On Integrated Chance Constraints in ALM for Pension Funds

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1. Introduction
2. Intoduction to stochastic programming
3. Application to ALM for pension funds

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Asset Liability Management (ALM) for pension fund

The pension fund ALM problem is an **optimization problem**.

Stochastic programming is a good tool for pension fund ALMs

- Ziemba and al. [1986], Carino and al. [1994], Consigli and Dempster [1998].

Constraint: Guarantee sufficient solvency over the studying period

- Expected amount of shortage: Integrated Chance Constraint (ICC): Vlerk and al. [2003]
- Novelty: Handleability, Comparability ⇒ scale free, time dependent.

Objective: Analyse and measure the impact of ICC by the means of an example.

1 Introduction

2 Intoduction to stochastic programming
- Stochastic linear program (SLP)
- Risk constraints

3 Application to ALM for pension funds

Stochastic linear program: Structure

Linear program (LP)	Stochastic LP (SLP)
$\min_X \quad C^\top X$	$\min_X \quad C^\top X$
s.t. $BX \quad \lesseqqgtr d$	s.t. $B(\omega)X \quad \lesseqqgtr d(\omega), \quad \omega \subset \Omega$
$X \in \mathcal{X} \subset \mathbb{R}^m$	$X \in \mathcal{X} \subset \mathbb{R}^m$
Algo: Simplex Interior point methods	**Algo:** L-shaped (two-stage) Bender's decomposition (multi-stage) **e.g.:** One period asset allocation.

Solution methods may significantly differ:

- LP is easily handleable in most cases
- In some cases, one may determine deterministic equivalent of the SLP
- SLP algorithms often take advantage of the problem structure

Scenarios and decisions: Kouwenberg [2001]

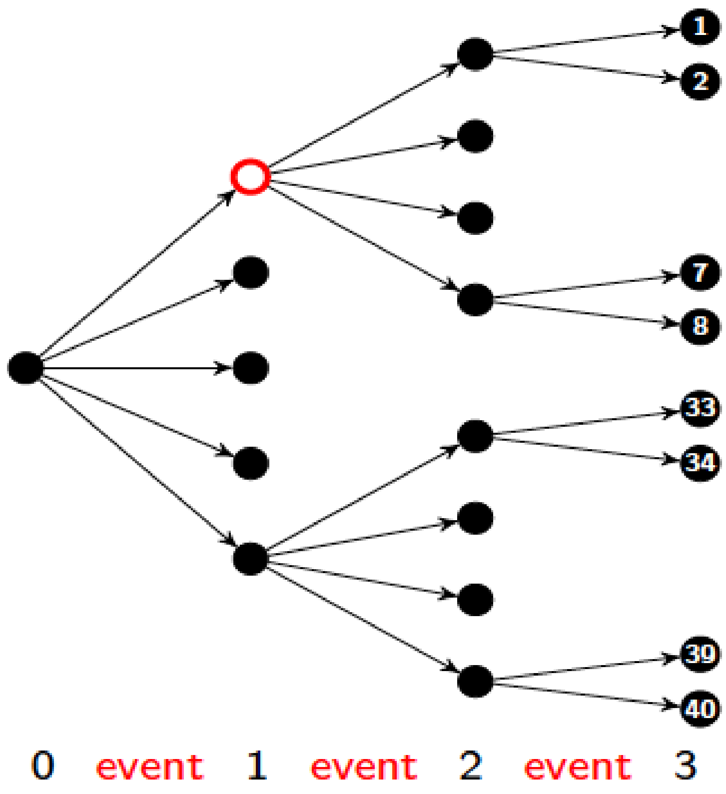


Figure: A scenario tree with 40 scenarios and 66 nodes.

- Discrete time period of one year.
- A node is a possible outcome of the stochastic event at a given time t, denoted (t, s) in scenario s.
- Each path of nodes from $t = 0$ to $t = 3$ is called a scenario.
- Decisions are made at the beginning of each period (node) with full knowledge of the past.
- Scenario trees are the workhorse of Multi-stage SP.
- **Optimality is defined in terms of current costs plus expected future costs.**
- **Only time 0 variables' values are of interest.**
- **Non-anticipativity...**

Multi-period SLP: Dempster and al. [1998]

When objectives and constraints are linear and randomness can be approached by a set of S scenarios, multiperiod linear programming is defined as:

$$
\begin{array}{llllll}
\min_{X_0} C_0^\top X_0 + \mathbb{E}_0 \left\{ \min_{X_1} C_{1,s}^\top X_{1,s} + \mathbb{E}_1 \left\{ \min_{X_2} C_{2,s}^\top X_{2,s} + \cdots + \mathbb{E}_{T-1,s} \left\{ \min_{X_T} C_{T,s}^\top X_{T,s} \right\} \cdots \right\}\right\} \\
\text{s.t. } B_0 X_0 & & & & & = b_0 \\
D_{11}^{S0} X_0 + & B_1^{S0} X_1 & & & & = b_1 \\
D_{21}^{S0} X_0 + & D_{22}^{S1} X_1 + & B_2^{S1} X_2 & & & = b_2 \\
 & & & \ddots & & \vdots \\
D_{T1}^{S0} X_0 + & D_{T2}^{S1} X_1 + & D_{T3}^{S1} X_2 & & D_T^{ST-1} X_{T-1} + B_T^{ST-1} X_T & = b_T \\
 & & & & l_t \le X_t \le u_t;\ t = 1, \cdots, T & \\
 & & & & X \in \mathcal{X} &
\end{array} \quad (1)
$$

with s_t is the history of the process up to time t and over the time horizon $[1, \ldots, T]$.

Integrated chance constraint (ICC): Klein [1986]

One may want to control the expected amount by which the goal (linear constraint) is not attained. We introduce the **Integrated Chance Constraint (ICC)**:

$$\mathbb{E}\left(B^s X - d^s\right)^- \leq \beta \tag{2}$$

where $(a)^- := \max\{-a, 0\}$ and β, a positive user-defined parameter, puts an upper bound on the expected shortfall.

Remark:

- ICC measures the magnitude of not reaching the condition, Klein [1986].
- when randomness is defined by scenarios, (2) is equivalent to $\sum_{s=1}^{S} p^s \left(B^s X - d^s\right)^- \leq \beta$.
  - linear optimization problem with linear feasibility set
  - 1^{st} application to ALM for pension fund: Vlerk and al. [2003]

1. Introduction
2. Intoduction to stochastic programming
3. Application to ALM for pension funds
   - The ALM problem
   - Integrated chance constraint in ALM
   - Results and Conclusion

Settings

We define by

- T the time horizon of the ALM study
- t the time index such that $t \in \{0, 1, \dots, T\}$

- cr_t the contribution rate of year $t + 1$ (decision)
- H_t the asset allocation at time t (decision)
- Z_t is the remedial contribution at time t (decision)

- (t, s) the node at time t in scenario s
- W_t^s the total level of salary during year t in scenario s
- A_t^s the total amount of asset at time t in scenario s
- L_t^s the total amount of liability at time t in scenario s

Funding ratio

The funding ratio (or cover ratio) is defined by

$$FR_t^s := \frac{A_t^s}{L_t^s} \tag{3}$$

- Good indicator of pension fund solvency
- Underfunding occurs when $FR_t^s \leq 1 \Leftrightarrow A_t^s - L_t^s \leq 0$
- The higher FR_t^s is, the healthier the fund is
- Target funding ratio γ s.t. $FR_t^s \geq \gamma \Leftrightarrow A_t^s - \gamma L_t^s \geq 0$

Remark: One can control FR_t^s by making the **right decision (H_0, cr_0 and Z_0)**.

Assumption

- Time horizon $T := 5$ split into five periods of one year
- Defined Benefit fund
- Five classes of assets: Stocks, Real estate, Bonds, deposits and cash
- Initial funding ratio $F_0 := 0.92$ and target funding ratio $\gamma := 1.05$

The ALM problem

Objective: Direct cost + Penalty cost

$$\min_{H,cr,Z} \mathbb{E}_0 \left[\sum_{t=0}^{T-1} v_{t+1} \left(cr_t W_{t+1} + \lambda_z Z_{t+1} \right) + \sum_{t=0}^{T-2} v_{t+1} \lambda_{\Delta_{cr}} \Delta_{cr_t} W_{t+1} \right] \quad (4)$$

where $\Delta_{cr_t} := \mid cr_{t+1} - cr_t \mid$ is the absolute variation of contribution rate from year t to $t+1$, v_t is the discount factor for a cash flow in year t, λ_z and $\lambda_{\Delta_{cr}}$ are, respectively, penalty parameters for remedial contribution and absolute variation of contribution rate.

Constraints:

- Risk constraints: ICC
- Other constraints: short selling, liquidity, Portfolio, budget, operating

Remark: Only H_0, cr_0 and Z_0 are of interest to the decision maker

ICC in ALM

We define the one-period ICC in ALM for pension fund:

$$\mathbb{E}_{t,s}\left(A^{*s}_{t+1} - \gamma L^{s}_{t+1}\right)^{-} := \sum_{s' \in \mathcal{S}} p^{s'}_{t,s} \left(A^{*s'}_{t+1} - \gamma L^{s'}_{t+1}\right)^{-} \leq \beta \tag{5}$$

where β is the risk parameter and $p^{s'}_{t,s}$ stands for the conditional probability to reach node $\left(t+1, s'\right)$ going from (t, s) and $p^{s'}_{t,s} = 0$ for any scenario s' of $t+1$ not descending from (t, s)

- Decisions are taken at each node (t, s) with respect to (5)
- ALM one-year term constraints in a multistage framework
- Linear constraint at each node (t, s)
- **Application:** See Vlerk and al. [2003]
- **Limit:** β is neither scale free, nor time dependent

ICC in ALM (cont'd)

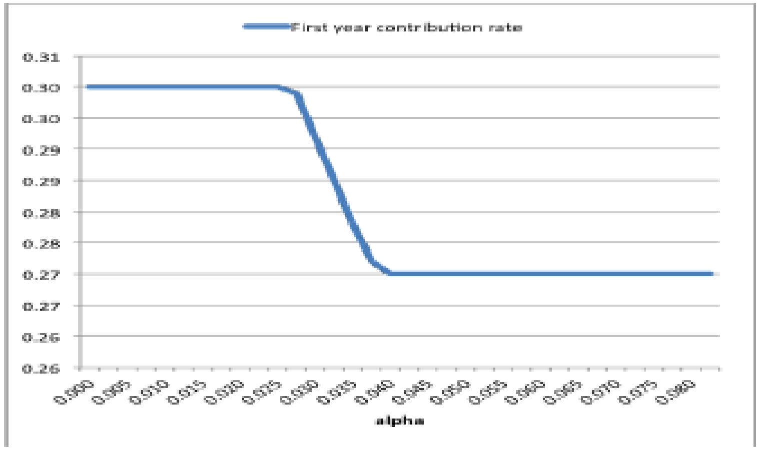
What if we define β as a function of obligations and depending on time: $\beta_t^s := f\left(A_t^s, L_t^s\right) := \alpha L_t^s$? Therefore, constraint 5 leads to

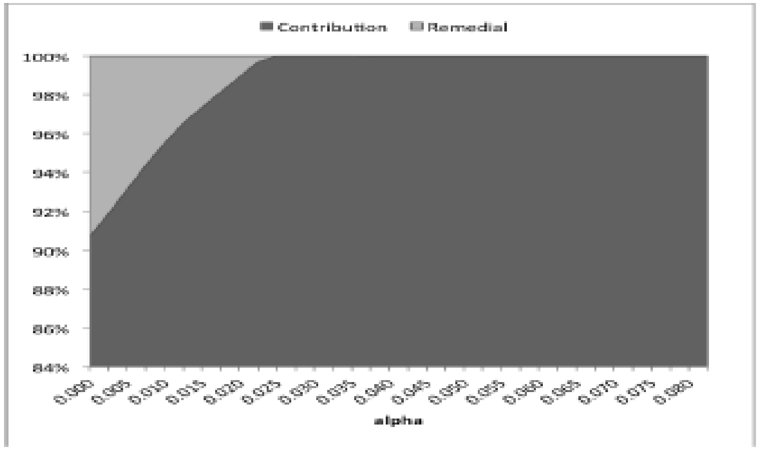
$$\mathbb{E}_{t,s}\left(A_{t+1}^{*s} - \gamma L_{t+1}^{s}\right)^{-} := \sum_{s' \in \mathcal{S}} p_{t,s}^{s'} \left(A_{t+1}^{*s'} - \gamma L_{t+1}^{s'}\right)^{-} \leq \alpha L_t^s \qquad (6)$$

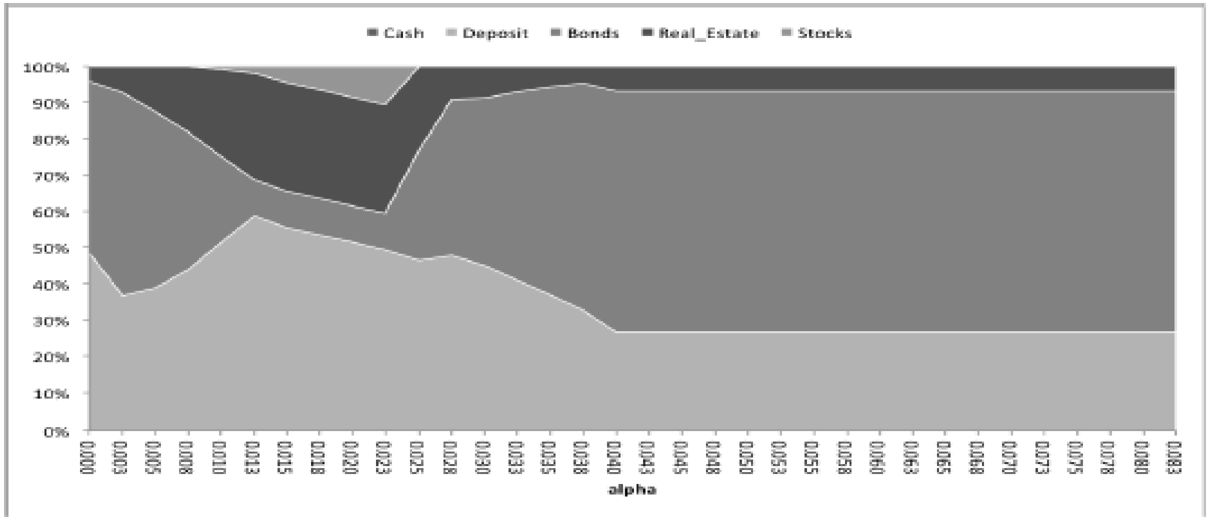
where β_t^s can be obtained at each node (t, s) and α is a scale free parameter

- **Question 1:** *How does the first stage optimal decision change with respect to* α ?
- **Question 2:** *How does the first stage optimal decision change with respect to the initial cover ratio* ?

Answer to question 1; $F_0 := 0.92$



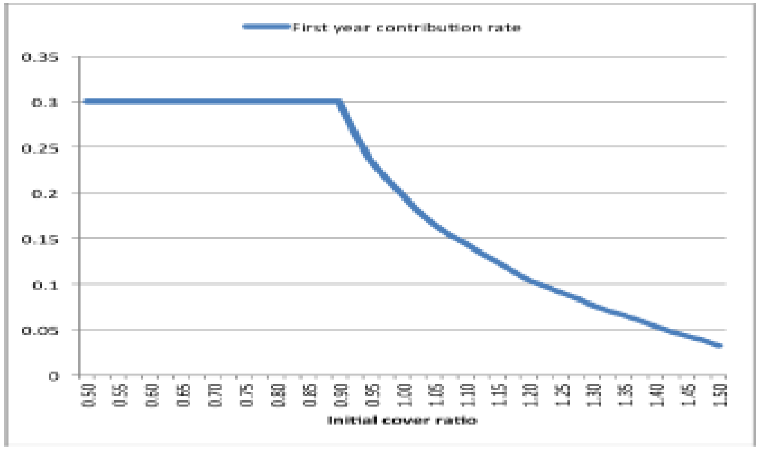


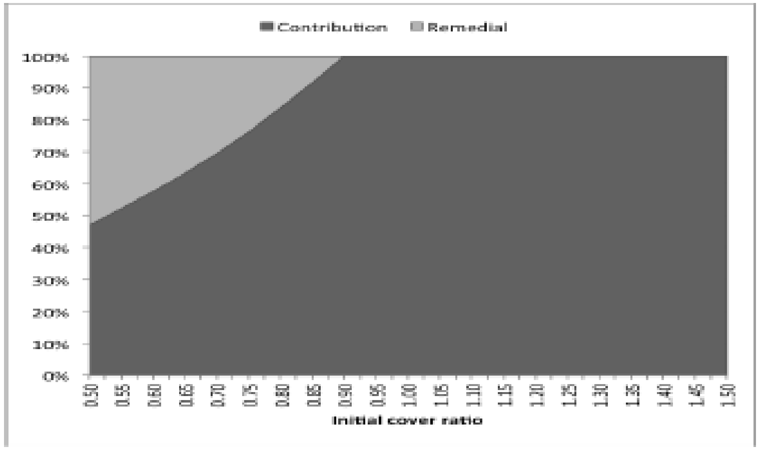


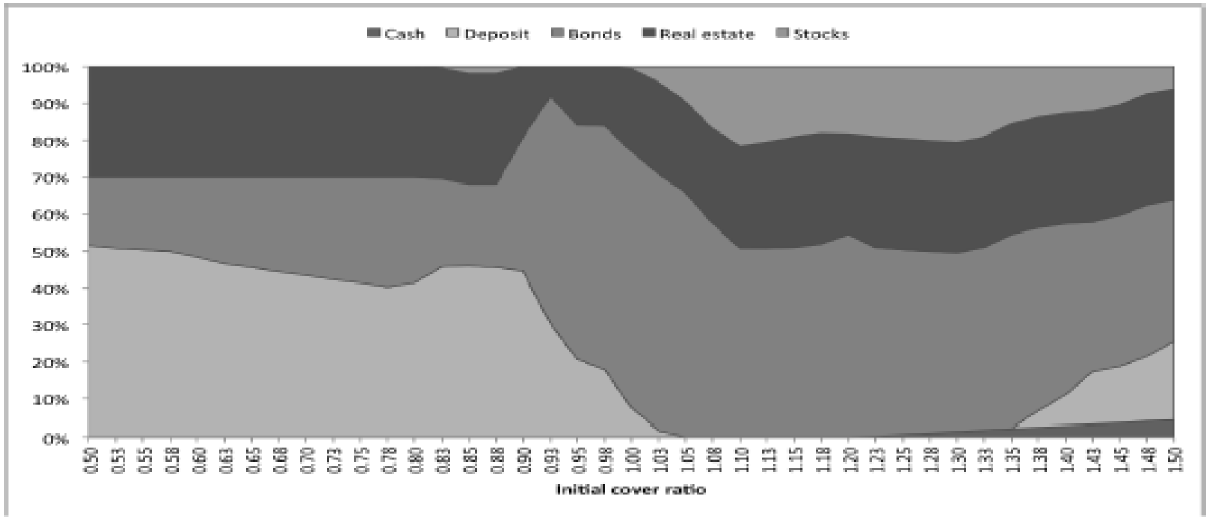
Answer to question 1 (cont'd)

- When $\alpha \leq 0.025$, cr_0 is at its maximum. From $\alpha = 0.025$, it decreases linealy until it reaches 0.27 at $\alpha = 0.04$ and remains unchanged thereafter.
- The proportion of remedial contribution linealy decreases from 9% at $\alpha = 0$ to reach 0% at $\alpha = 0.025$ and stays constant for $\alpha \geq 0.025$.
- From $\alpha = 0.025$, the fund can no more receive remedial contribution.
- The model parameters have been set for this purpose.
- The risk exposition increases with α.
- From $\alpha = 0.04$, the ICC has no more influence on the model.

Answer to question 2, $\alpha := 0.035$







Answer to question 2 (cont'd)

- When $F_0 \leq 0.9$, cr_0 is at its maximum 0.3. From $F_0 = 0.9$, it continuously decreases.
- The proportion of remedial contribution linealy decreases from 52% at $F_0 = 50\%$ to reach 0% at $F_0 = 0.9$ and stays constant for $F_0 \geq 0.9$.
- From $F_0 = 0.9$, the fund can no more receive remedial contribution.
- The model parameters have been set for this purpose.
- When $F_0 < 0.9$, the optimal asset allocation is stable: approximatively 30% in riskier asset.
- From $F_0 = 0.9$, the funding ratio is high enough and the risk exposition increase progressively until it reach 50% at $F_0 = 1.275$.
- For $F_0 \geq 1.275$, the risk exposition decreases in order to guarantee the target funding ratio $\bar{F}$.

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