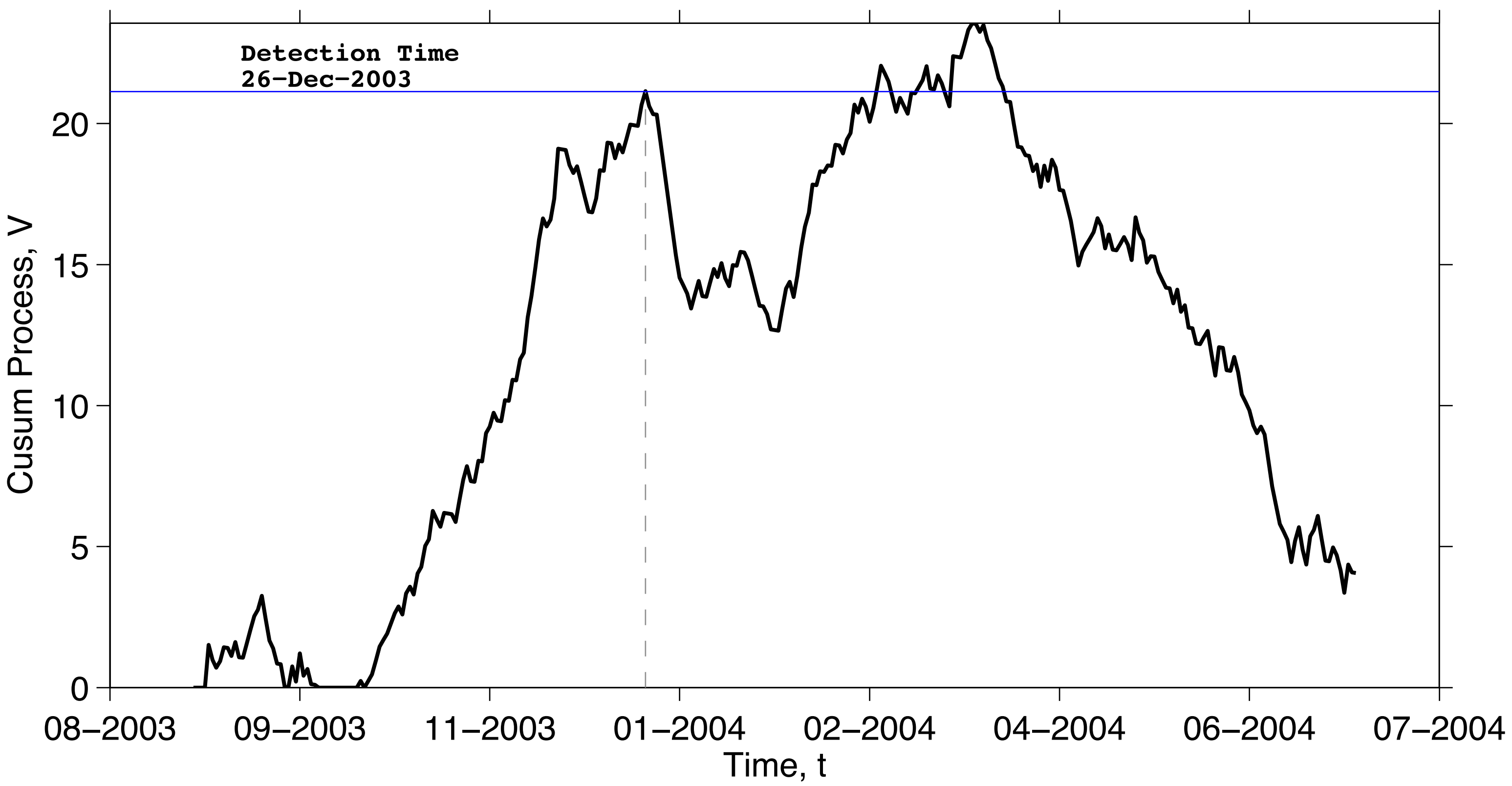

P1
P2
P3
P4
P5
P6
P7
P8
P9
P10
P11
P12
P13
P14
Integer Age
Exposures
90
80
70
60
−0.05
0.00
0.05
Gender
Female
Male



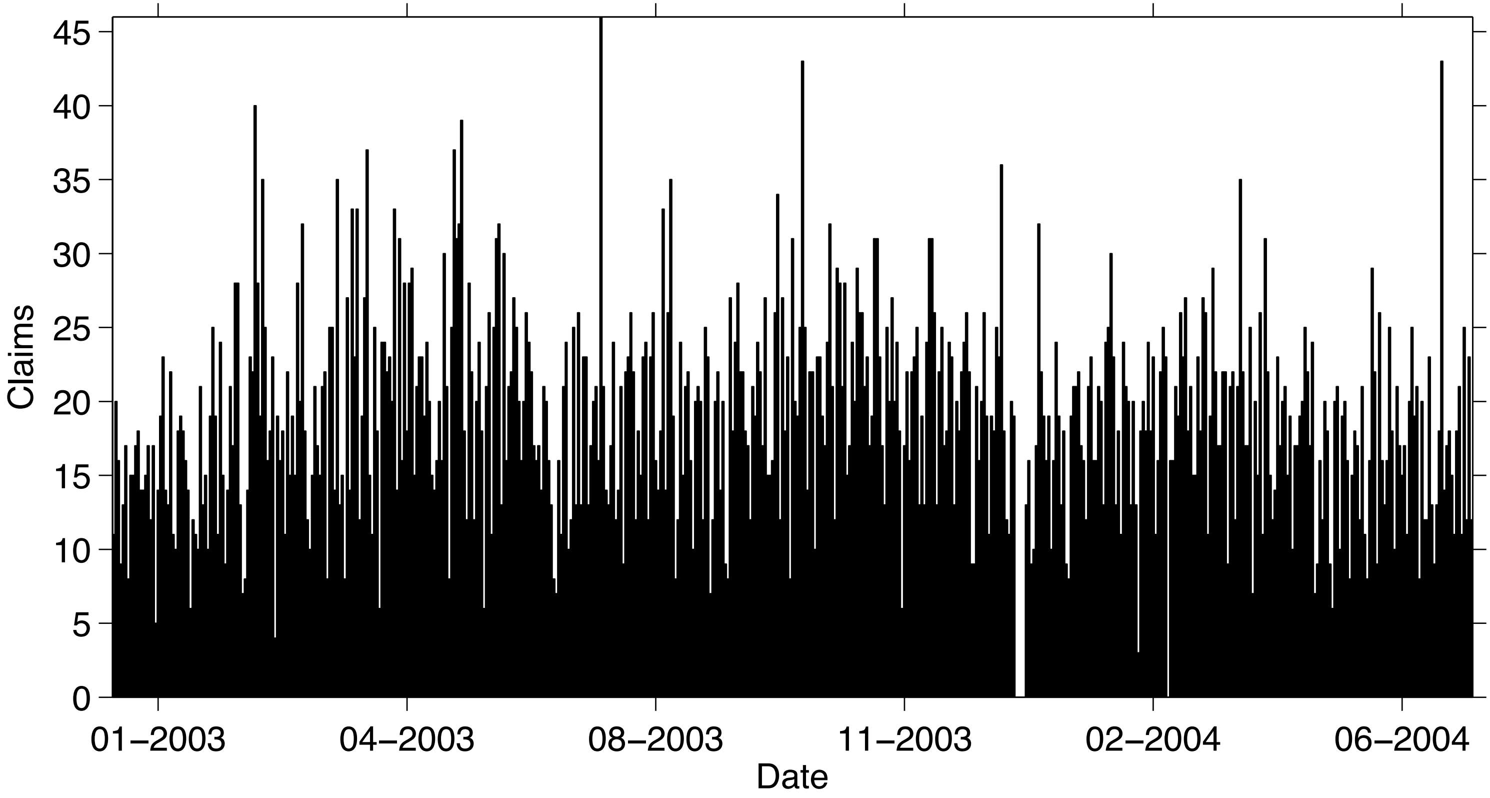
P1
P2
P3
P4
P5
P6
P7
P8
P9
P10
P11
P12
P13
P14
Value
Type
All
Surv
Train
Gender
Female
Male

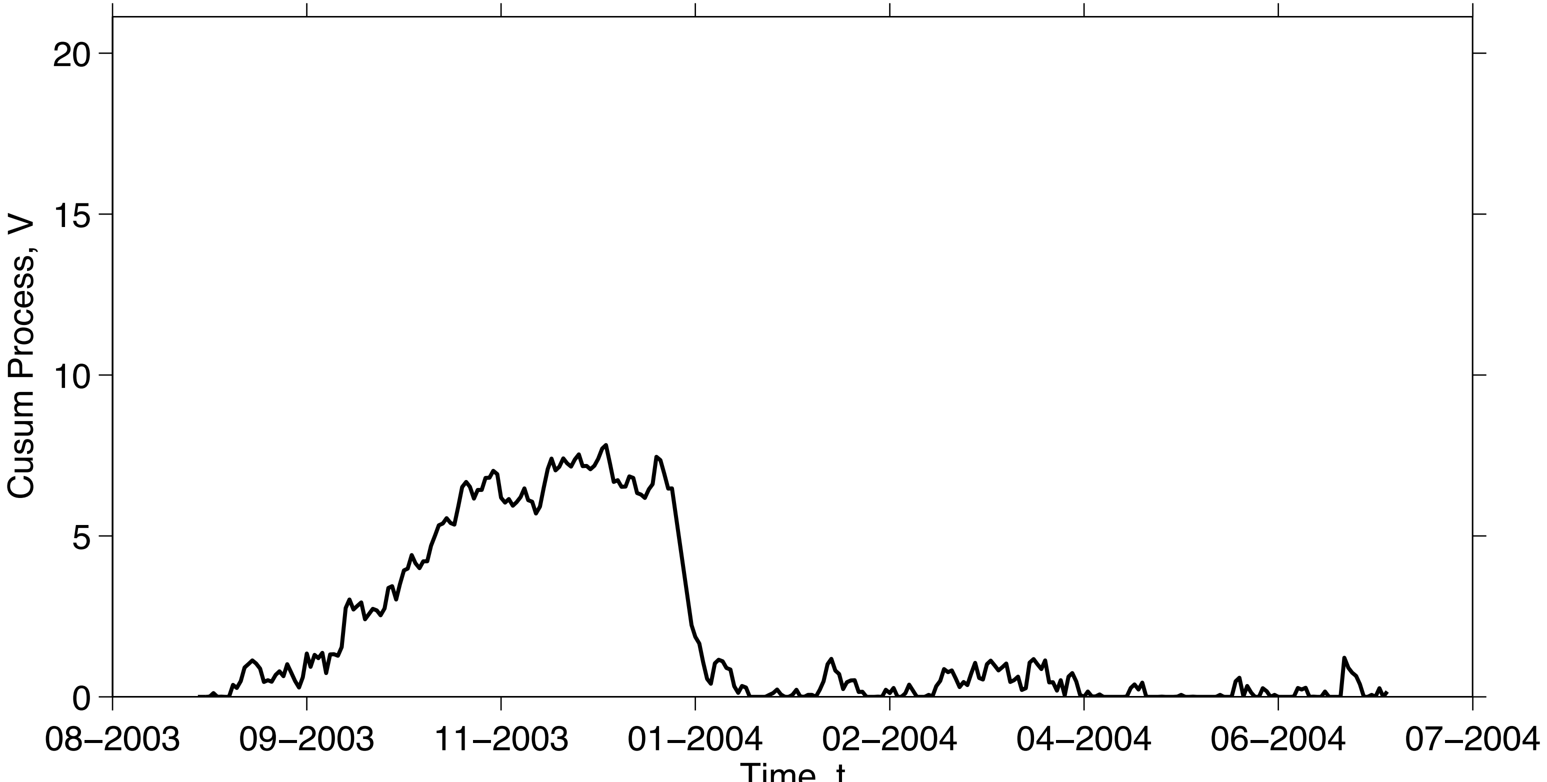

P1
Target = 1.08 | Real = 1.29
P2
Target = 1.08 | Real = 1.21
P3
Target = 0.92 | Real = 0
P4
Target = 0.92 | Real = 0.56
P5
Target = 1.08 | Real = 1.4
P6
Target = 0.92 | Real = 0
P7
Target = 1.08 | Real = 1.03
P8
Target = 0.92 | Real = 0.28
P9
Target = 0.92 | Real = 0.46
P10
Target = 1.08 | Real = 1.08
P11
Target = 0.92 | Real = 0.42
P12
Target = 0.92 | Real = 0.66
P13
Target = 0.92 | Real = 0.55
P14
Target = 0.92 | Real = 0.95
Cusum Process
Time
Jan 2011
Apr 2011
Jul 2011
Oct 2011
Jan 2012


12
10
8
6
4
2
0
Claims
01−2003
04−2003
08−2003
11−2003
02−2004
06−2004
Date



Detection Time
26–Dec–2003
Cusum Process, V
0
5
10
15
20
08–2003
09–2003
11–2003
01–2004
02–2004
04–2004
06–2004
07–2004
Time, t


Claims
0
5
10
15
20
25
30
35
40
45
01−2003
04−2003
08−2003
11−2003
02−2004
06−2004
Date



Cusum Process, V
20
15
10
5
0
08−2003
09−2003
11−2003
01−2004
02−2004
04−2004
06−2004
07−2004
Time, t

Perspectives

- LoLitA closing international conference, Paris, Jan. 15-16-17, 2018
- LoLitA Lecture Notes
- Own Longevity of LoLitA: after 2018