Surrenders in a competing risks framework, application with the Fine and Gray model

LIFE Colloquium of the International Actuarial Association
"Long-Term Saving in an Ageing World"
Barcelona, october 23-24, 2017

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Outline

1. Introduction
2. Competing risks and the subdistribution approach
3. Description of the product considered in the study
4. Application of survival analysis to our Whole Life contracts database

Two words on the surrender risk

First, what is the **surrender risk** in life insurance? [**?**], [**?**]

Some key points:

1. major or minor topic ? depending on the business line...
2. risk factors are "market-specific" [**?**]:

   *clear need to integrate product and country characteristics as risk factors into the surrender behaviours modelling [**?**].*

3. timing is a key-point to recover administration costs...

$\Rightarrow$ **Regressions** (avoid GLM, whose use introduce a selection bias) that aim at predicting the **timing** of the surrender.

2 Competing risks and the subdistribution approach

Survival analysis: theoretical background [?]

→ T: unobservable lifetime, with density f (survival function S).
→ C: contract duration until censorship (administrative here).

The **actual observation** is given by $\bar{T} = \min(T, C)$.
For right censored data, the corresponding counting process follows

$$N(t) = \sum_{i=1}^{n} N_i(t) \qquad \text{where} \quad N_i(t) = \mathbb{1}_{\{\bar{T}_i \leq t \,;\, T_i \leq C_i\}}.$$

To $N_i(t)$ is associated the so-called "intensity process" $A_i(t)$ s.t.

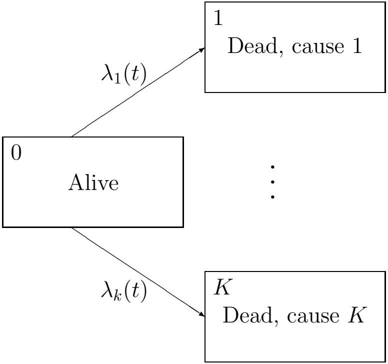
$$A_i(t) = \int_0^t Y_i(s)\,\lambda(s)\,ds, \qquad \text{where}$$

$Y_i(t)$: at-risk process ($\simeq$ exposure), $\lambda(t)$: hazard rate such that

$$\lambda(t) = \frac{f(t)}{S(t)} = \lim_{\Delta \to 0} \frac{\mathbb{P}(t \leq T < t + \Delta \mid T \geq t)}{\Delta}.$$

More seriously... consider $\bar{T} = \min(T_1, ..., T_K, C)$

T_j: lifetime before death from cause j.



- $(J_t)_{t>0}$ is the competing risks process. It tells us in which state the i^{th} policyholder is at time t ($J_t \in \{0, 1, ..., K\}$).
- τ is given by $\tau = \inf\{t > 0 \mid J_t \neq 0\}$.

Main quantities of interest

1. The cause-specific hazard functions: $\forall j \in \{1, ..., p\}$,

$$\lambda_j(t) = \lim_{\Delta \to 0} \frac{\mathbb{P}(t \leq T < t + \Delta\,,\, J_T = j \mid T \geq t)}{\Delta}.$$

$$\lambda(t) = \sum_{j=1}^{p} \lambda_j(t), \quad \text{et} \quad S(t) = \mathbb{P}(T > t) = e^{-\int_0^t \sum_{j=1}^p \lambda_j(s)\, ds}.$$

2. The cumulative incidence functions (CIF):

$$F_j(t) = \mathbb{P}(T \leq t,\, J_T = j) = \int_0^t f_j(s)\, ds = \int_0^t S(s^-)\, \lambda_j(s)\, ds,$$

$$= \int_0^t e^{-\int_0^s \sum_{j=1}^p \lambda_j(u)\, du}\, \lambda_j(s)\, ds \quad \text{``} = 1 - e^{-\int_0^t \alpha_j(s)\, ds} \text{''}$$

The subdistribution approach [?]

Context: $J_t \in \{0,1,2\}$ ($K = 2$, event of interest is labeled "1").

Idea: study a new process $(\xi_t)_{t>0}$, derived from $(J_t)_{t>0}$ and obtained by stopping adequately the latter:

$$\xi_t = \mathbb{1}_{\{J_t=2\}} \, J_{\tau^-} + \mathbb{1}_{\{J_t\neq 2\}} \, J_t.$$

Interpretation: $\{J_t = 0\} \simeq$ nothing happened until time t, whereas $\{\xi_t = 0\} \simeq$ there was no event of interest until t.

Tool: consider $\nu = \inf\{t > 0 : \xi_t \neq 0\}$, the new random lifetime before the occurence of the event of interest (surrender).

$$\nu = \begin{cases} \tau & \text{if } J_\tau = 1, \\ \infty & \text{if } J_\tau = 2. \end{cases}$$

Trick: $\forall t \in [0, \infty), \quad \mathbb{P}(\nu \leq t) = \mathbb{P}(T \leq t,\, J_T = 1) = F_1(t).$

Then, the subdistribution hazard of the event of interest follows

$$F_1(t) = 1 - S_1(t) = 1 - e^{-\int_0^t \alpha_1(s)\, ds}$$

and is finally given by

$$\alpha_1(t) = \lim_{\Delta \to 0} \frac{\mathbb{P}(t < T \leq t + \Delta\,,\, J_t = 1 \mid \{T > t\} \cup \{T \leq t,\, J_t \neq 1\})}{\Delta}.$$

Novelty: $\forall t$, at-risk policyholders consist now in insureds still in state $\{0\}$ at time t added to policyholders who have undergone a competing risk before t.

Pros/Cons: not necessary to model every cause of failure / at-risk set is not really realistic, and not always known.

3 Description of the product considered in the study

Description of Whole Life products

- lump sum at death of the insured,
  (embedding a guaranteed return during the contract lifetime),
- fiscality constraints: TAMRA law,
- cyclical level premiums, whose amount depends on
  - policyholder's gender, age, and health;
  - the tobacco consumption.
- commission equals 0 after 2 years of contract duration,
- Surrender option can be exercised at any time, with surrender value embedding 3 components
  - a lump sum corresponding to the guarantee,
  - final capped dividends depending on the sum insured,
  - stochastic dividends during the contract lifetime
    [Financial markets are likely to impact the behaviours.]

The contract can be partially or totally surrendered: **we focus here on total surrenders** (also other lapse causes: maturity, death, ...).

History: potential impact of financial markets

29 531 contracts, from 01/1995 to 05/2010.

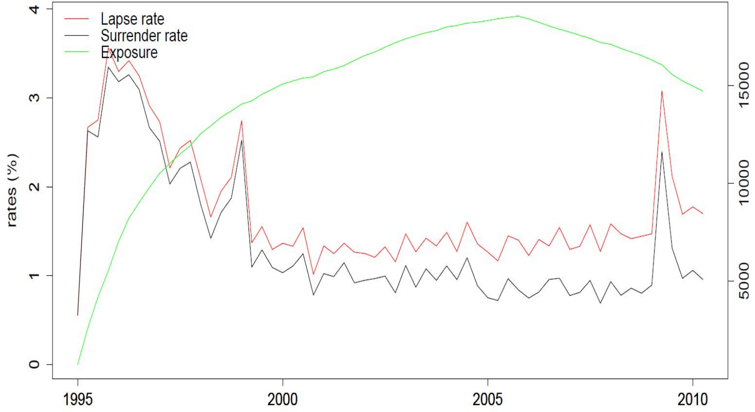


Figure: Exposure (green), lapses (red), and surrender rate (black).

Summary and main effects

→ Descriptive statistics show impacts of risk factors on lifetimes.
→ Correlation between the variable of interest and some risk factors: non-parametric and parametric tests.

Factor	Age	Health diagnostic	Gender	Living place	UW year	Prem. freq.
H_0	rejected	rejected	rejected	**not rejected**	rejected	rejected

Table: χ^2 tests (binary surrender decision VS categorical risk factors).

Factor	Age class	Health diagnostic	Gender	Living place	Acc. rider	Prem.freq.
Test	KW	KW	Wilcoxon	KW	Wilcoxon	KW
H_0	rejected	rejected	rejected	rejected	rejected	rejected

Table: Independence tests (Kruskal-Wallis: KW) on contract lifetimes.

p-values suggest the following most discriminating features: health diagnostic ($\simeq$ *premium*), accidental death rider and premium freq.

4 Application of survival analysis to our Whole Life contracts database

Profile of hazard rates for our competing risks

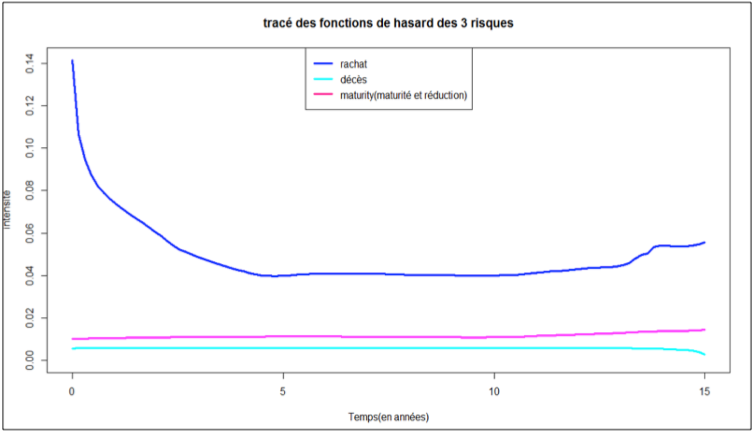


Figure: Adjusted non-parametric Nelson-Aalen estimator of the subdistribution hazards depending on the cause of lapse. ▸ Baseline hazard

Cox model for the surrender subdistribution

We calibrate an extended Cox model for the subdistribution hazard associated to the lifetime before surrender: for policyholder i,

$$\lambda_i(t) = \lambda_0(t)\, \exp(X_i^T\, \beta + Z(t)\, \eta).$$

- $\lambda_0(t)$: baseline hazard, non-parametric and unspecified.
- $X_i^T = (X_{i1}, ..., X_{ik})$ stands for the constant risk factors;
- $\beta^T = (\beta_1, ..., \beta_k)$: corresponding regression coefficients;
- $Z(t)$: variation of the Dow Jones, and η its effect on $\lambda_i(t)$.

$\surd$ Correlation between covariates has initially been checked.
$\surd$ Assumption of PH was first validated (Schoenfeld residuals).
$\surd$ Estimated regression coefficients in line with empirical results.

Impacts and goodness-of-fit

```
  n= 302508, number of events= 3584

                                   coef exp(coef) se(coef) robust se      z Pr(>|z|)
DJIAtend                        1.83029   6.23571  0.29456   0.29946  6.112 9.84e-10 ***
acc.death.amountRider          -0.23535   0.79030  0.04947   0.04937 -4.767 1.87e-06 ***
premium.frequencyMonthly        0.21651   1.24173  0.04553   0.04559  4.748 2.05e-06 ***
premium.frequencyOther         -0.28789   0.74985  0.06264   0.06298 -4.571 4.86e-06 ***
premium.frequencyQuarterly      0.26376   1.30181  0.05160   0.05143  5.128 2.92e-07 ***
premium.frequencySemi-annual    0.47925   1.61487  0.06445   0.06396  7.493 6.74e-14 ***
ageYoung                        0.31802   1.37440  0.04720   0.04745  6.703 2.05e-11 ***
premiumprim2                    1.13071   3.09786  0.12834   0.12862  8.791  < 2e-16 ***
premiumprim3                    1.67849   5.35747  0.11215   0.11229 14.948  < 2e-16 ***
premiumprim4                    1.80629   6.08783  0.11763   0.11768 15.349  < 2e-16 ***
risk.state2Standard            -0.31817   0.72748  0.04072   0.04063 -7.832 4.77e-15 ***
genderMale                      0.14041   1.15075  0.03350   0.03340  4.204 2.62e-05 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

                             exp(coef) exp(-coef) lower .95 upper .95
DJIAtend                        6.2357     0.1604    3.4673   11.2147
acc.death.amountRider           0.7903     1.2653    0.7174    0.8706
premium.frequencyMonthly        1.2417     0.8053    1.1356    1.3578
premium.frequencyOther          0.7498     1.3336    0.6628    0.8484
premium.frequencyQuarterly      1.3018     0.7682    1.1770    1.4399
premium.frequencySemi-annual    1.6149     0.6192    1.4246    1.8306
ageYoung                        1.3744     0.7276    1.2524    1.5083
premiumprim2                    3.0979     0.3228    2.4076    3.9861
premiumprim3                    5.3575     0.1867    4.2991    6.6764
premiumprim4                    6.0878     0.1643    4.8338    7.6671
risk.state2Standard             0.7275     1.3746    0.6718    0.7878
genderMale                      1.1507     0.8690    1.0778    1.2286

Concordance= 0.626  (se = 0.005 )
Rsquare= 0.003   (max possible= 0.188 )
Likelihood ratio test= 851  on 12 df,   p=0
Wald test            = 672.9  on 12 df,   p=0
Score (logrank) test = 721.4  on 12 df,   p=0,   Robust = 834.9  p=0

  (Note: the likelihood ratio and score tests assume independence of
     observations within a cluster, the Wald and robust score tests do not).
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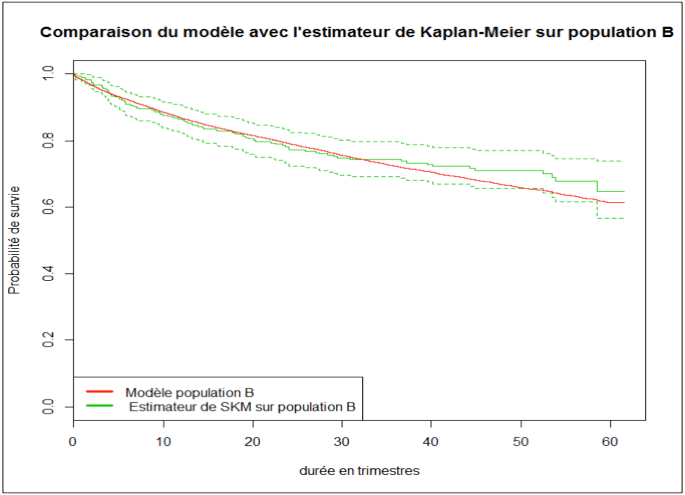
Validation: empirical VS modeled survival curves



Accurate modelling in the first 8 years. Impact of risk factors: OK

Issue and suggested improvement

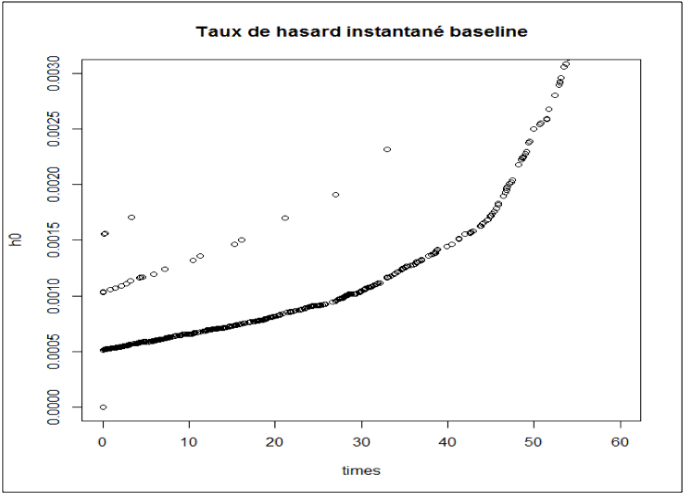


Figure: Baseline hazard after the calibration of a Cox subdistribution hazard type for the surrender risk. To compare to ▸ Nelson-Aalen est.

Comments and perspectives

$\rightarrow$ This framework seems to be the most realistic for this problem, was not really investigated for life insurance lapses previously.

$\rightarrow$ The subdistribution approach clearly allows us to reduce the model risk, as it does not rely on modelling other causes of failure.

Nevertheless, it requires

- more work to do on the specification of the baseline hazard;
- to perform further studies on the simulation of stochastic counting processes in the subdistribution approach;
- to better integrate correlation between behaviours, [?]:
    - common shocks model,
    - adding a frailty variable into the hazard definition,
    - use survival mixtures.

Final goal: should improve the day-to-day ALM of the company.

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