Population Structure and Asset Values

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Abstract

With the large baby-boom cohort entering retirement, many are concerned that the expected drop in saving and investment will result in substantially diminished asset prices and compromised pension plans. This paper contributes to the quantification of the link between population structure and asset values, by generating returns on assets in the presence of demographic change. We carry this out in the context of a large scale computable Overlapping Generations Model (OLG) with endogenous labour supply, aggregate risk, and two asset classes. Our model generates typical age specific asset holdings and consumption patterns, and results in age specific portfolio allocations consistent with the data. We use counterfactuals to predict the outcome of changes in demographic structure, and find that asset prices are moderately lower with an older population. Specifically, a 4% increase in the survival probability of households over age 65 results in a 4.16% drop in the return on capital, and a 3.02% drop in the return on bonds. While our baseline model employs a two-pillar pension system (a pay-go public provision plus private saving), we also explore a three-pillar pension system (adding a publicly administered, partially funded, employment-related plan) and consider the implications of tax and pension policy on economic outcomes. This framework will be helpful to assess the implications of the proposed expansion of the Canada Pension Plan. Additional modifications include a bequest motive and age dependent healthcare costs.

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1 Introduction

The large baby-boom cohort, which has affected economic growth for six decades, has just started to enter retirement. With a higher old-age dependency ratio, economies may expect significant implications for the asset markets, the labour market, and long term growth. Specifically, a major concern is that with an aging population, there will be less saving and investment (and a shift in asset allocation), which could severely diminish asset prices. These concerns have spurred research on the impact of population aging on asset prices (e.g. Mankiw and Weil, 1989; Poterba, 2001 Börsch-Supan et al., 2006; Cornell, 2012; Kang, 2013). In addition to concerns over asset prices, large-scale retirement is associated with reduced labour force growth, and with dissaving (Beach, 2008). Policy makers and academics alike have raised concerns over the ability of pension plans and private savings in Canada to meet comfortable retirement income targets (Ambachtsheer, 2009; Horner, 2009). The ability of savings to meet targets could be further compromised in the event of depressed asset prices, or an asset price meltdown.[1]

This paper, seeks to quantify the impact of population structure on asset values and to consider the effects of different tax and pension parameters on pension outcomes. Because the high old-age dependency ratio is at the heart of policy makers concerns, we develop a computable Overlapping Generations Model (OLG) to explore the implications of an older demographic structure on economic outcomes. We calibrate and simulate a 20-period OLG life-cycle model with endogenous labour supply, aggregate uncertainty, two asset classes (risky and risk-free), and a simple two pillar pension system (a public pay-go plan, and private savings). This model generates typical age specific asset holdings and consumption patterns, and results in age specific portfolio allocations consistent with the data. We construct counterfactuals to consider what happens if the population structure is altered, and find that asset prices are moderately lower with an older population. Specifically, a 4% increase in the survival probability of households over age 65 results in a 4.16% drop in the return on capital, and a 3.02% drop in the return on bonds.

Within the retirement literature, there is also an ongoing discussion about what policies could secure better retirement prospects for Canadians (Ambachtsheer, 2009). Ideas

[1]However, we note that there is little consensus in the literature on the potential impact of population aging on asset values. Study conclusions range from catastrophic impact, described as "asset meltdown" (Brook, 1998; Mankiw and Weil, 1989; Kang, 2013); to moderate effect on the markets but no meltdown (Abel, 2003; Andrews et al., 2014; Börsch-Supan et al., 2006; Börsch-Supan, 2006; Campbell, 2001; Geanakoplos et al., 2004; Liu and Spiegel, 2011; Poterba, 2004; Schieber and Shoven, 1994); to a total rejection of the asset meltdown hypothesis (Kedar-Levy, 2006; Green and Hendershott, 1996; Cornell, 2012; Bovbjerg and Scott, 2006).

range from lower tax rates on investments to stimulate saving (for example, tax-preferred accounts with low or deferred tax rates), combinations of defined benefit and defined contribution structures, increased contributions to the publicly administered employment related Canada/Quebec Pension Plans[2], and increased outlays of the pay-as-you-go public pension system (Old Age Security and Guaranteed Income Supplement). These policy tools could influence the saving and investment decisions of households as well as market returns and economic growth.

As such, we explore alternative pension systems within the framework of our model. Our baseline model employs a two-pillar pension system (a pay-go public provision plus private saving), subsequently we explore a three-pillar pension system by adding a publicly administered, partially funded, employment-related plan. Compared to the two-pillar system, the three-pillar system generates lower private investment, and reduced total asset holdings among older cohorts, but has a smaller impact on consumption.

Both the two pillar and three pillar models predict a standard life-cycle decumulation of assets at end of life. However, data from the 2013 Survey of Consumer Finances suggests only weak asset decumulation in older ages. Similar findings are reported in De Nardi et al. (2010) who note that many elderly retain substantial assets. Controlling for cohort effects, Poterba (2001) also reports scant decumulation of net financial assets among older ages. Moreover, Poterba et al. (2006) find that, contrary to the recommended reduction in portfolio risk in later life-stages, households do not substantially reduce equity exposure with age.

Recent literature suggests that healthcare costs (out of pocket medical expenses and nursing home costs), and voluntary bequests may be key to explaining the lack of significant asset decumulation among older ages (e.g. De Nardi et al., 2010; De Nardi and Yang, 2014; De Nardi and Fella, 2016; Kopecky and Koreshkova, 2014). To match better to the weak decumulation of assets and the retention of portfolio risk near the end of life, we expand on our three pillar model to include a simple voluntary bequest motive. We further restrict the household budget by requiring payment of medical expenses that increase exponentially in age, past retirement.

With a simple voluntary bequest motive, the three-pillar model predicts weaker asset decumulation, in particular for non-risky assets. With both bequest motive and healthcare costs, we see a further increase in the amount of risky and risk free assets retained among

[2] Indeed, in 2016, the Canada Revenue Agency announced changes to the Canada Pension Plan that aim to increase the targetted replacement rate from 25% to 33% of pre-retirement earnings. This increase will be funded by higher contribution rates as well as an increase in the maximum pensionable earnings.

retired households. The model does not yet quantitatively match portfolio risk among the eldest cohorts. We can generate the observed increase in risk-free asset holding, however, the model predicts a steeper decumulation of risky-assets than is observed in the data.

This paper makes three specific contributions to the literature. First, while a handful of empirical studies have found associations between demographics and specific asset classes (Ang and Maddaloni, 2003; Bakshi and Chen, 1994; Goyal, 2004; Poterba, 2001; Poterba, 2004), few studies integrate demographic structure in a model with more than one asset class (see Brooks, 2000; Bucciol and Beetsma, 2011; Černỳ et al., 2006; Hasanhodzic and Kotlikoff, 2015; Muto et al., 2012; Reiter, 2015; Xu, 2013). Incorporating both a risk-free and a risky asset, with endogenous returns, help us better estimate the effect of a high old-age dependency ratio on asset prices, and on the financial health of various cohorts within the population. To the best of our knowledge, this has not been done previously.

Second, our baseline model employs a standard two-pillar pension system; however we then expand this framework to a three-pillar pension system which includes a publicly administered, partially funded, employment-related pension. This framework will be helpful to assess the implications of the proposed expansion of the Canada Pension Plan. Finally, we devote considerable attention to generating age-specific portfolio allocations that are consistent with the data. Our model does a reasonable job in this regard, with the exception of the oldest cohorts, and we explore additional mechanisms to address this concern (e.g. bequest motive, and healthcare costs).

This paper[3] follows the methodology used in Hasanhodzic and Kotlikoff (2015), which use a simulation approach in the spirit of the generalized stochastic simulation algorithm (GSSA) by Judd et al. (2009, 2011). We calibrate and simulate a 20-period overlapping generations (OLG) life-cycle model with aggregate productivity shocks.

The base model includes a pay-go pension system. A second section will additionally include a partially funded pension. The pension payment and government portfolio of this system are both exogenous. We calibrate (on public asset allocation) the model such that at the steady state, the magnitude of the pay-go versus funded system approximates the country's current levels of pay-go versus funded pension relative to the size of the economy. We then contrast results for the expanded pension model under different replacement rates (to investigate impact of CPP expansion).

[3]The detail of the algorithm used in this paper is available upon request.

2 Model environment

2.1 Demographics

Time is discrete and goes forever. During each time period, the household sector is made of J overlapping generations, of age between 18 and 97. We use $j \in \{1, 2, \ldots, J\}$ to denote cohorts' age. Moreover, households are generally categoried into five life stages: young-working (YW), middle-working (MW), mature-working (W), semi-retired (SR), and retirement (R), corresponding to age groups $\{18-33, 34-49, 50-65, 66-81, 82-97\}$ respectively.

Let $N_{j,t}$ represent the size of generation j in period t. In our baseline model, there is no heterogeneity within each cohort, later versions of our model will incorporate heterogeneity (e.g. in terms of gender, productivity and education). We use a representive household, which has a size of $N^i_{j,t}$, to characterize type i households at age j in period t, where $i \in \{1, \ldots, I\}$. In the model with no intra-cohort heterogeneity, $i = 1$ and is representative of the average household of age j in period t.

In each period t, a new generation aged $j = 1$ is born into the economy, while the other existing generations each shifts forward by one. The exogenous growth rate of the new generation $j = 1$ is denoted by n. Each type i household at age j has an exogenous marginal probability ϕ^i_j of reaching age $j+1$ in period $t+1$. The oldest generation, $j = J$, dies out deterministically in the subsequent period, i.e. $\phi^i_J = 0$. Then, the demographic stucture in period t is expressed as below:

$$N^i_{j,t} = \begin{cases} (1+n)\chi^i N_{0,t-1}, & \text{if } j = 1, \\ \phi^i_{j-1}\chi^i N_{j-1,t-1}, & \text{if } 1 < j \leq J. \end{cases}$$

where χ^i is the proportion of type i households within a generation. In our basic model, χ^i is constant across generations.

2.2 Households

At each age, each household has a fixed constant H units of time to spend on labour and leisure. In addition, at their YW and MW stages, a household at age j mandatorily spends FC_j percent of H units of time per period on fertility (which can be thought of as time required for child rearing). Similarly, the household is required to take FE_j percent of H units of time on education. Let H_j denote the total available time that can be allocated between labour and leisure for households at age j.

$$H_j = \begin{cases} H\left(1 - FC_j - FE_j\right), & \text{if } j \in \{YW, MW\}, \\ H, & \text{if } j \in \{W, SR, R\}. \end{cases} \quad (2.1)$$

In all working ages, a household decides how much labour to supply to firms and earns wage income according to its labour efficiency ε_j^i, which is exogenously given. Starting from the SR stage, the household receives pension income. At its SR stage, in addition to receiving the pension, the household determines how much labour to supply out of a restricted $\iota_p H$ units of time. Thus, ι_p is the maximum fraction of the period that an SR household may work. Retirees supply zero labour and enjoy all available time as leisure with pension income.

Households value both consumption and leisure according to the following periodic utility function:

$$u^i\left(c, h\right) = \frac{c^{1-\gamma_c}}{1-\gamma_c} + \Psi \frac{\left(H_j - h\right)^{1-\gamma_h}}{1-\gamma_h}$$

where c and h denote consumption and labour supplied, respectively. γ_c represents the relative risk-aversion and γ_h represents the parameter that regulates the Frisch elasticity of labour supply. Ψ represents the utility weight of leisure relative to market consumption.

Following Neusser (1993), we assume that for the oldest cohort, leaving wealth to other generations generates utility irrespective of the well-being of the heirs. This is the simplest way of introducing a bequest motive in a model in which no such motive can arise endogenously. We introduce this motive to match more closely to observed asset holdings at end of life.

2.3 Assets

Households can save and invest in two financial assets: one is a one-period risk-free bond and the other is risky capital (stock). Let $\theta_{j,t}^i$ denote a household's total demand for assets (savings) and $\eta_{j,t}^i$ the share of saving invested in risk-free bonds at the end of period t. There is neither a borrowing constraint on bonds nor a short sale constraint on stock in the basic model. Households who invest one unit of consumption in bonds in period t receive $1 + \bar{r}_t$ units in period $t + 1$with certainty. Note $\bar{r}_t$ is known in period t although it is received in the next period. On the other hand, the return of one unit of consumption invested in capital in period t is r_{t+1}, which is realized in period $t + 1$. Households enter period t with $\theta_{j-1,t-1}$ in assets, which corresponds to the total assets they demanded in the prior period.

Holding risk-free assets can be negative, which reflects the fact that households may borrow. In this basic model, risk-free bonds are in zero net supply, therefore we have:

$$\sum_j \sum_i \eta^i_{j,t} \theta^i_{j,t} N^i_{j,t} = 0. \tag{2.2}$$

Because households' investment decisions are made at the end of each period, the total capital used in production in period t, K_t, is given by

$$K_t = \sum_j \sum_i \left(1 - \eta^i_{j,t-1}\right) \theta^i_{j,t-1} N^i_{j,t-1}. \tag{2.3}$$

New born young workers $j = 1$ enter the economy with zero asset holding, i.e. $\theta^i_{0,t} = 0$. The oldest generation consumes and leaves the economy with asset holdings as a voluntary bequest.

2.4 Production

In each period, a representative firm uses labour H_t, in efficiency units, and physical capital K_t to produce total final goods Y_t. We assume a Cobb-Douglas production function and no adjustment cost on capital:

$$Y_t = z_t K_t^\alpha H_t^{1-\alpha},$$

where $\alpha \in (0, 1)$ is the capital share.

The total factor productivity (TFP) z_t follows a simple AR(1) process:

$$\ln(z_t) = \rho \ln(z_{t-1}) + \nu_t,$$

where $\nu_t \sim N(0, \sigma_z^2)$.

The investment-specific technology shock follows a simple AR(1) process.

$$K_{t+1} = (1 - \delta)K_t + q_t I_t,$$

$$\ln(q_t) = \rho_t \ln(q_{t-1}) + v_{q,t},$$

where $v_{q,t} \sim N(0, \sigma_t^2), \sigma_z$ is uncorrelated with σ_q.

The aggregate amount of efficiency labour in period t, H_t, is given by:

$$H_t = \sum_j \sum_i \varepsilon_j^i h_{j,t}^i N_{j,t}^i, \tag{2.4}$$

where ε_j^i represents age- and type-specific labour productivity. Therefore, $\varepsilon_j^i h_{j,t}^i$ is the efficiency labour supplied by a type i household at age j in period t.

2.5 Government

2.5.1 2-pillar pension system

We first consider a two-pillar pension system. In addition to households' private saving, old households get funds from a pay-as-you-go proportional pension scheme. For the pay-as-you-go scheme, the government takes a fixed percentage, τ_s, of wage from each current worker, and this income is distributed uniformly among the retirees. Let p_t represent the pension income for a retiree in period t:

$$p_{j,t} = \begin{cases} 0, & \text{if } j \in \{YW, MW, W\}, \\ \frac{\tau_s w_t H_t}{\sum_{j\in\{OW,R\}} \sum_i N_{j,t}^i}, & \text{if } j \in \{SR, R\}, \end{cases} \tag{2.5}$$

where w_t is the wage rate paid by the firm in period t.

2.5.2 3-pillar pension system

Now on top of the above 2-pillar pension system, we introduce a partly funded pension system. For this system, we assume the government holds a pool of assets θ_G, with η_G proportion of risk-free bonds and $(1 - \eta_G)$ proportion of risky capital. Note here (θ_G, η_G) are both exogenously given, i.e. we set θ_G to be 10% of steady state GDP and η_G to be 10%. Every period, the government pays the income from its asset holdings to the households, plus a fraction of the tax, τ_s^G, imposed on the working cohorts, to pay out the ratio κ_j of pre-retirement income to the retired cohorts. At this moment, we set the payout to be exactly κ_j percent of the average wage income of the age $j = SR - 1$ generation at the steady state. We think this is reasonable because we use a stationary population structure here and the economy just fluctuates around the steady state. Note κ_j is a flat rate (25%) for all retirees. Therefore we have the funded pension payout as

$$p_j^G = \begin{cases} 0, & \text{if } j \in \{YW, MW, W\}, \\ \kappa_j \left(\frac{w_{SS} \sum_i \varepsilon^i_{SR-1} h^i_{SR-1,SS} N^i_{SR-1,SS}}{\sum_i N^i_{SR-1,SS}} \right), & \text{if } j \in \{SR, R\}, \end{cases} \quad (2.6)$$

and the government's budget (for the funded pension system) as

$$\sum_{j=SR}^{W} p_j^G N_{j,t}^i = \left[\eta_G \left(1 + \left(1 - \tau_r\right) \bar{r}_{t-1}\right) + \left(1 - \eta_G\right) \left(1 + \left(1 - \tau_r\right) r_t\right)\right] \theta_G + \tau_s^G w_t H_t + B_t^G, \quad (2.7)$$

where τ_s^G is therefore the exogenous tax rate imposed on working cohorts in order to pay out the funded pension. B_t^G represents the endogenous amount of bonds that the government issues to balance its budget (2.7) every period. Note if there is population growth, the government needs to maintain θ_G such that it grows at the same pace with the total population.

With the partly funded pension, we modify aggregate assets holdings (2.2) and (2.3) as follows.

$$\sum_j \sum_i \eta_{j,t}^i \theta_{j,t}^i N_{j,t}^i + \eta_G \theta_G = B_t^G,$$

$$K_t = \sum_j \sum_i \left(1 - \eta_{j,t-1}^i\right) \theta_{j,t-1}^i N_{j,t-1}^i + \left(1 - \eta_g\right) \theta_G.$$

2.5.3 Other taxes and accidental bequest

The government also collects taxes from households to be spent on other items, which are not modelled here. These taxes include a labour income tax τ_h (other than τ_s and τ_s^G), a proportional consumption tax (τ_c), an investment tax (τ_r), and a tax on pension income (τ_p).

If a household dies accidentally before the highest age J, its net wealth is collected by the government rather than being inherited. The government collects all residual assets from the fraction of the population that dies and transfers this sum equally to all remaining households. Let ξ_t be the lump-sum transfer associated with accidental bequests that are left by households who die at the end of period $t-1$.

$$\xi_t = \frac{\sum_j \sum_i \left(1 - \phi_{j-1}^i\right) \left[\eta_{j-1,t-1}^i \left(1 + \left(1 - \tau_r\right) \bar{r}_{t-1}\right) + \left(1 - \eta_{j-1.t-1}^i\right) \left(1 + \left(1 - \tau_r\right) r_t\right)\right] \theta_{j-1,t-1}^i N_{j-1,t-1}^i}{\sum_j \sum_i N_{j,t}^i}$$

3 Agent problems

3.1 Household decisions

The timing of household decisions is as follows. At the beginning of each period t, a type i age j representative household holds assets $\theta^i_{j-1,t-1}$, which are brought from period $t-1$. During the period, the household supplies labour, $h^i_{j,t}$, to the firm and earns an income commensurate with their efficiency hours and the market wage. At the end of period t, the household's total available resources include the gross return on risk-free bonds and risky capital, wage income, and pension income, less taxes. Then the household decides how to allocate these resources on consumption, $c^i_{j,t}$, asset holdings for the next period, $\theta^i_{j,t}$, and the share of investment on bonds, $\eta^i_{j,t}$. Deaths occur at the end of the period and the residual assets from the fraction of the population that dies are collected by the government.

The state of the economy is given by $(s_t; z_t) = \left(x^1_{2,t}, \cdots, x^i_{j,t}, \cdots, x^I_{J,t}; z_t\right)$, where $x^i_{j,t}$ is the value of asset holding of a representative type i households at age j in period t :

$$x^i_{j,t} = \left[\eta^i_{j-1,t-1}\left(1 + (1-\tau_r)\,\bar{r}_{t-1}\right) + \left(1 - \eta^i_{j-1,t-1}\right)\left(1 + (1-\tau_r)\,r_t\right)\right]\theta^i_{j-1,t-1} \tag{3.1}$$

Note $x^i_{1,t} = 0$ since newly born young workers enter the economy with zero asset holding.

Let $V^i_j(s_t; z_t)$ be the value of the representative household:

$$V^i_j(s_t; z_t) = \max_{\left\{c^i_{j,t}, h^i_{j,t}, \theta^i_{j,t}, \eta^i_{j,t}\right\}} u^i\left(c^i_{j,t}, h^i_{j,t}\right) + \beta\phi^i_j E_t\left[V^i_{j+1}\left(s_{t+1}; z_{t+1}\right)\right] \tag{3.2}$$

subject to the following budget constraint:

$$(1+\tau_c)\,c^i_{j,t} + \theta^i_{j,t} \leq \left\{ \left(1 - \tau_s - \tau^G_s - \tau_h\right) w_t \varepsilon^i_j h^i_{j,t} + x^i_{j,t} + (1-\tau_p)\left(p_{j,t} + p^G_j\right) + \xi_t \right\}. \tag{3.3}$$

and the time constraint of labour:

$$h^i_{j,t} \leq H^c_j = \begin{cases} H_j, & \text{if } j \in \{YW, MW, W\}, \\ \iota_p H, & \text{if } j \in \{SR\}, \\ 0, & \text{if } j \in \{R\}, \end{cases} \tag{3.4}$$

β is the households' discount factor. Households of the oldest generation, $j = J$, have the following problem.

$$V_J^i(s_t; z_t) = \max_{\{c_{J,t}^i, \theta_{J,t}^i, \eta_{J,t}^i\}} u^i\left(c_{J,t}^i, 0\right) + \beta E_t\left[v^i\left(X_{J+1,t+1}^i\right)\right]$$

where

$$X_{J+1,t+1}^i = \left[\eta_{J,t}^i\left(1 + (1-\tau_r)\bar{r}_t\right) + \left(1-\eta_{J,t}^i\right)\left(1 + (1-\tau_r)r_{t+1}\right)\right]\theta_{J,t}^i$$
$$v(X) = \Gamma\frac{X^{1-\gamma_b}}{1-\gamma_b}$$

Γ denotes the intensity of the bequest motive. To make the analysis simple, it is assumed that wealth is distributed equally (the same as incidental bequest) to all existing cohorts.

For generations $j < J$, the first order conditions (FOCs) with respect to the four control variables, $\left\{c_{j,t}^i, h_{j,t}^i, \theta_{j,t}^i, \eta_{j,t}^i\right\}$ are given as follows:

$$u_1^i\left(c_{j,t}^i, h_{j,t}^i\right) - \lambda_{j,t}^1(1+\tau_c) = 0, \tag{3.5}$$
$$u_2^i\left(c_{j,t}^i, h_{j,t}^i\right) + \lambda_{j,t}^1\left(1 - \tau_s - \tau_s^G - \tau_h\right)w_t\varepsilon_j^i - \lambda_{j,t}^2 = 0, \tag{3.6}$$
$$\beta\phi_j^i E_t\left[\frac{\partial V_{j+1}^i(s_{t+1}; z_{t+1})}{\partial\theta_{j,t}^i}\right] - \lambda_{j,t}^1 = 0, \tag{3.7}$$
$$\beta\phi_j^i E_t\left[\frac{\partial V_{j+1}^i(s_{t+1}; z_{t+1})}{\partial\eta_{j,t}^i}\right] = 0, \tag{3.8}$$

where $\lambda_{j,t}^1$ and $\lambda_{j,t}^2$ are the Lagrange multipliers for budget and time constraints, respectively.

As for $j = J$, the FOCs are

$$u_1^i\left(c_{J,t}^i, 0\right) - \lambda_{J,t}^1(1+\tau_c) = 0,$$
$$\beta E_t\left[\Gamma\left(X_{J+1,t+1}^i\right)^{-\gamma_b}\left[\eta_{J,t}^i\left(1+(1-\tau_r)\bar{r}_t\right) + \left(1-\eta_{J,t}^i\right)\left(1+(1-\tau_r)r_{t+1}\right)\right]\right] = \lambda_{J,t}^1,$$
$$\beta E_t\left[\Gamma\left(X_{J+1,t+1}^i\right)^{-\gamma_b}(1-\tau_r)(\bar{r}_t - r_{t+1})\theta_{J,t}^i\right] = 0,$$

Envelope Theory implies

$$\frac{\partial V_{j+1}\left(s_{t+1};z_{t+1}\right)}{\partial \theta^i_{j,t}} = \lambda^1_{j+1,t+1}\left[\eta^i_{j,t}\left(1+\left(1-\tau_r\right)\bar{r}_t\right)+\left(1-\eta^i_{j,t}\right)\left(1+\left(1-\tau_r\right)r_{t+1}\right)\right], \quad (3.9)$$

$$\frac{\partial V^i_{j+1}\left(s_{t+1};z_{t+1}\right)}{\partial \eta^i_{j,t}} = \lambda^1_{j+1,t+1}\left(1-\tau_r\right)\left(\bar{r}_t-r_{t+1}\right)\theta^i_{j,t}. \quad (3.10)$$

Then substituting (3.9) and (3.10) into (3.7) and (3.8), and using (3.5) to eliminate $\lambda^1_{j,t}$, we get the following non-linear equation system that solves the households' problems.

$$\left(c^i_{j,t}\right)^{-\gamma_c} = \beta\phi^i_j E_t\left[\left(1+\left(1-\tau_r\right)r_{t+1}\right)\left(c^i_{j+1,t+1}\right)^{-\gamma_c}\right], \quad (3.11)$$

$$0 = \beta\phi^i_j E_t\left[\left(1-\tau_r\right)\left(\bar{r}_t-r_{t+1}\right)\left(c^i_{j+1,t+1}\right)^{-\gamma_c}\right], \quad (3.12)$$

$$\frac{\Psi^i\left(H_j-h^i_{j,t}\right)^{-\gamma_h}+\lambda^2_{j,t}}{\left(c^i_{j,t}\right)^{-\gamma_c}} = \frac{1-\tau_s-\tau^G_s-\tau_h}{1+\tau_c}w_t\varepsilon^i_j. \quad (3.13)$$

$$\lambda^2_{j,t}\left(H^c_j-h^i_{j,t}\right) = 0 \quad (3.14)$$

where E_t is the conditional expectation of z_{t+1} given z_t. (3.11) and (3.12) are Euler equations that characterize returns on risk-free bonds and risky capital. Equation (3.13) indicates the intra-temporal substitution between consumption and labour supply. Equation (3.14) is the complementary slackness condition. Note for $j = J$, (3.11) and (3.12) are replaced by the following two equations.

$$\beta\Gamma E_t\left[\left(1+\left(1-\tau_r\right)r_{t+1}\right)\left(X^i_{J+1,t+1}\right)^{-\gamma_b}\right] = \left(c^i_{J,t}\right)^{-\gamma_b},$$

$$\beta\Gamma E_t\left[\left(1-\tau_r\right)\left(\bar{r}_t-r_{t+1}\right)\left(X^i_{J+1,t+1}\right)^{-\gamma_b}\right] = 0.$$

3.2 Firm decisions

The profit-maximizing behavior of the firm gives rise to first order conditions that determine the real net-of-depreciation rate of return to capital and the real wage rate per unit of efficiency labour, respectively:

$$r_t = \alpha z_t K^{\alpha-1}_t H^{1-\alpha}_t - \delta, \quad (3.15)$$

$$w_t = \left(1-\alpha\right)z_t K^{\alpha}_t H^{-\alpha}_t. \quad (3.16)$$

where $\delta \in [0, 1]$ is the depreciation rate.

4 Recursive competitive equilibrium

At the beginning of each period, the state of the economy is given by $\{s_t; z_t\}$, where $s_t = \left(x^1_{2,t}, \ldots x^i_{j,t}, \ldots x^I_{J,t}\right)$ represents the distribution of values of asset holdings in period t. Given the initial state of the economy $(s_0; z_0) = \left(x^1_{2,0}, \ldots, x^i_{j,0}, \ldots, x^I_{J,0}; z_0\right)$, the recursive competitive equilibrium is defined as follows:

Definition: the **Recursive Competitive Equilibrium (RCE)** consists of value functions $V^i_j(s_t; z_t)$; the household policy functions for consumption $c^i_{j,t}(s_t; z_t)$, labour supply $h^i_{j,t}(s_t; z_t)$, total saving $\theta^i_{j,t}(s_t; z_t)$, and the share of savings invested in risk-free bonds $\eta^i_{j,t}(s_t; z_t)$ for each age and type (j, i); the inputs for the representive firm $K_t(s_t; z_t)$ and $H_t(s_t; z_t)$; the government policy $p_t(s_t; z_t)$, $B^G_t(s_t; z_t)$; and prices $\bar{r}_t(s_t; z_t)$, $r_t(s_t; z_t)$, and $w_t(s_t; z_t)$ such that:

(i) Given the prices, the value function $V^i_j(s_t; z_t)$ solves the recursive problem (3.2) of the representative type i households at age j, subject to the budget constraint (3.3) and time constraint (3.4). $c^i_{j,t}(s_t; z_t)$, $h^i_{j,t}(s_t; z_t)$, $\theta^i_{j,t}(s_t; z_t)$ and $\eta^i_{j,t}(s_t; z_t)$ are the associated policy functions for all generations and states;

(ii) The firm maximizes its profits in each period given prices, i.e. wages and rates of returns. In future versions of this paper, we will incorporate the intra-cohort heterogeneity, allowing different types of i, in the sense that workers have different wage levels. We have several ways to introduce intra-cohort heterogeneity;

(iii) All markets clear: Labour, capital and risk-free bond market clearing conditions are implied by (2.4), (2.3) and (2.2). These market clearing conditions and binding household budget constraints imply market clearing in consumption.

5 Parameterization

This section discusses parameter values for our baseline model with $J = 20$ and $i = 1$, i.e. each period represents 4 years and there is no intra-cohort heterogeneity. We have several parameters and constraints that are fixed and exogenous in our initial model, and we draw on the existing literature, as well as Canadian data, to set reasonable baseline values. Parameter values are summarized in Table 4 in Appendix A.

For the discount factor, we employ the standard 0.99 quarterly value, which is 0.8515 in our 4-year cohorts model. Capital's share is also standard in the literature at around 0.3. We follow Prescott (1986) and set the autocorrelation coefficient for TFP at a quarterly value of 0.95 (0.4401 for our model). The standard deviation of the error term in the TFP process (0.00763 quarterly, 0.0305 for our model) is also drawn from Prescott. We set the depreciation rate to be 0.048 quarterly (0.192 for our model).

Estimates of relative risk-aversion between one and two are common in the consumption literature, so we set $\gamma_c = 2$. Following Heathcote et al. (2010), we set the reciprocal of the intertemporal elasticity of substitution for households' non-market time, γ_l, to 3.0. We calibrate the utility weight of leisure relative to market consumption, Ψ, to 21.833 such that the average hours worked in the market for households at YW, MW and W, are estimated to be 30% of the time endowment H.

The following parameters we derive from the data. Survival probabilities, shown in Table 5 in Appendix A, are derived using Statistics Canada's 2009-2011 complete life table. We set the annual population growth rate at 1.2% (4.8% for our model), since Canadian population growth has fluctuated around 1% from the 1970s to the present and sat at 1.2% for 2012. Sales tax rates vary substantially across Canadian provinces, ranging from 5% to 15%. Using provincial population shares, we construct a weighted average tax that is about 12.3%[4]. To estimate average labour income tax, we use the 2011 Survey of Labour and Income Dynamics (SLID)[5]. For individuals aged 16-65, the total average income tax paid is about \$7,000, dividing this amount by the average total income (approximately \$42,000) the effective income tax rate ($\tau_s + \tau_s^G + \tau_h$) is 16.7%. In the baseline (2-pillar model) parameterization, we assume 25% of labour taxes go to social security. That is, the percentage of labour taxes that goes to pension, $ratio_s$, equals 25% such that $\tau_s = 4.2\%$ and $\tau_h = 12.5\%$. The effective tax rate on income for those aged 65-81 is 9.9%[6]. In the 3-pillar pension system, we set $\tau_s^G = 5.0\%$ such that $\tau_s = 4.2\%$ and $\tau_h = 7.5\%$.

The SLID does not separately identify tax rates on various sources of income. So we

[4]In addition to sales tax, Canada has additional consumption related taxes (e.g. on liquor, fuel and residence). OECD (2015) reports that taxes on income sources represent 47% of the tax burden, social security another 16%, taxes on goods and services 25% and taxes on property 11%. Combining the latter two Canadians have a tax burden of 36%. If Canadians consume approximately 90% of their income, then these tax burden rates roughly match up with Canada's estimated average tax rates for income and consumption (77% of 16.7 is 12.9 of which 90% is 11.6, within range of 12.3), and a social security tax that represents approximately 1/3 of a 16.7% income tax is 5.5% (within range our 3.2%).

[5]The SLID is the primary source for income statistics in Canada. All estimates, from the SLID (as well as the General Social Survey), are weighted using survey weights.

[6]Income tax rates for seniors should be lower because of pension income splitting, age credits, and other tax credits.

cannot produce effective tax rates on earnings versus investment income. However, we do know that interest income is taxed identically to labour income, eligible dividends get a small tax break, capital gains are taxed on 50% of gains, and investments (up to a limit) in Tax Free Savings Accounts are not taxed at all. We begin with the assumption of the same tax rate for labour and investment income, and then do a sensitivity analysis with a two percentage point lower tax rate on investment income.

At this point, we assume that a household's productivity remains the same, at unit, over its life-cycle and do not take any experience effect into consideration. We leave the analysis of age-specified productivity to future study.

Finally, we have three time constraints that limit the amount of labour that workers can provide. First, we impose a time constraint on the semi-retired workers to reflect the large proportion retiring after age 65. The labour force participation rate of those age 66-81, as a fraction of the participation rate of those age 16-65, is just under 8%. Our oldest workers are therefore constrained against using 92% of their time.

Next, we consider that children and education both take up substantial amounts of time, limiting the hours available for labour market activities and leisure. We turn to the General Social Survey (GSS), cycle 19: time use (2005), to estimate the average hours spent on own-education activities, and on childcare. For the latter, we include time spent directly caring for children, as well as time spent on activities when childcare is a secondary activity. The age-specific constraints are given in Table 6 in Appendix A.

6 Primary results

6.1 Baseline model

The life-cycle patterns of consumption, asset holdings and labour supply are depicted in Figure 1. Consumption is clearly hump shaped over the life cycle - a fact that has been well documented. The upper-right panel shows how households provide labour during life cycle. In their earlier working ages, a household's labour supply critically depends on its time constraints of child rearing and education. This is because when the household is young, its leisure is quite stable and none of the time constraints are binding. When entering the semi-retired stage ($j > 12$), households work the maximum available units of time, i.e. $h_{j\in SR} = \iota_p H$. After retirement, the households do not work and enjoy all time as leisure. We also see the typical pattern of asset accumulation in working periods and decumulation in retirement (bottom-left panel), although the decumulation is steeper than observed in the data. This decumulation is problematic given that our focus is an exploration of the impact of population aging on asset prices, and retirement outcomes. To this end, we incorporate a bequest motive, and increasing healthcare costs, in subsequent sections of this paper.

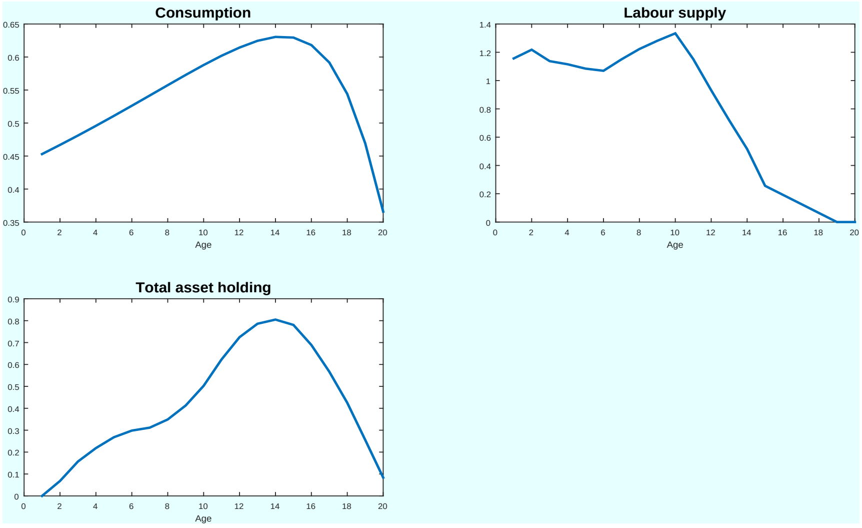


Figure 1: Lifecycle patterns of consumption, labour supply and total asset holding.

In order to understand and predict the impact of population aging on asset outcomes, it is important that our model closely reflects the observed portfolio allocation behaviour. Figure 2 shows the households' age-specific portfolio allocation. Similar to total asset holding, a household's capital holding also follows (roughly) a hump-shape pattern. The short sale on capital, i.e. negative capital holding, never happens and households invest in risky capital at all ages. On the other hand, the bond holding is negative among younger cohorts. On average, households sell bonds (borrow) in early ages and demand bonds in old ages. The curvature in the youngest cohorts is caused by household's time constraints on child rearing and education. In fact, relaxing these time constraints, the model exhibits a pattern of monotonically increasing bond demand with age. That is, the young borrow against their future labour income and insure the old by selling the bonds.

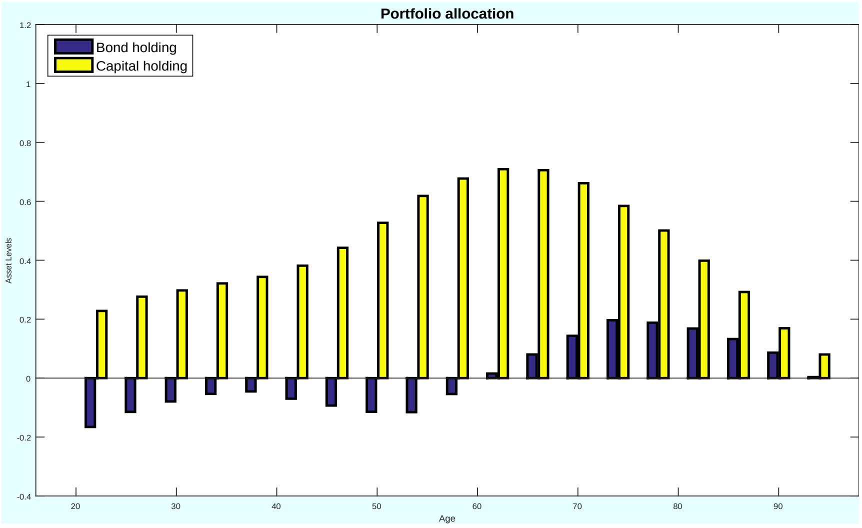


Figure 2: Lifecycle portfolio allocation under a two-pillar pension system.

Our results reconcile the empirical observation quite well qualitatively. Figure 3 is compiled from a representative survey of U.S. households' portfolios, the 2013 Survey of Consumer Finance (SCF), where we count as risky assets any high risk stocks, and any IRAs, mutual funds and other savings vehicles held in stocks. We also count as risky assets any non-stock IRAs, mutual funds and saving vehicles along with mixed funds, corporate bonds, other bonds, and money owed to respondent. Low risk assets are net of all debt and include checking accounts, t-bills, government and other savings bonds and certificates of deposit. One possible reason for the observed increase in assets among the oldest cohorts in Figure 3 is that households have bequest motives. Other possible reasons include increasing and uncertain healthcare costs. Moreover, if longevity is greater for higher income households, we might expect survival bias to generate an uptick in assets for the oldest cohort. In order to explore these possibilities, we have implemented different versions of the model by considering bequest motives, introducing a constraint for healthcare costs that increase with age. Our next step is to implement intra-cohort heterogeneity.

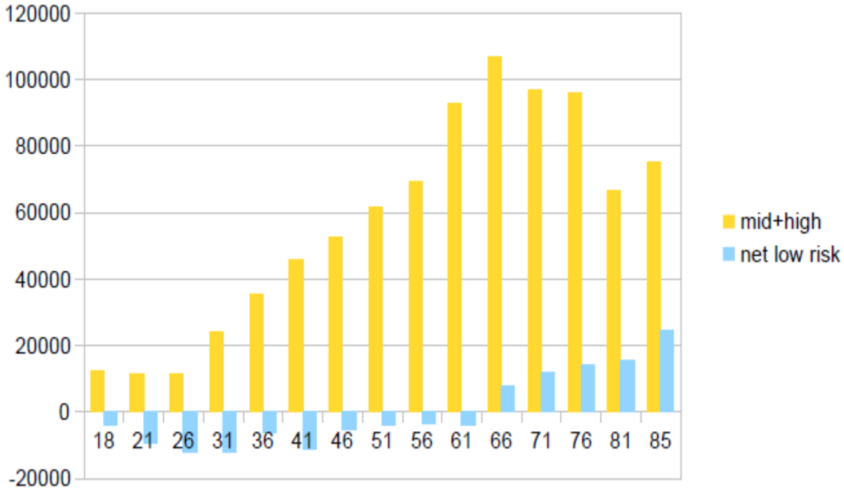


Figure 3: SCF summary: high + med risk versus low risk net of financial debt.

Table 1 provides the mean of asset returns in the baseline model with 2-pillar pension system.[7] We note that expected capital returns and bond returns are very close, resulting in an equity premium of less than 1/2 percent. The size of private pensions is just under 60% of GDP, which is close to what we observe empirically. Because we have no partially funded public pillar in this model, the public holdings of risky assets and the funded tax rate are both zero.

Table 1: Variable mean values: two-pillar pension model.

Variable	**Mean**
Expected capital return $(E_t(r_{t+1}))$	0.3052
Bonds return $(\bar{r}_t)$	0.3012
Equity premium $(E_t(r_{t+1}) - \bar{r}_t)$	0.0040
Public holdings risky assets/GDP	N/A
Private holdings risky assets/GDP	0.5964
Private holding bonds/GDP	0.0000
$\tau^G_{s,t}$	N/A
$c^i_{20,t}$	0.4528

[7] The corresponding business cycle statistics are available upon request.

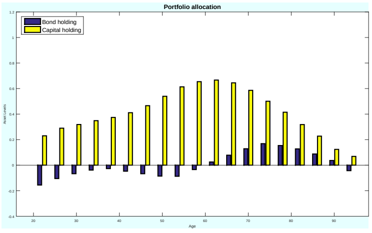
6.2 Impact of population structure on portfolio allocation and asset returns

Next we examine the impact of demographic factors on economic activity. At this point, we focus on how population aging affects asset returns. To do so, we conduct counterfactual simulations in which population aging varies, in the sense of changing survival probability for old households. Suppose the survival probability linearly increases (decreases) form 2% (-2%) for generation $j = 13$ to $\triangle^O$ for generation $j = J$. Simulation outcomes are listed in Table 2. Portfolio allocation changes are presented in Figure 4.

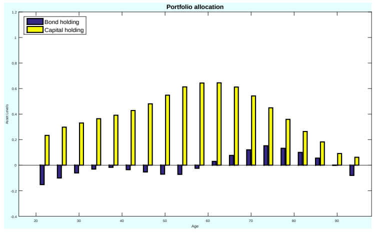
Table 2: Means with various $\triangle^O$: two-pillar pension model

$\triangle^{\mathbf{O}}$	**Dep Ratio**	$E_t(r_{t+1})$	$\bar{r}_t$	$E_t(r_{t+1}) - \bar{r}_t$	Risky asset holdings over GDP (K_t/Y_t)
-20%	0.2633	0.3252	0.3249	0.003934	0.5926
-10%	0.2736	0.3180	0.3177	0.003971	0.5933
Baseline	0.2933	0.3052	0.3012	0.004006	0.6084
+10%	0.3150	0.2926	0.2922	0.004026	0.6331
+20%	0.3293	0.2842	0.2838	0.004030	0.6444

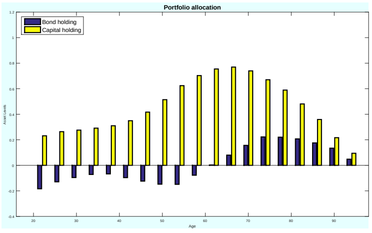
The model predicts a clear relationship between population aging and asset returns: The higher the survival rate of the old, the lower the returns. Two reasons may explain why increased longevity results in lower asset returns. One reason is that increased longevity influences households' precautionary motive. Households expecting to survive longer have an added incentive to save so as to insure themselves against outliving their assets. This results in larger capital stock and lower asset returns. The other reason is the scarcity of labour relative to capital. In our model, there is no child generation, thus, higher values of $\triangle^O$ imply a higher old-age dependency ratio. A larger share of dependents is associated with a lower proportion of labour available for the firm, and therefore decreases the return on capital. Moreover, not surprisingly, the more that the population is aging, the higher ratio of risky asset holdings relative to GDP.



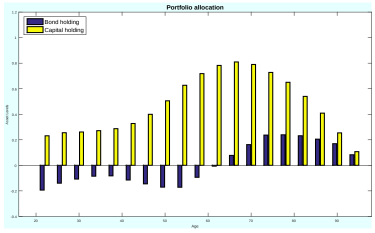
(a) $\Delta^o = -10\%$.



(b) $\Delta^o = -20\%$.



(c) $\Delta^o = 10\%$.



(d) $\Delta^o = 20\%$.

Figure 4: Portfolio allocation with various Δ^O: two-pillar pension model.

6.3 Three pillar pension model

Canada has a three pillar federal pension system. The first pillar is a universal benefit (Old Age Security, Guaranteed Income Supplement, and spousal allowance), the second pillar is an earnings-related contributory program (Canada/Quebec Pension Plan, which has similarities to the U.S. social security system), and the third pillar is private retirement savings (which include individual savings as well as employer plans). Our two pillar model includes the first pillar and private savings. Our three pillar model incorporates an earnings-related contributory program.

The predictions of this model are presented below. Assuming a 20% income replacement ratio for the employment-related pension plan, we note that the introduction of the earnings-related retirement program results in an increase in the magnitude of total retirement savings. Now private investment in risky assets constitute about 56% of GDP, and the earnings-related pillar comprises just under 10% of GDP. The equity premium increases a little in this model, and the mean returns to our risky and risk free assets are down. Note that consumption, for the oldest cohort, has increased (from 0.45 to 0.49) going from the two to the three pillar model

Figure 5 shows the lifecycle portfolio allocation under a three pillar pension system. There is a slight shift in the asset holdings for younger generations, but the lifecycle pattern remains substantively similar.

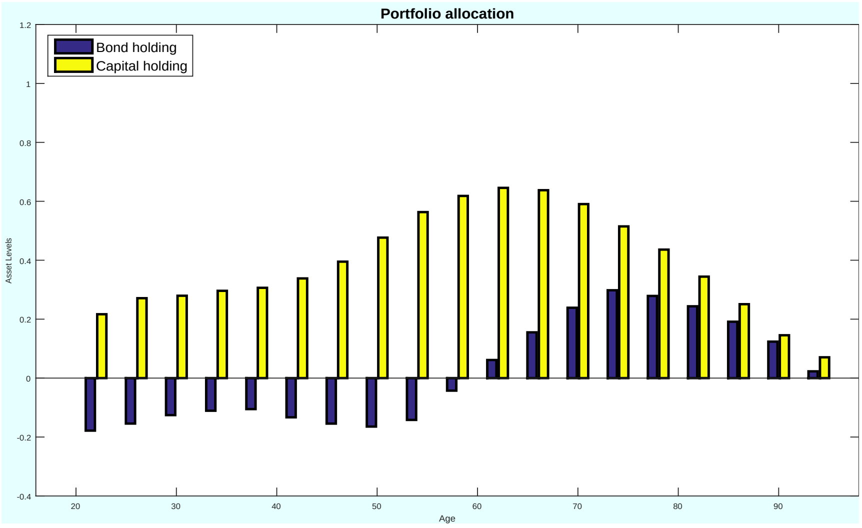


Figure 5: Lifecycle portfolio allocation under a three-pillar pension system.

Table 3: Variable mean values: three-pillar pension model.

Variable	Mean
Expected capital return $(E_t(r_{t+1}))$	0.2826
Bonds return $(\bar{r}_t)$	0.2780
Equity premium $(E_t(r_{t+1}) - \bar{r}_t)$	0.0046
Public holdings risky assets/GDP	0.0968
Private holdings risky assets/GDP	0.5581
Private holding bonds/GDP	-0.0645
$\tau^G_{s,t}$	0.0300
$c^i_{20,t}$	0.4918

6.4 Further modifications: bequest, healthcare costs and old working

As noted earlier, we aim to match asset holding across cohorts, and the decumulation among the oldest groups is steeper than shown in the data. As such, we consider the addition of a simple bequest motive, and of healthcare costs that increase exponentially for the oldest cohorts.

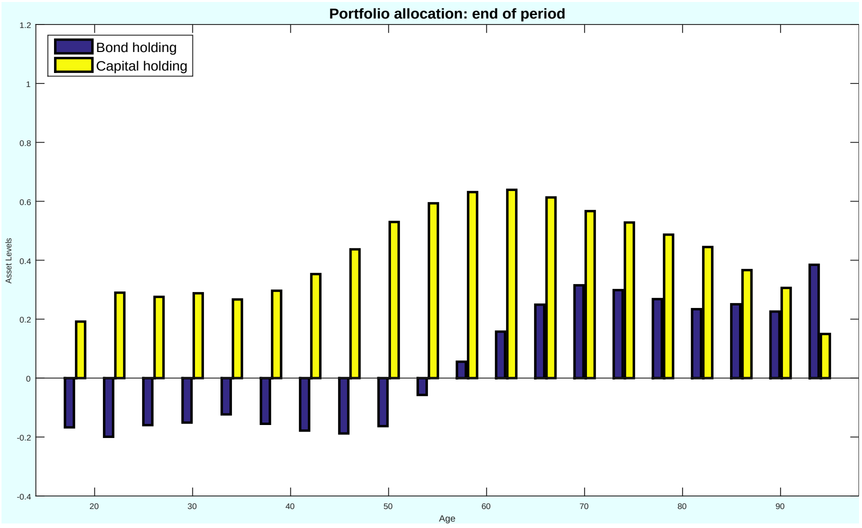


Figure 6: Lifecycle portfolio allocation in baseline model with bequest motive.

Figure 6 presents the lifecycle portfolio allocation in the model with a simple voluntary bequest motive, we note that households do retain more assets in older ages, and in particular, bond holding scarcely decreases, and even rises in the oldest cohorts, consistent with what is observed in the data. Decumulation of capital is less severe in the model with voluntary bequests.

We add healthcare costs to the model and present portfolio allocation results in Figure 7. With both a bequest motive and healthcare costs, we see a further increase in the amount of risky and risk free assets retained among retired households. Indeed, the model now does a better job of generating an increase in bond holding, qualitatively similar to what we observe in the data. However, the model does not yet do a good job of matching the magnitude of the portfolio allocations, nor the uptick in risky asset holding among the oldest.

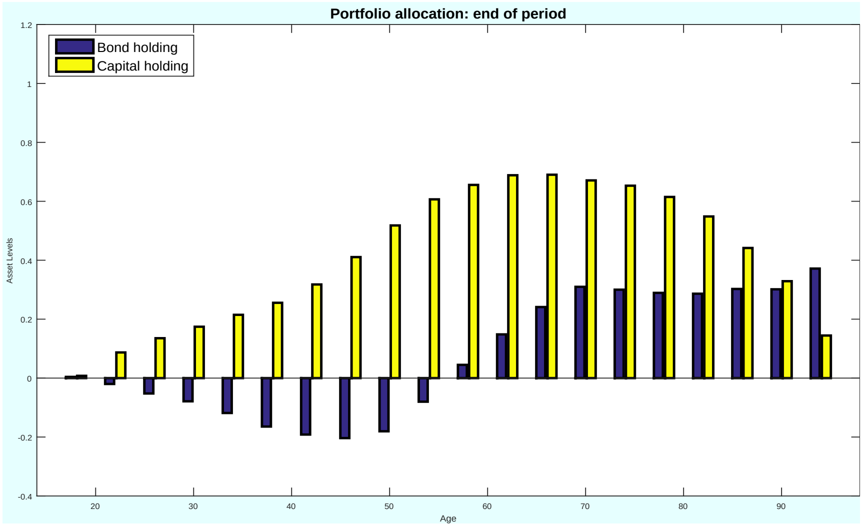


Figure 7: Lifecycle portfolio allocation in baseline model with bequest motive and exponential increasing healthcarecosts for retirees.

In order to match better to the portfolio allocation among the oldest, we consider two further tests. First, we reduce the curvature of the bequest motive. Second, we allow the oldest cohorts to work. The interpretation of a lower curvature on the bequest motive is that uncertainty in the amount of the bequest is less important (for the utility of the giver), akin to lower risk aversion. Our test indicates that a lower curvature on the bequest motive yields less decumulation of both assets, but in particular, the risky asset.

Our intention in relaxing the time constraint for the oldest cohorts, is that if these cohorts could work, labour market income could insure against negative shocks in asset returns. Results do indicate that the portfolio allocation shifts to have proportionately more risky assets, and retains a low decumulation rate near end of life, but we do not see the uptick in risky asset holding at the oldest ages, and overall asset accumulation is somewhat lower with labour market participation in the oldest cohorts.

Our next step is to consider intra-cohort heterogeneity as we expect that differences in survival rates might generate the uptick in risky asset holding in the last periods of life since higher income and wealth are associated with greater longevity. Moreover, we expect that retirement outcomes will vary broadly across different demographic groups, and policy makers should be aware of the implications of demographic structure across the population, not just on aggregate.

7 Conclusion and future research

With the large baby-boom cohort entering retirement, many are concerned that the expected drop in saving and investment will result in substantially diminished asset prices and compromised pension plans. This paper develops a large scale computable Overlapping Generations model to quantify the impact of population structure on asset values. Results from our counterfactual excercises suggest that asset prices are moderately lower with an older population. Specifically, a 4% increase in the survival probability of households over age 65 results in a 4.16% drop in the return on capital, and a 3.02% drop in the return on bonds.

This paper makes three specific contributions to the literature. First, we investigate the impact on asset prices of a higher old-age dependency ratio in a model that incorporates both a risk-free and a risky asset, with endogenous returns. Second, we test the implications of demographic structure under a three-pillar pension system that includes a publicly administered, partially funded, employment-related pension. This framework can also be used to assess the implications of the proposed expansion of the Canada Pension Plan. Finally, we generate age-specific portfolio allocations that are consistent with the data, with the exception of the oldest cohort. We explore additional mechanisms to address this concern (e.g. bequest motive, healthcare costs and, as a next step, intra-cohort heterogeneity). Both the bequest motive and healthcare costs modestly increase the asset holdings at the oldest cohort, but not to the extent depicted in the data. Our next step is to consider intra-cohort heterogeneity with productivity and longevity differences.

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A Parameter values

Table 4: ***Parameters List: Baseline Model***

Parameter	Value	Description
J	20	Each period represents 4 years
I	1	No intra-cohort heterogeneity
H	4.0	Available time to spend for households
β	0.8515	Discount factor
α	0.3	Capital share of production
ρ_z	0.4401	Autocorrelation coefficient for the TFP process
σ_z	0.0305	The standard deviation of error term in the TFP process
ρ_q	0.4401	Autocorrelation coefficient for the TFP process
σ_q	0.0305	The standard deviation of error term in the TFP process
δ	0.192	Depreciation rate
n	0.0489	Population growth rate
γ_c	2.0	Relative risk-aversion on consumption
γ_b	2.0	Relative risk-aversion on bequest
γ_l	3.0	Reciprocal of the intertemporal elasticity of substitution for household's non-market time
Ψ	21.833	Utility weight of non-market time relative to market consumption
τ_c	0.123	Consumption tax rate
τ_r	0.167	Tax on investment income
$\tau_s + \tau_s^G + \tau_h$	0.167	labour income tax
$ratio_s$	1.0	Percentage of labour tax to pension (Social security deduction)
τ_p	0.099	Tax on pension income
ι_p	0.08	Labour time constraint at stage SR, % of H
ε_j	1.0	Age-specific productivity (efficiency labour) profile
χ	1.0	proportion of type i households within a generation.

Table 5: ***Survival Probabilities from age*** j ***to*** $j+1$***:*** ϕ_j

	Age (year)	Model Age	Survival Prob.
Young-working (YW)	18 - 21	1	0.9982
	22 - 25	2	0.9979
	26 - 29	3	0.9980
	30 - 33	4	0.9977
Middle-working (MW)	34 - 37	5	0.9971
	38 - 41	6	0.9961
	42 - 45	7	0.9947
	46 - 49	8	0.9927
Mature-working (W)	50 - 53	9	0.9895
	54 - 57	10	0.9849
	58 - 61	11	0.9781
	62 - 65	12	0.9681
Old-working (SR)	66 - 69	13	0.9533
	70 - 73	14	0.9316
	74 - 77	15	0.8995
	78 - 81	16	0.8527
Retirement (R)	82 - 85	17	0.7848
	86 - 89	18	0.6887
	90 - 93	19	0.5589
	94 - 97	20	0.0000

Table 6: ***Time on Child Rearing and Education at age*** j***:*** FC_j ***and*** FE_j

	Age (year)	Model Age	FC_j (% of H)	FE_j (% of H)
Young-working (YW)	18 - 21	1	0.00	0.16
	22 - 25	2	0.03	0.11
	26 - 29	3	0.07	0.03
	30 - 33	4	0.14	0.02
Middle-working (MW)	34 - 37	5	0.14	0.01
	38 - 41	6	0.12	0.01
	42 - 45	7	0.11	0.01
	46 - 49	8	0.05	0.01