



Population Structure: Impact on Asset Values


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Motivation

- Baby boomers entering retirement
 - concerns about diminished returns, compromised pensions
- Higher old-age dependency ratio may lead to
 - less saving (dissaving) and investment
 - shift of individuals' asset allocations toward low risk / low return assets
 - reduced labour force growth
- All with implications for asset returns and retirement outcomes

Model Framework

- Overlapping Generations Model (OLG) with
 - aggregate uncertainty
 - two asset classes (risky and risk-free)
 - multi-pillar pension systems (savings, pay-go, earnings based)
 - endogenous labour supply
- Generates standard age-specific labour, consumption, asset holdings, and portfolio allocation qualitatively consistent with the data
- Older population  moderately lower asset returns

Demographics

- Overlapping generations, $j \in \{1, 2, \dots, 20\}$, ages 18 - 97
- Five life stages: YW, MW, W, SR, R
- Intra-cohort heterogeneity, $i \in \{1, 2\}$, baseline $i = 1$
- Fertility rate: n
- Survival probability: $\phi_j^i \in [0, 1), \phi_J^i = 0$

$$N_{j,t}^i = (1+n)X^i N_{0,t-1}, \quad \text{if } j = 1$$

$$N_{j,t}^i = \phi_{j-1}^i X^i N_{j-1,t-1}, \quad \text{if } 1 < j \leq J$$

Household Time Endowment

- $$H_j = H(1 - FC_j - FE_j), \quad j \in \{YW, MW\},$$
$$H_j = H, \quad j \in \{W, SR, R\}.$$

- Fixed constant H units of time
- Education (FE) and child rearing (FC)
- SR can work maximum of $\iota_p H$

Household Preferences

- Periodic utility from Consumption and Leisure

$$u^i(c, h) = \frac{c^{1-\gamma_c}}{1-\gamma_c} + \Psi \frac{(H_j - h)^{1-\gamma_h}}{1-\gamma_h}$$

- Coefficient of relative risk aversion: γ_c
- Parameter that regulates the Frisch elasticity of labour supply: γ_h
- Utility weight of leisure relative to consumption: Ψ

Assets

- Total asset holdings: $\theta_{j,t}^i$

Risk Free Bonds

- Return in period t+1: \bar{r}_t
- Share of total assets in risk free: $\eta_{j,t}^i$
- Zero net supply: $\sum_j \sum_i \eta_{j,t}^i \theta_{j,t}^i N_{j,t}^i = 0$

Risky Capital

- Return in period t+1: r_{t+1}
- Share of total assets: $1 - \eta_{j,t}^i$
- Total capital: $K_t = \sum_j \sum_i (1 - \eta_{j,t}^i) \theta_{j,t-1}^i N_{j,t-1}^i$

Production

- $Y_t = z_t K_t^\alpha H_t^{1-\alpha}$ and $K_{t+1} = (1 - \delta)K_t + q_t I_t$

$$\ln(z_t) = \rho \ln(z_{t-1}) + v_t \text{ where } v_t \sim N(0, \sigma_z^2)$$

$$\ln(q_t) = \rho_q \ln(q_{t-1}) + v_{q,t} \text{ where } v_{q,t} \sim N(0, \sigma_q^2)$$

$$\text{corr}(\sigma_q^2, \sigma_z^2) = 0$$

- Aggregate efficient labour is: $H_t = \sum_j \sum_i \epsilon_j^i h_{j,t}^i N_{j,t}^i$
- Baseline: $\epsilon_j^i = 1$ \longrightarrow no age or type-specific labour productivity

Pay-as-you-go Pension

- Pay-as-you-go proportional pension scheme

$$p_{j,t} = 0, \quad \text{if } j \in \{YW, MW, W\},$$

$$p_{j,t} = \frac{\tau_s w_t H_t}{\sum_{j \in \{OW, R\}} \sum_i N_{j,t}^i} \quad \text{if } j \in \{SR, R\}.$$

- Fixed tax, τ_s , on labour income uniformly distributed to retirees

Partially Funded Pension

- Partially funded, employment earnings based pension

$$p_{j,t}^G = 0, \quad \text{if } j \in \{YW, MW, W\},$$

$$p_{j,t}^G = \kappa_j \left(\frac{w_{ss} \sum_i \epsilon_{SR-1}^i h_{SR-1,SS}^i N_{SR-1,SS}^i}{\sum_i N_{SR-1,SS}^i} \right) \quad \text{if } j \in \{SR, R\}.$$

- Government taxes working cohorts at rate, τ_s^G , and pays out a fraction, κ_j , of pre-retirement income

Government Budget

- *In the two pillar model:*

$$\sum_{j=SR}^W p_j^G N_{j,t}^i = [\eta_G (1 + (1 - \tau_r) \bar{r}_{t-1}) + (1 - \eta_G) (1 + (1 - \tau_r) r_t)] \theta_G \\ + \tau_s^G w_t H_t + B_t^G$$

In the three pillar model:

$$\sum_j \sum_i \eta_{j,t}^i \theta_{j,t}^i N_{j,t}^i + \eta_G \theta_G = B_t^G$$

$$K_t = \sum_j \sum_i (1 - \eta_{j,t}^i) \theta_{j,t-1}^i N_{j,t-1}^i + (1 - \eta_G) \theta_G$$

- Government holds a pool of assets, θ_G , with proportion, η_G , in risk-free bonds, and issues bonds, B_t^G , to balance the budget.

Taxes and Bequests

- *Taxes*

- Consumption tax: τ_c
- Labour income tax: τ_h
- Investment income tax: τ_r
- Tax on pension income: τ_p
- Tax for pay-go pension and social security: τ_s and τ_s^G

- *Bequests*

- Base model has accidental bequests only
- Bequest motive – utility from leaving bequest $v(X) = \Gamma \frac{X^{1-\gamma_b}}{1-\gamma_b}$

Household Decision

- $V_j^i(s_t; z_t) = \max [c_{j,t}^i, h_{j,t}^i, \theta_{j,t}^i, \eta_{j,t}^i] \left\{ u^i(c_{j,t}^i, h_{j,t}^i) + \beta \phi_j^i E_t [V_{j+1}^i(s_{t+1}; z_{t+1})] \right\}$

subject to

$$(1 + \tau_c) c_{j,t}^i + \theta_{j,t}^i \leq \left\{ (1 - \tau_s - \tau_s^G - \tau_h) w_t \epsilon_j^i h_{j,t}^i + x_{j,t}^i + (1 - \tau_p)(p_{j,t} + p_j^G) + \xi_t - HC \right\}$$

where

$$h_{j,t}^i \leq H_j^c = \begin{cases} H_j & \text{if } j \in \{YW, MW, W\} \\ \tau_p H_j & \text{if } j \in \{SR\} \\ 0 & \text{if } j \in \{R\} \end{cases}$$

$$HC_j = 0 \quad \text{if } j \in \{YW, MW, W\}$$

$$HC_j = 0.2 \exp\left(\frac{4(j-12)}{J-12} - 4\right) \quad j \in \{SR, R\}$$

Household Decision - Oldest Generation

- $V_J^i(s_t; z_t) = \max [c_{j,t}^i, \theta_{j,t}^i, \eta_{j,t}^i] \{ u^i(c_{j,t}^i, 0) + \beta E_t [v^i(X_{J+1,t+1}^i)] \}$

where

$$X_{J+1,t+1}^i = [\eta_{j,t}^i (1 + (1 - \tau_r) \bar{r}_t) + (1 - \eta_{j,t}^i) (1 + (1 - \tau_r) r_{t+1})] \theta_{j,t}^i$$

and

$$v(X) = \Gamma \frac{X^{1-\gamma_b}}{1-\gamma_b}$$

Firm Decision

- Firm maximizes profits, resulting in:

$$r_t = \alpha z_t K_t^{\alpha-1} H_t^{1-\alpha} - \delta$$

$$w_t = (1 - \alpha) z_t K_t^\alpha H_t^{-\alpha}$$

where $\delta \in [0,1]$

Recursive Competitive Equilibrium

- Value functions
- Household policy functions for consumption, labour supply, total saving, and share of savings invested in risk-free bonds
- Inputs (capital and hours of labour) for the representative firm
- Government policy for pensions and bond issuance
- Rates of return for risk-free bonds and risk capital, and wages

Such that in each period, the:

- household problems are solved,
- the competitive firm maximizes profits, and
- all markets clear.

Lifecycle Consumption, Labour, Assets

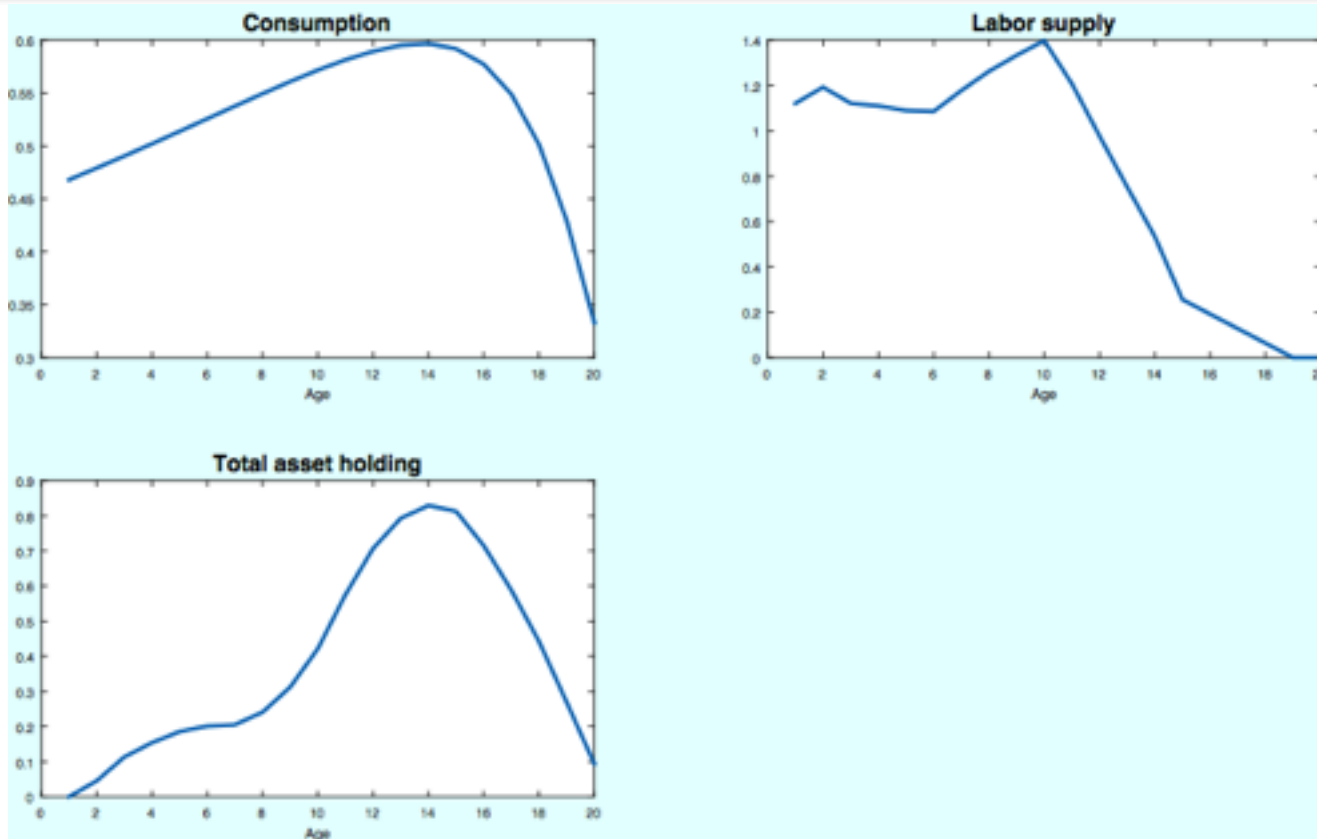


Figure 1 - Lifecycle consumption, labour and asset profiles

Observed Age-Specific Portfolio Allocation

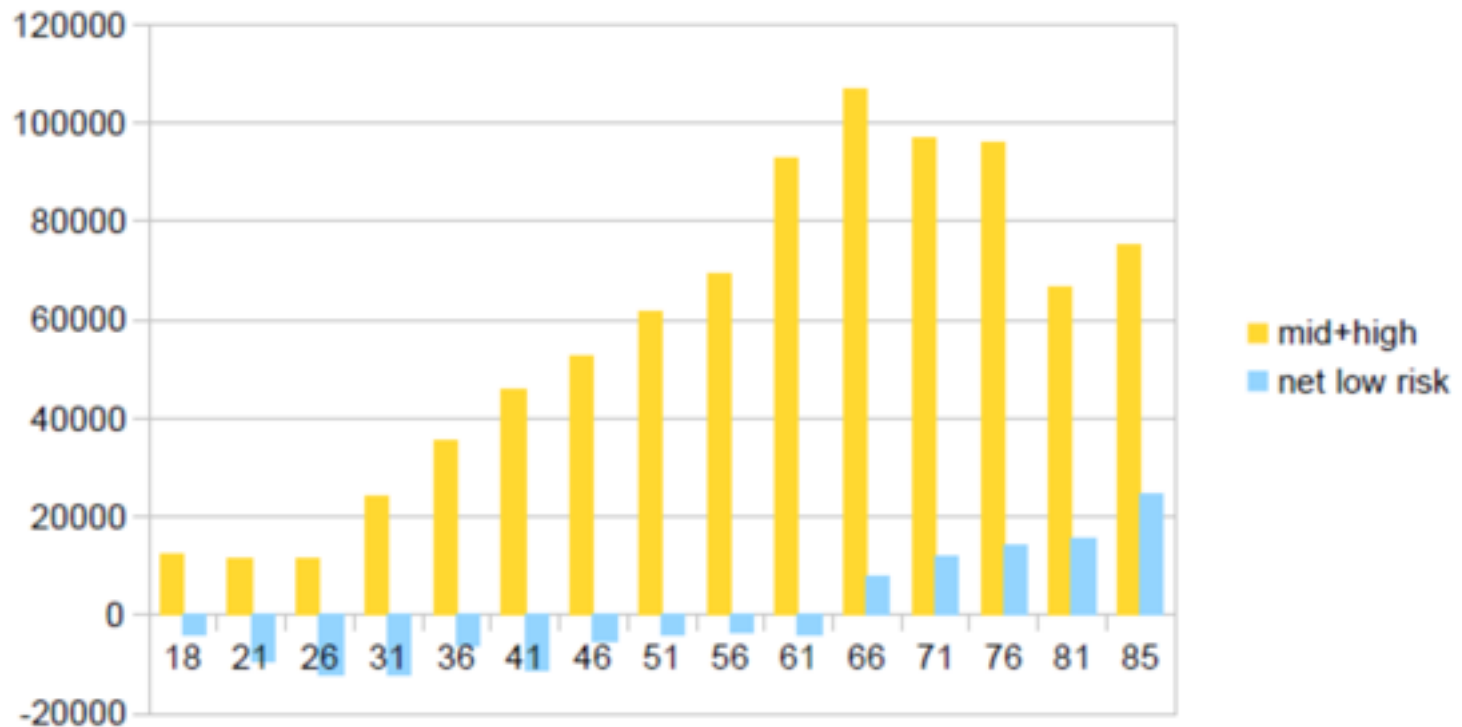


Figure 2 - Portfolio allocation by age: risky vs net low-risk financial assets

Portfolio Allocation – 2 Pillar Pension Model

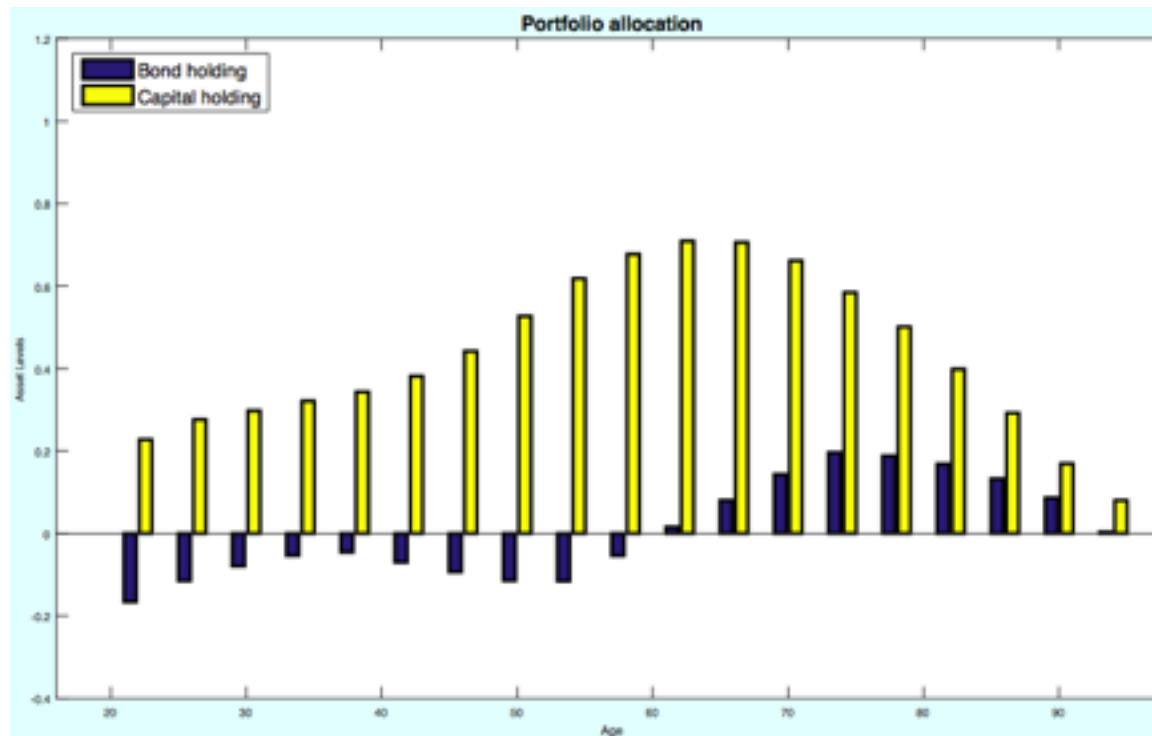


Figure 3 - Portfolio allocation in 2 pillar model

Portfolio Allocation

3 Pillar Pension Model – Baseline

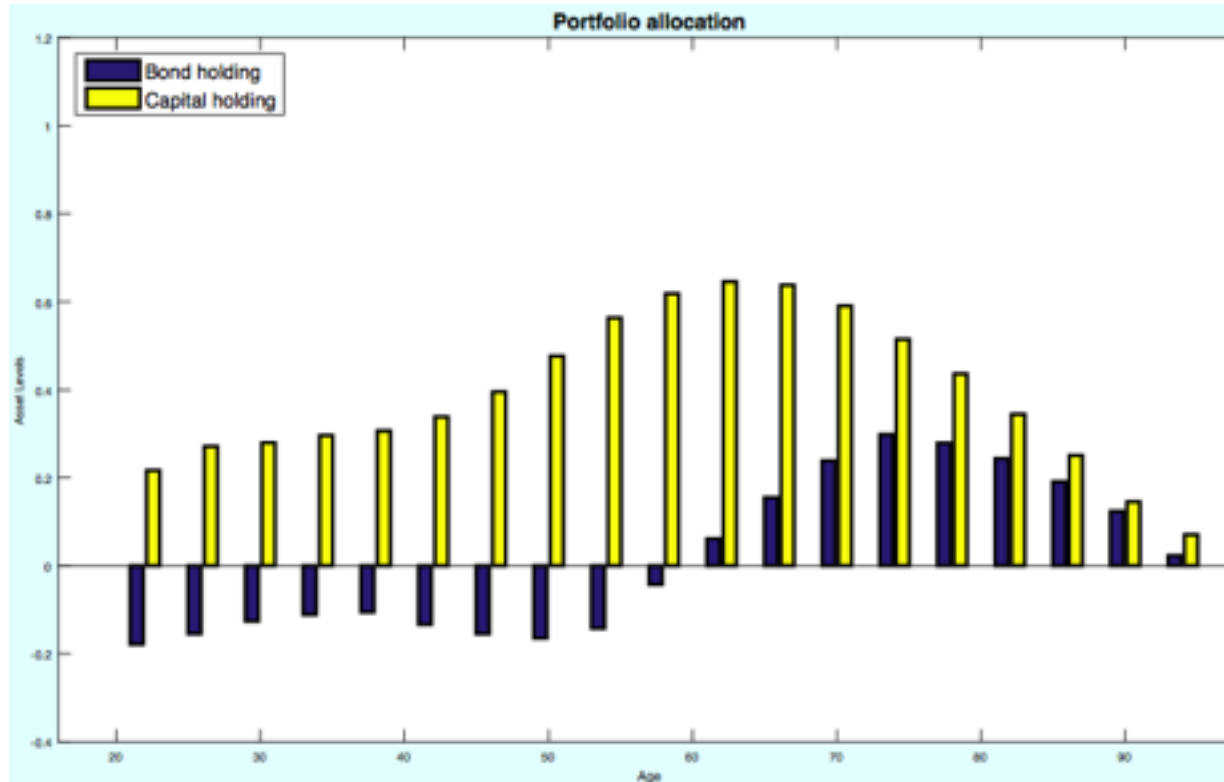


Figure 4 - Portfolio allocation in 3 pillar model

3-Pillar Model Results under Alternative Demographic Structures

Variable	Base-3 Pillar	+10%	+20%	-10%	-20%
$E_t(r_{t+1})$	0.2855	0.2788	0.2735	0.2919	0.2965
\bar{r}_t	0.2851	0.2784	0.2730	0.2915	0.2961
Priv. risky assets / GDP	0.5223	0.5233	0.5362	0.5214	0.5206
$C_{20,t}$	0.3327	0.3771	0.4183	0.2984	0.2512

Portfolio Allocation

3 Pillar + Health Costs + Bequests

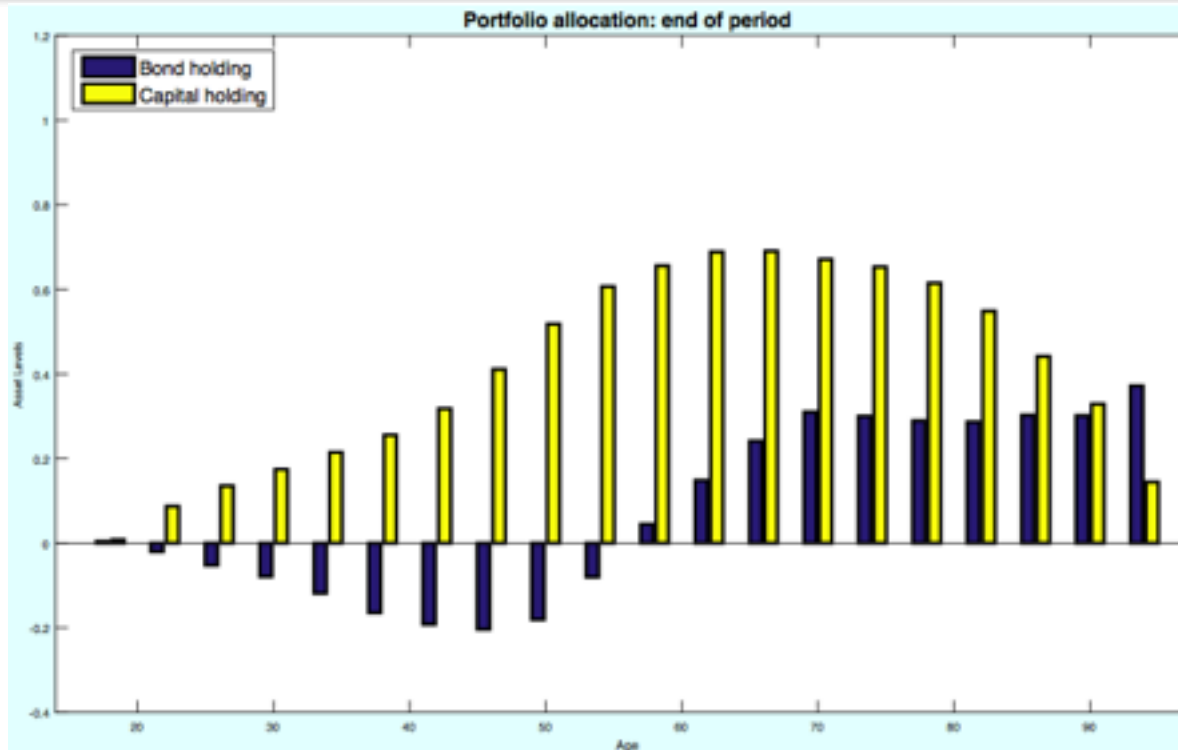


Figure 5 - Portfolio allocation 3 pillar + health costs + bequests

Discussion and Next Steps

- Asset prices are moderately lower with older population: Higher survival probability for age 65+ (max 20% at $j=J$)
→ approximately 4% lower returns on capital and on bonds
- Higher replacement ratio → lower asset accumulation

Next Steps:

- Improve portfolio allocation match
 - consumption saturation
 - intra-cohort heterogeneity
- Explore further intra-cohort heterogeneity models

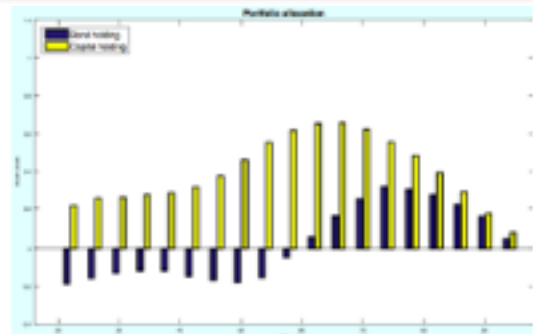
Appendix



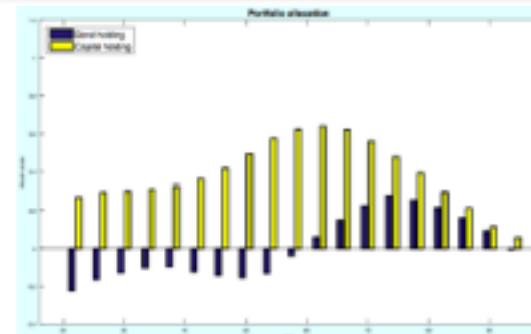
Parameterization

Parameter	Value	Description
H	4	Time available to household (one period represents 4 yrs)
β	0.8515	Discount factor (0.95 annual)
α	0.3	Capital's share of production
ρ_z	0.4401	Autocorrelation coefficient for TFP
σ_z	0.0305	Std. Deviation of error for TFP process
ρ_q	0.4401	Autocorrelation coefficient for IST
σ_q	0.1221	Std. Deviation of error for IST process
δ	0.192	Depreciation Rate
n	0.0489	Population Growth rate
γ_c	2.0	Relative risk aversion – consumption
γ_b	2.0	Relative risk aversion - bequest
γ_l	3.0	Inverse of intertemporal elasticity of substitution of non-market time
Ψ	21.833	Utility weight of non-market time relative to consumption
τ_c, τ_i, τ_p	0.123, 0.167, 0.167	Tax rates on consumption, investment income, pension,
$\tau_h + \tau_s + \tau_s^G$	0.167	Tax on labour income
ratio _s	1.0	Proportion of labour tax to social security
τ_p	0.08	Labour constraint for SR

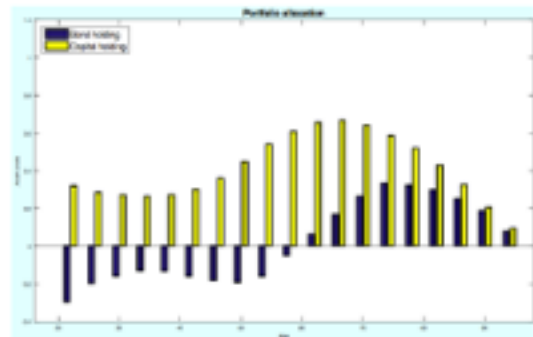
Portfolio Allocation under Alternative Demographic Structures



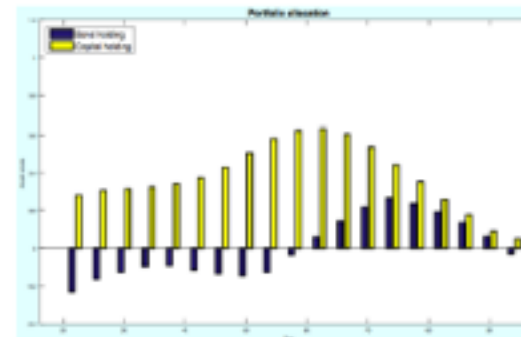
(a) Baseline + 10% maximal higher survival probability



(b) Baseline - 10% maximal higher survival probability



(c) Baseline + 20% maximal higher survival probability



(d) Baseline - 20% maximal higher survival probability

Figure 6 - Portfolio allocation alternative demographics



**THANK YOU FOR YOUR
ATTENTION!**