Large Loss Pricing Tool for non-working layers in casualty insurance

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Overview ILF EVT

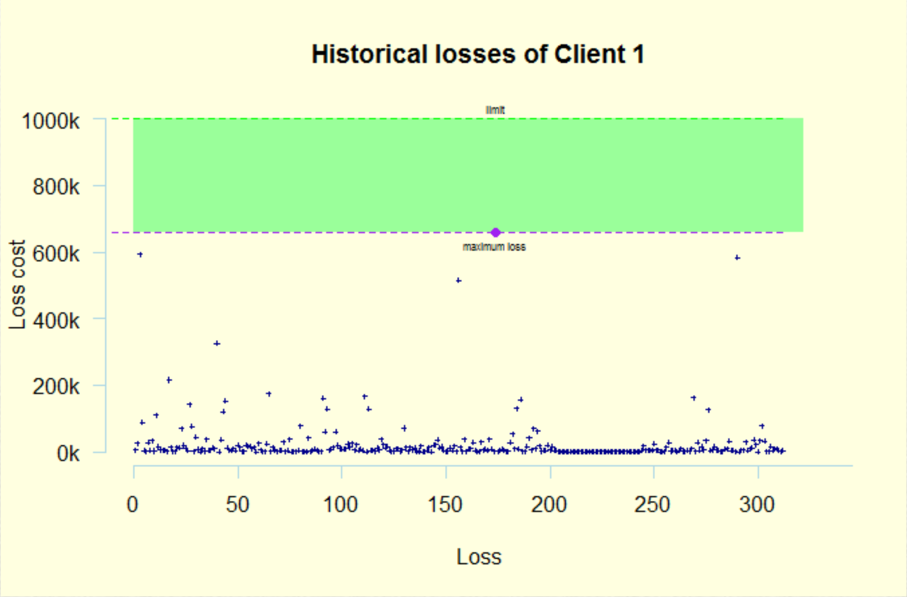
Need for a tool to price limits in casualty insurance

Property Insurance	Casualty Insurance
Damage on the insured product	Damage on a third-party
Probable maximum loss known	Probable maximum loss unknown
→Known and limited exposure	→Unknown and unlimited exposure

Need to price non-working layers:
→To give a limit a cost

1. Vertical exposure: need for a tool focusing on severity

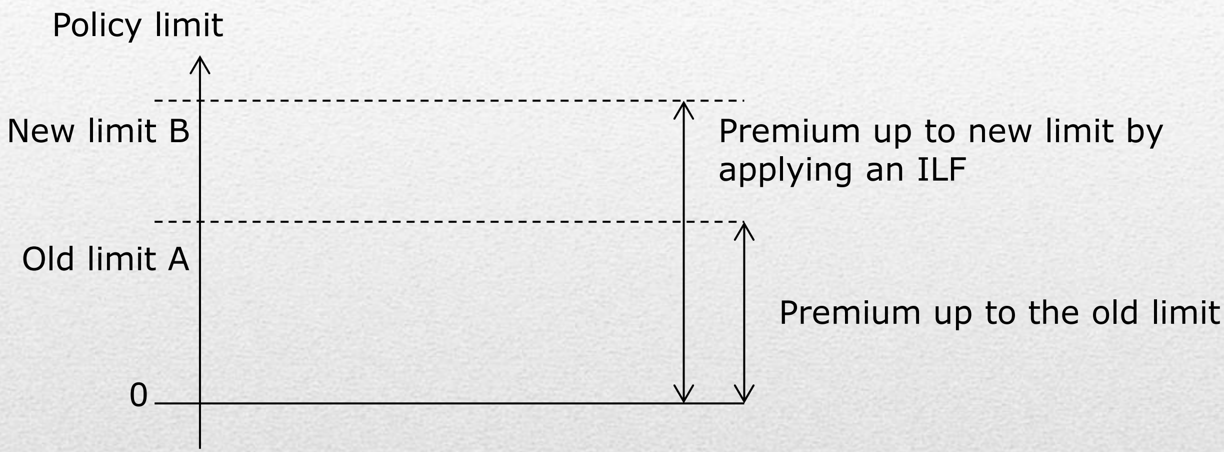
2. Apply a loading to a premium calculated using another method (e.g. Burning Cost)



ILF : Price the increase of a limit

Increased Limit Factors Theory:
Factor enabling to give a price to an increase in the limit:

$$\Pi_B = ILF_{A \to B} * \Pi_A$$



Calibrated ILF:

$$ILF_{A \to B} = \frac{E[loss\ cost_{limit\ B}]}{E[loss\ cost_{limit\ A}]} = \frac{E[min(X,B)]}{E[min(X,A)]}$$

1. ILF depends on severity only
2. Factor to be applied to determine the cost of large losses

ILF : Price the increase of a limit

ILF is often underestimated. To consider the « fair » premium, we use:

$$ILF_{A \to B} = \frac{E[min(X,B)]}{E[X|X < A]}$$

given that the experience method used (Burning Cost for example) captures the risk up to the maximum loss. In this case, A stands for the maximum loss observed.

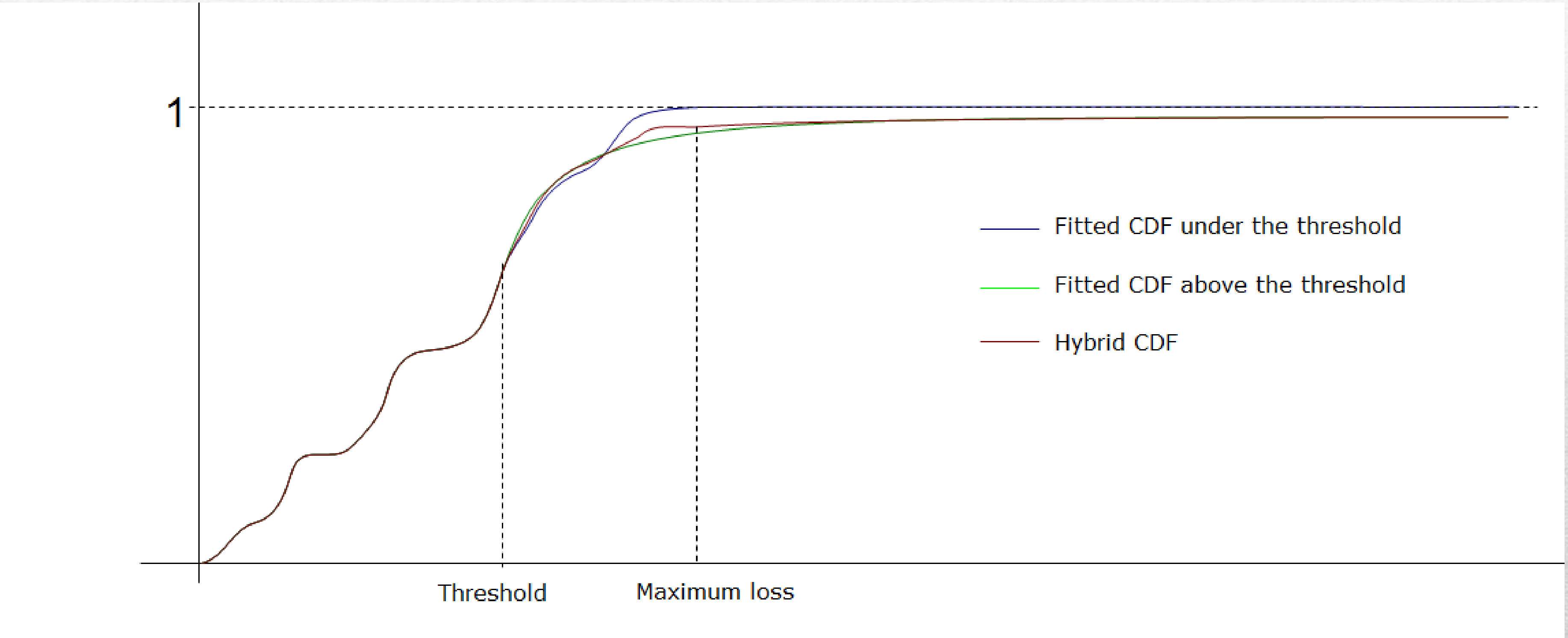
We use distribution fitting to determine the law of X.

Example:	**Kolmogorov-Smirnov**				**Anderson-Darling**			
	Gamma	LogNormal	Pareto	Weibull	Gamma	LogNormal	Pareto	Weibull
Client 1	<0.001	0.71	<0.001	0.002	<0.001	0.38	<0.001	0.007

→ We use a Lognormal distribution (whose fit isn't rejected at a 10% level).

Hybrid law:

Improve the distribution fitting and capture the behaviour of the tail:
→split the cumulative distribution function



Extreme Value Theory:

We want to fit the tail of the distribution. We focuse on the excess function:

$$F_u(x) = P(X - u < x | X > u)$$

For u large enough and under conditions satisfied by most common laws, F_u can be approximated by a single tri-parametric distribution: the Generalized Pareto Distribution :

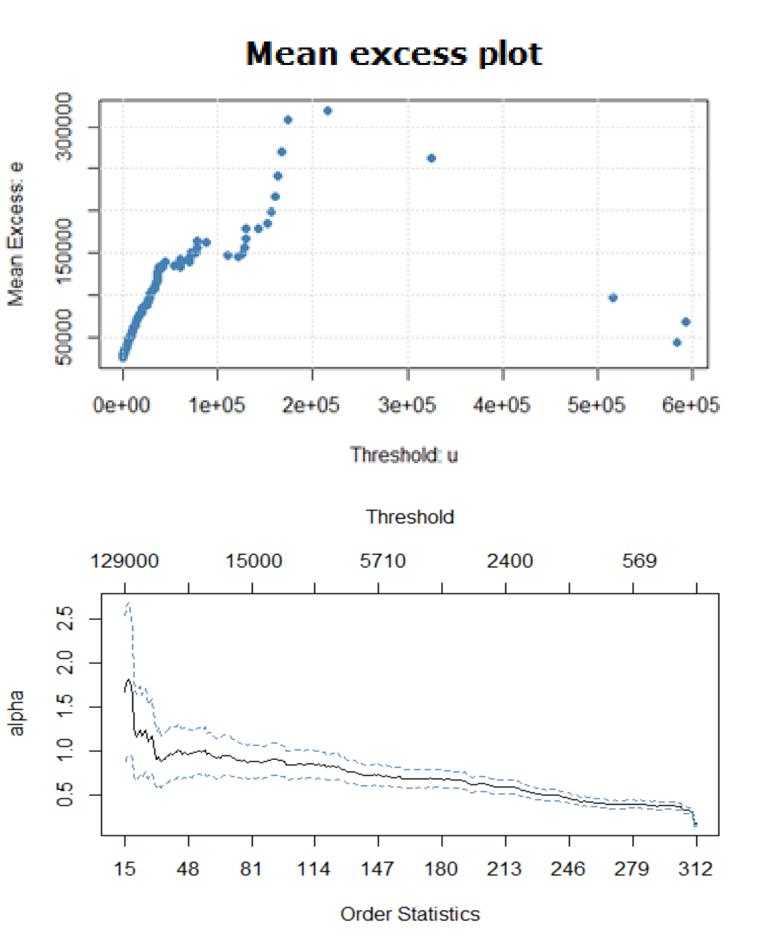
$$\begin{cases} G_{\xi,\sigma}(x) = \left(1 - \left(1 + \xi\frac{x}{\sigma}\right)^{-\frac{1}{\xi}}\right) \; pour\ \xi < 0\ et\ x \in \left]0; -\frac{\sigma}{\xi}\right[\rightarrow \begin{cases} \text{DA of Weibull} \\ \text{upper-bounded} \\ \text{support} \end{cases} \\ G_{\xi,\sigma}(x) = \left(1 - \left(1 + \xi\frac{x}{\sigma}\right)^{-\frac{1}{\xi}}\right) \; pour\ \xi = 0\ et\ x > 0 \rightarrow \begin{cases} \text{DA of Gumbel} \\ \text{low-tail laws} \end{cases} \\ G_{\xi,\sigma}(x) = \left(1 - \left(1 + \xi\frac{x}{\sigma}\right)^{-\frac{1}{\xi}}\right) \; pour\ \xi > 0\ et\ x > 0 \rightarrow \begin{cases} \text{DA of Fréchet} \\ \text{heavy-tail laws} \end{cases} \end{cases}$$

→Given a sample of losses, we can modelize the tail using the GPD in most cases.

EVT fitting: find the domain of attraction and the threshold

- <u>Mean excess plot:</u>
  $e(u) = E[X - u|X > u]$ is a linear function of u. The trend gives the domain of attraction.
  → *Client 1: positive trend: domain of attraction of Fréchet*

- <u>Hill Plot:</u> when truncating the sample and fitting a GPD in the extreme value zone, the Hill estimator stays the same. We seek a point from which the estimator is stable
  → *Client 1: estimator stable from 50 excesses*



Results on loss distribution for Client 1

Fitting	**Results**	**Comments**
Initial (1 law)	Log-Normal distribution	Central trend fitting. Log-Nomal distribution passes adequacy tests.
Final (2 laws)	Empirical distribution + GPD above the threshold	Tail-fitting. GPD test* rejects Log-Normal distribution.

*see Garrido's PhD thesis

The tail of the distribution would have been underestimated using one distribution fitting (Log-Normal belonging to domain of attraction of Weibull).

→Need for a tail-distribution fitting.

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