Solvency Analysis of Deferred Annuities

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Motivation and the Problem

- What role do assumptions about underlying mortality and interest rate dynamics play in the solvency assessment of deferred annuity contracts?
- What is the difference in using deterministic and stochastic mortality models for these purposes?
- How solvency margins are affected by the increase in deferred periods?

Literature Review

- Solvency requirements for annuity/pension portfolios
  - Olivieri and Pitacco [2003]
  - Hari et al. [2008]
  - De Waegenaere et al. [2010]
- Projected life tables and stochastic mortality models
  - Cairns et al. [2009]

General model

- Portfolio reserve

$$\mathcal{V}_t = N_t V_t, \quad t = 0, ..., \omega - x_0,$$

where N_t = random number of insureds alive at time t. V_t = individual reserve

- Random portfolio fund

$$Z_0 = P + C,$$

$$Z_t = Z_{t-1}(1+i) - R_t, \quad t = 1, 2, ..., \omega - x_0,$$

where P = premium and C = initial capital, R_t = random amount of payments.

- Required Solvency Margin

$$Pr\left\{\bigwedge_{t=1}^{T}(Z_t - \mathcal{V}_t \geq 0)\right\} = 1 - \epsilon$$

Deterministic and Stochastic models

- Model of the evolution of the number of survivors

$$D_t|N_t, q_{x_0+t} \sim Bin(N_t, q_{x_0+t}).$$

- $q^{[med]}$ only for simualtion of random number of survivors in deterministic model
- $q^{[min]}$, $q^{[med]}$ and $q^{[max]}$ for simulation of random number of survivors in Stochastic model.
- Full stochastic mortality model q_{x_0+t}

Data

- England and Wales male mortality data from 1965 to 2011
- Identical immediate and d-years deferred ($d = 5, 10, 15, 20$) annuities
- Initial age $x_0 = 65$
- Annual payment $R = 100$.
- Constant interest rate $i = 3\%$ to calculate premium and reserve

Deterministic model.

Table: Required relative solvency margins $\frac{Z_0 - V_0}{\mathcal{V}_0}$. $\epsilon = 0.025$.

	Immediate annuities		Deferred annuities ($d = 5$)	
N_0	$T = n$	$T = 5$	$T = n$	$T = 5$
100	7.47%	4.55%	10.21%	5.24%
1,000	2.39%	1.54%	3.26%	1.84%
10,000	0.77%	0.50%	1.05%	0.60%
100,000	0.24%	0.16%	0.33%	0.19%
1,000,000	0.08%	0.05%	0.10%	0.06%

Deterministic model.

Table: Required relative solvency margins $\frac{Z_0 - V_0}{\mathcal{V}_0}$ for different deferred periods d. $\epsilon = 0.025$.

	$N_0 = 100$			$N_0 = 10,000$			$N_0 = 1,000,000$		
d	$T = n$	$T = 10$	$T = 5$	$T = n$	$T = 10$	$T = 5$	$T = n$	$T = 10$	$T = 5$
0	7.47%	6.02%	4.55%	0.77%	0.64%	0.50%	0.08%	0.06%	0.05%
5	10.21%	7.95%	5.24%	1.05%	0.83%	0.60%	0.10%	0.08%	0.06%
10	13.60%	8.88%	5.24%	1.39%	0.96%	0.60%	0.14%	0.10%	0.06%
20	18.22%	8.88%	5.24%	1.84%	0.96%	0.60%	0.18%	0.10%	0.06%
30	39.54%	8.88%	5.24%	3.81%	0.96%	0.60%	0.38%	0.10%	0.06%

Stochastic model

- A mixture of idiosyncratic and longevity risk
- Weights $\rho^{[min]} = 0.2$, $\rho^{[med]} = 0.6$ and $\rho^{[max]} = 0.2$
- Single premium and individual reserve are still calculated using the function $q^{[med]}$ and the interest rate $i = 0.03$.

Stochastic model.

Table: Required relative solvency margins $\frac{Z_0 - V_0}{\mathcal{V}_0}$. $\epsilon = 0.025$.

	Immediate annuities		Deferred annuities ($d = 5$)	
N_0	$T = n$	$T = 5$	$T = n$	$T = 5$
100	8.56%	4.75%	11.78%	6.38%
1,000	5.00%	2.14%	6.86%	2.61%
10,000	4.04%	1.53%	5.56%	1.86%
100,000	3.74%	1.34%	5.16%	1.64%
1,000,000	3.65%	1.29%	5.03%	1.57%

Stochastic model.

Table: Required relative solvency margins $\frac{Z_0 - V_0}{\mathcal{V}_0}$ for different deferred periods d. $\epsilon = 0.025$.

	$N_0 = 100$			$N_0 = 10,000$			$N_0 = 1,000,000$		
d	$T = n$	$T = 10$	$T = 5$	$T = n$	$T = 10$	$T = 5$	$T = n$	$T = 10$	$T = 5$
0	8.56%	6.51%	4.75%	4.04%	2.69%	1.53%	3.65%	2.37%	1.29%
5	11.78%	8.58%	5.24%	5.56%	3.57%	1.86%	5.03%	3.16%	1.57%
10	15.72%	10.12%	5.24%	7.31%	4.13%	1.86%	6.61%	3.66%	1.57%
20	21.66%	10.12%	5.24%	8.99%	4.13%	1.86%	8.04%	3.66%	1.57%
30	41.70%	10.12%	5.24%	12.74%	4.13%	1.86%	10.61%	3.66%	1.57%

Full Stochastic Mortality Models

	Time indices	Cohort effect	Age categorical
Lee Carter	1	-	+
Renshaw Haberman	1	+	+
Age Period Cohort	1	+	+
Cairns Blake and Dowd (M5)	2	-	-
Cairns Blake and Dowd (M7)	3	+	-
Plat	3	+	-/+

Full Stochastic Mortality Models

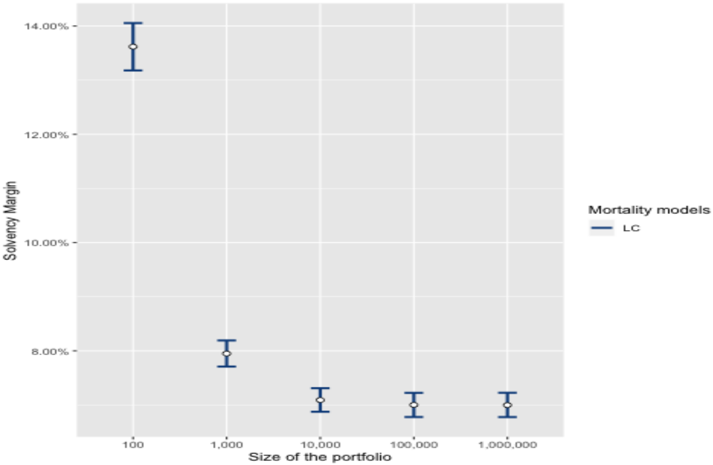

Figure: Solvency margins for different stochastic mortality models for $d = 5$ and $T = n$ with 95% confidence intervals

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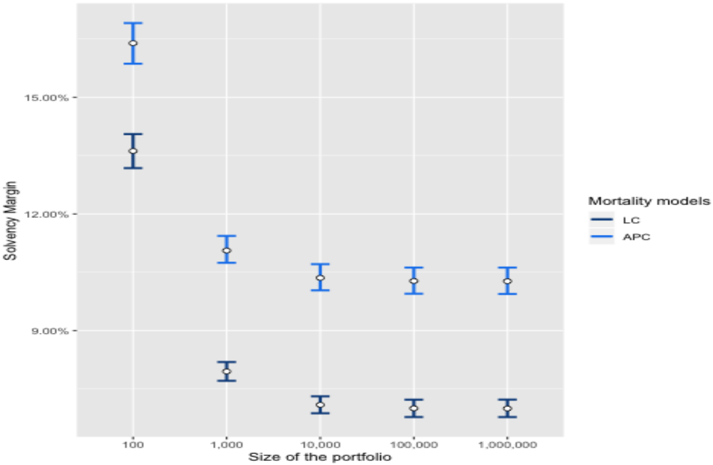

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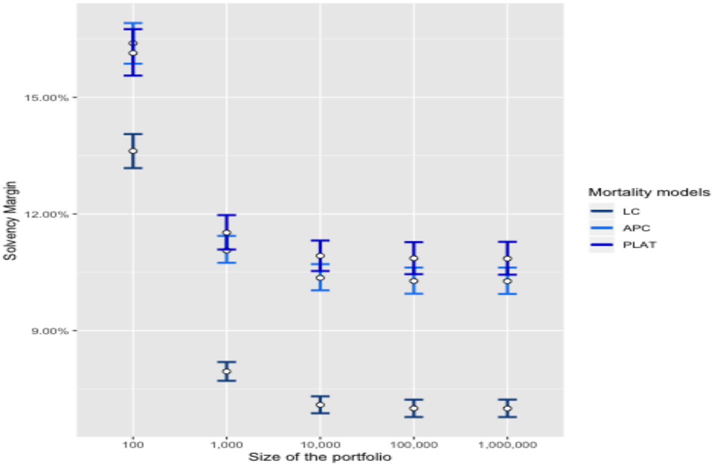

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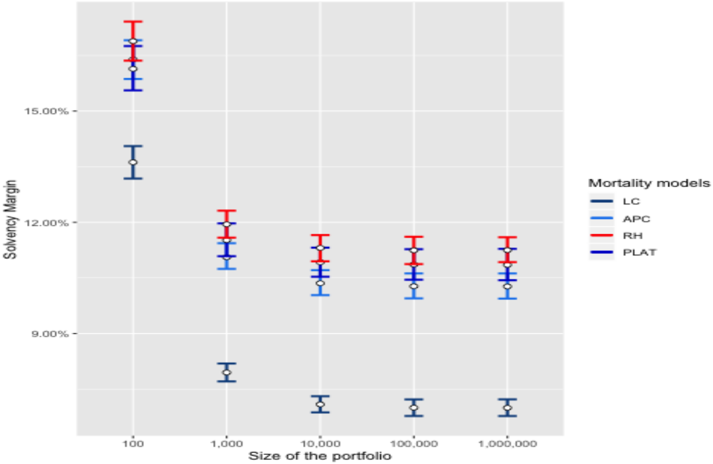

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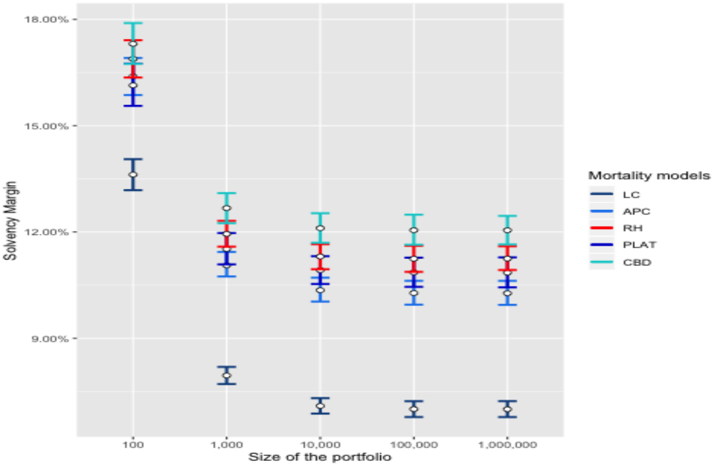

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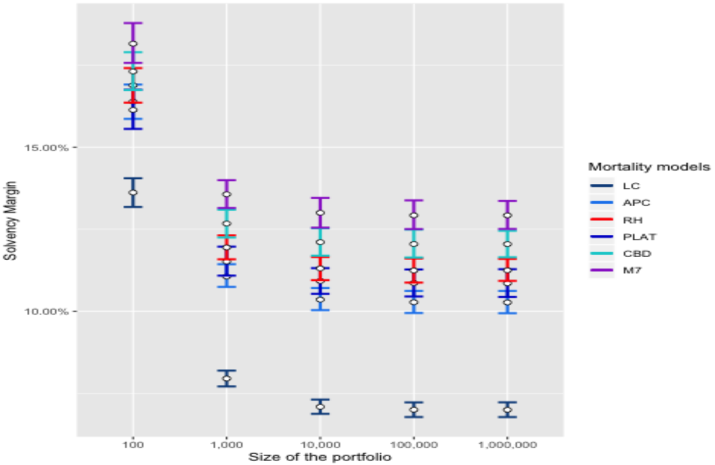

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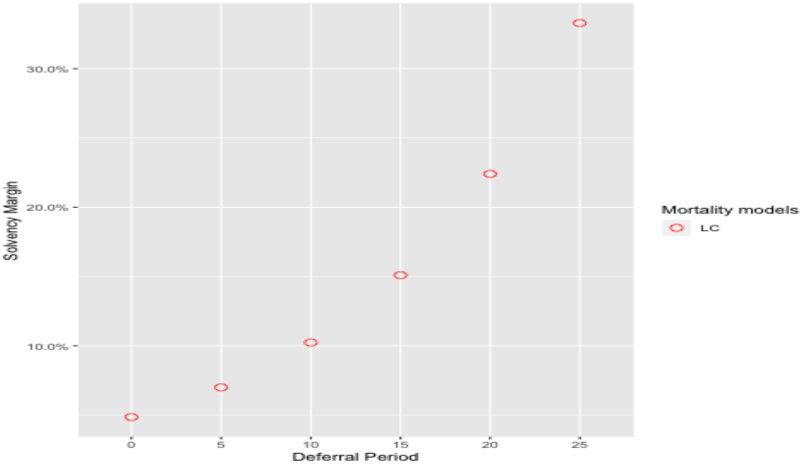

Figure: Solvency margins for different stochastic mortality models and different deferred periods when $T = n$

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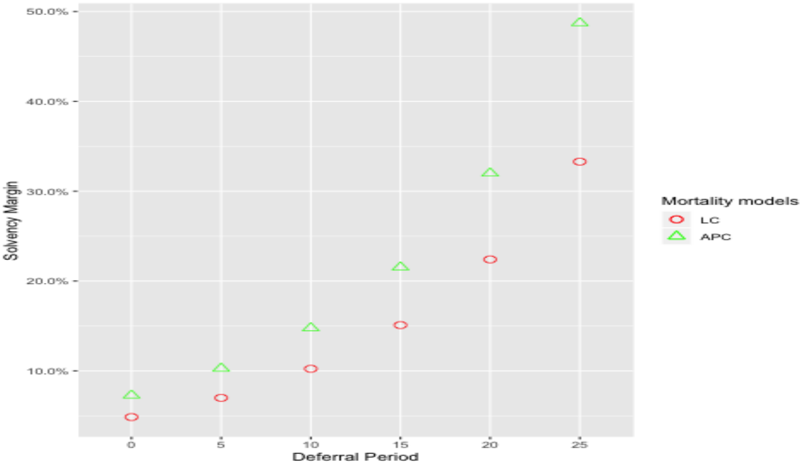

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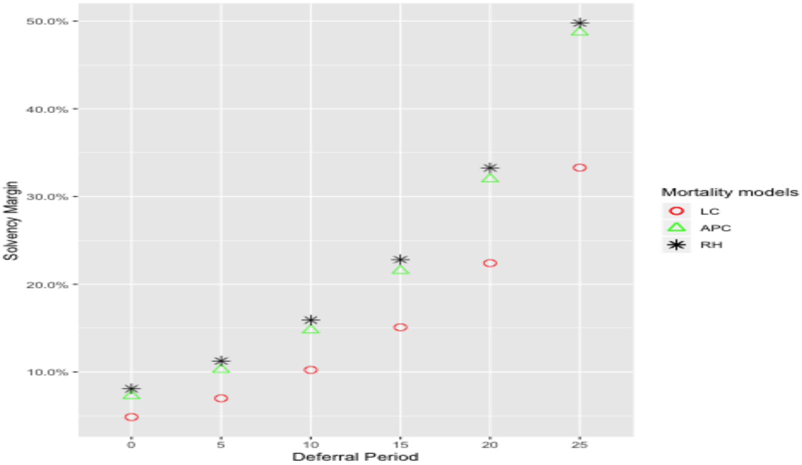

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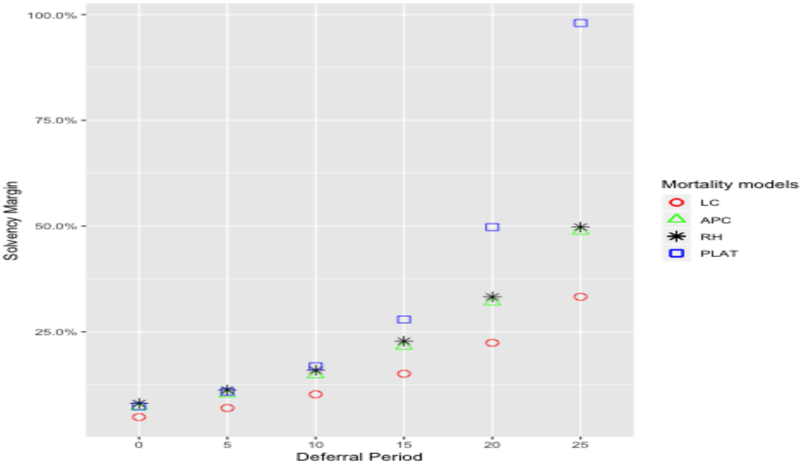

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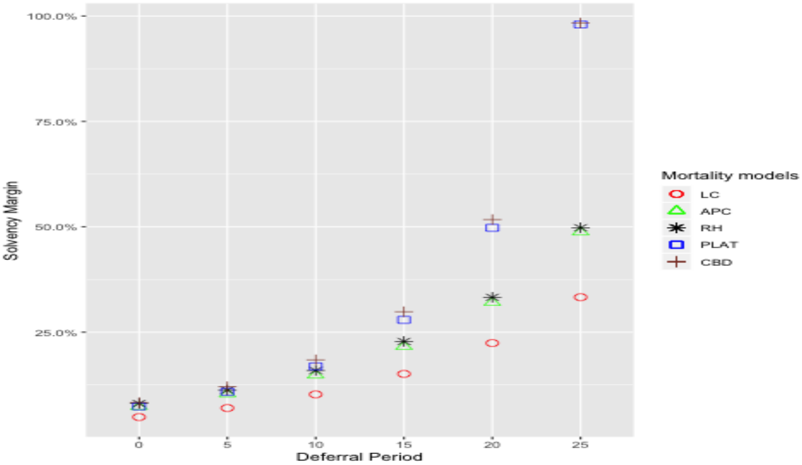

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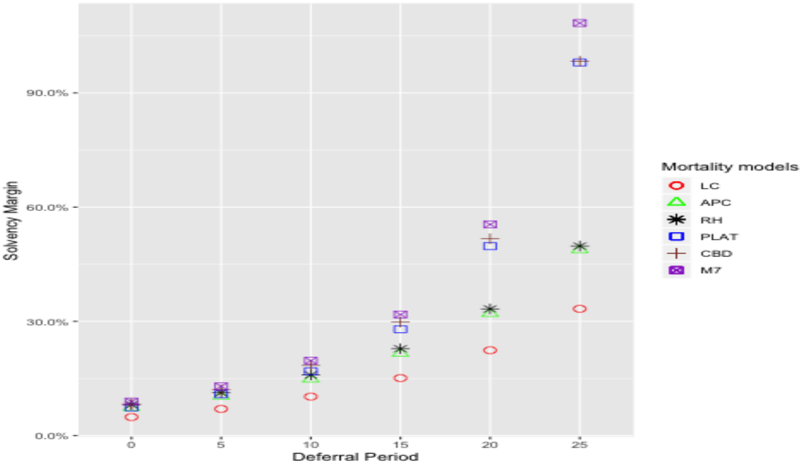

Figure: Solvency margins for different stochastic mortality models and different deferred periods when $T = n$

Vasicek model for simulation of interest rates

- $dr_t = a(\gamma - r_t)d_t + \sigma dW_t$
- The random portfolio fund

$$Z_t = Z_{t-1}(1 + i_t) - R_t$$

where $i_t = e^r - 1$ is the random return.

- Inputs

r_0	0.03	a	0.3
γ	0.03	σ	0.05, 0.1, 0.2

Full Stochastic Mortality Models combined with Vasicek interest rate model

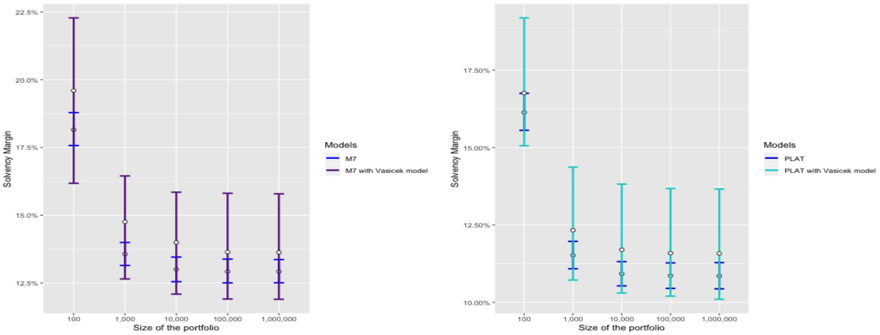

Figure: Solvency margins for different stochastic mortality models combined with Vasicek stochastic interest rate model with $\sigma = 0.05$ for $d = 5$ and $T = n$ with 95% confidence intervals.

Full Stochastic Mortality Models combined with Vasicek interest rate model

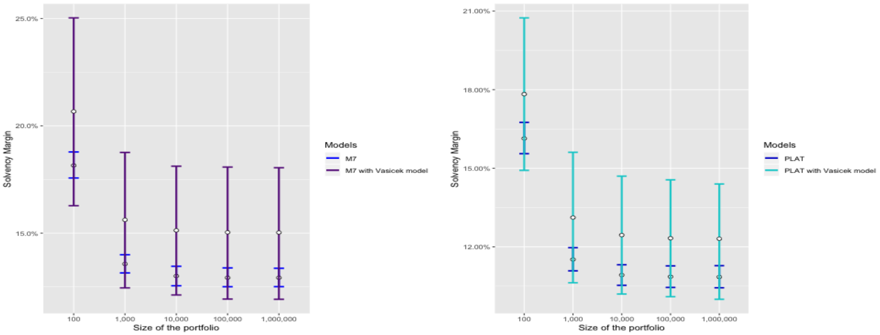

Figure: Solvency margins for different stochastic mortality models combined with Vasicek stochastic interest rate model with $\sigma = 0.1$ for $d = 5$ and $T = n$ with 95% confidence intervals.

Full Stochastic Mortality Models combined with Vasicek interest rate model

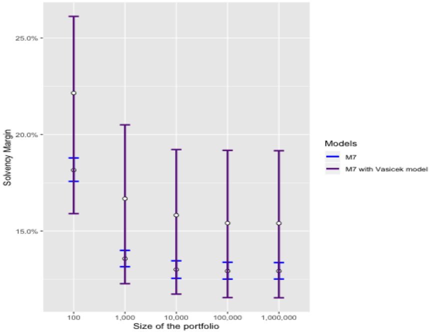

Figure: Solvency margins for different stochastic mortality models combined with Vasicek stochastic interest rate model with $\sigma = 0.2$ for $d = 5$ and $T = n$ with 95% confidence intervals.

Extensions

- Effect of guarantees in annuity period.
- The effect of annual premiums rather than single premium
- $CV[X] = \frac{\sqrt{Var[X]}}{E[X]}$ = coefficient of variation (or risk index), where X = random NPV (Net Present Value) of the portfolio outflows for annuity payments
- Relative and absolute solvency margins, i.e. $[\frac{Z_0 - \mathcal{V}_0}{\mathcal{V}_0}]''$ and $[Z_0 - \mathcal{V}_0]''$ according to the stress scenario (Solvency 2), consisting in a permanent 20% decrease of all the q_x, with relation to the probabilities adopted in premium and reserve calculations

Thank you!