Dynamic and granular loss modeling embracing dependencies

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Agenda

1. Aims
2. Methodology
3. Results
4. Conclusions

Aims

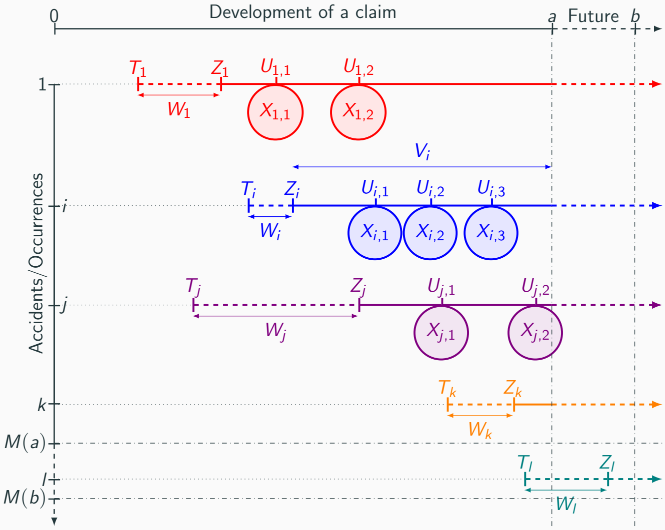
Two Main Goals

- Primary: predict future claim cash flows in non-life insurance and their uncertainty
- Secondary: back-predict incurred but not reported claims due to truncated data

Aggregated vs Granular

- Pitfalls of the conventional reserving techniques:
  - loss of information from the policy and the claim's development due to the aggregation, cf. Norberg (1993)
  - usually small number of observations in the triangle
  - only few observations for recent accident years
  - sensitivity to the most recent paid claims
- How to possibly overcome the issues:
  - individual/claim-by-claim/micro-level/granular data, which do not represent a mainstream in the reserving field, e.g., Antonio & Plat (2014)

Illustration



Methodology

Reporting Dates

- The time ordered reporting dates $\{Z_i\}_{i\in\mathbb{N}}$ are arrival times of a non-homogeneous Poisson process $\{M(t)\}_{t\geq 0}$ with a parametric intensity $\psi(t;\boldsymbol{\rho})$ such that

$$M(t) = \sum_{i=1}^{\infty} \mathbb{1}\{Z_i \leq t\}$$

- The cumulative intensity $\Psi(t;\boldsymbol{\rho}) := \int_0^t \psi(v;\boldsymbol{\rho})\mathrm{d}v$ diverges if $t \to \infty$

Reporting Delays

- The reporting delays W_i's are independent random variables
- Given $Z_n = z$, W_n has a parametric density $f_W(\cdot, z; \boldsymbol{\theta})$

Payment Dates

- The time ordered payment delays $\{U_{i,1} - Z_i, U_{i,2} - Z_i, \dots\}$ of the ith claim are arrival times of a non-homogeneous Poisson process $\{N_i(t)\}_{t \geq 0}$
- $N_i(t) | W_i, Z_i$
- Processes $\{N_i(t)\}_{t \geq 0}$, $i = 1, 2, \dots$ are independent with a parametric intensity $\lambda_i(t; \boldsymbol{\nu}, \boldsymbol{\beta})$ such that

$$N_i(t) = \sum_{k=1}^{\infty} \mathbb{1}\{U_{i,k} - Z_i \leq t\}$$

- The cumulative intensity $\Lambda_i(t; \boldsymbol{\nu}, \boldsymbol{\beta}) := \int_0^t \lambda_i(v; \boldsymbol{\nu}, \boldsymbol{\beta}) \mathrm{d}v$ converges if $t \to \infty$

Payment Amounts

- Sets of the payment amounts $\{X_{ij}\}_j$'s are independent random sequences
- Sequence $\{X_{ij}\}_j$ forms an AR process
- Given $Z_n = z$, the first payment X_{n1} has a parametric density $f_X(\cdot, z; \zeta)$

Accident Dates

- Accident dates as displacement of the reporting dates
- Reporting dates are fully observed, accident dates are truncated
- The displacement theorem (Kingman, 1993) provides that accident dates T_i's are arrival times of another non-homogeneous Poisson process with a parametric intensity

$$\mu(t; \boldsymbol{\rho}, \boldsymbol{\theta}) = \int_{\mathbb{R}} \psi(z; \boldsymbol{\rho}) f_W(t, z; \boldsymbol{\theta}) \mathrm{d}z$$

- Back-fit the incurred but not reported claims

Copula

- Suppose that $Y_t^{(\ell)}$ is a loss amount for a time time period t (e.g., month) and for a LoB ℓ
- Assume that the dependence between lines of business (LoBs) is modeled via a parametric copula (possibly time-varying in order to capture dynamic behavior)
- E.g., $\ell \in \{1, 2\} \ldots$ two LoBs (material damage and bodily injury)

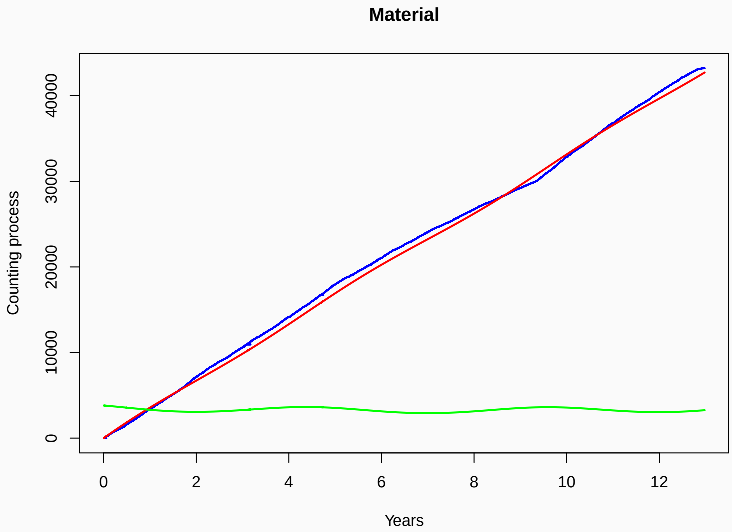
$$\mathbb{P}\left[Y_t^{(1)} \leq y^{(1)}, Y_t^{(2)} \leq y^{(2)}\right] = C\left(\mathbb{P}\left[Y_t^{(1)} \leq y^{(1)}\right], \mathbb{P}\left[Y_t^{(2)} \leq y^{(2)}\right]; \alpha(t)\right)$$

By-product for Stochastic Theory

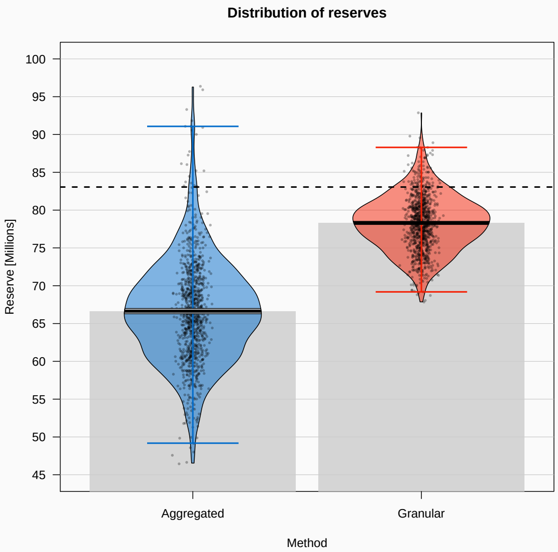
- Probabilistic framework for the n.i.n.i.d. observations
- Time-varying models in an unbalanced panel data setup
- Maximum likelihood estimators derived
- Proved consistency and asymptotic normality of the estimators
- Justification for usage of the method

Results

$M(t)$ as NHPP (Material)

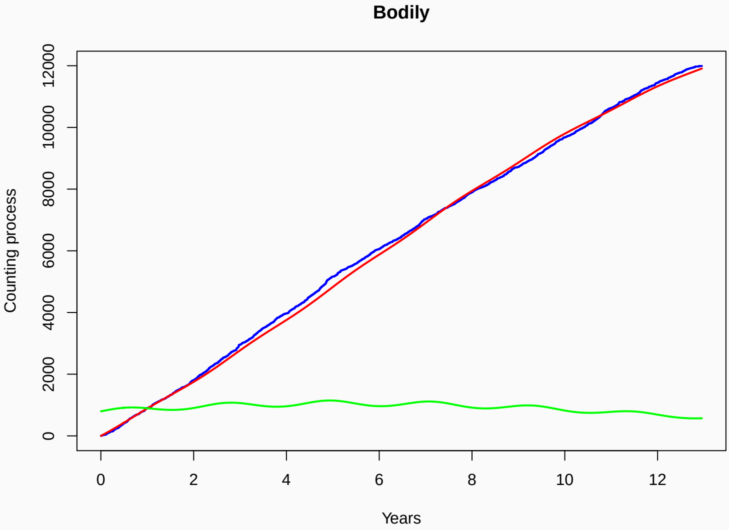


Material

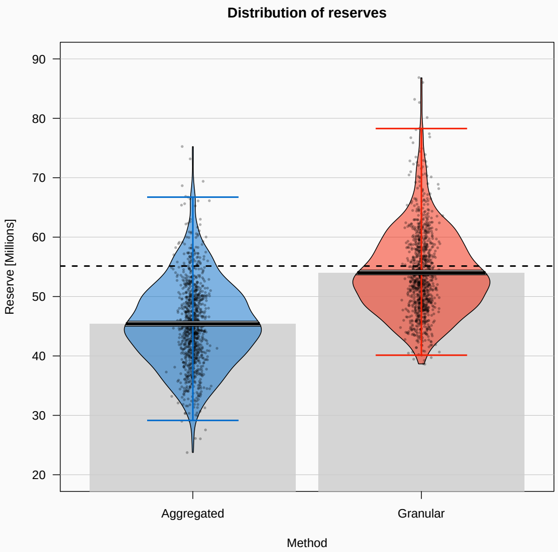


Distribution of reserves
Reserve [Millions]
100
95
90
85
80
75
70
65
60
55
50
45
Aggregated
Granular
Method

$M(t)$ as NHPP (Bodily)

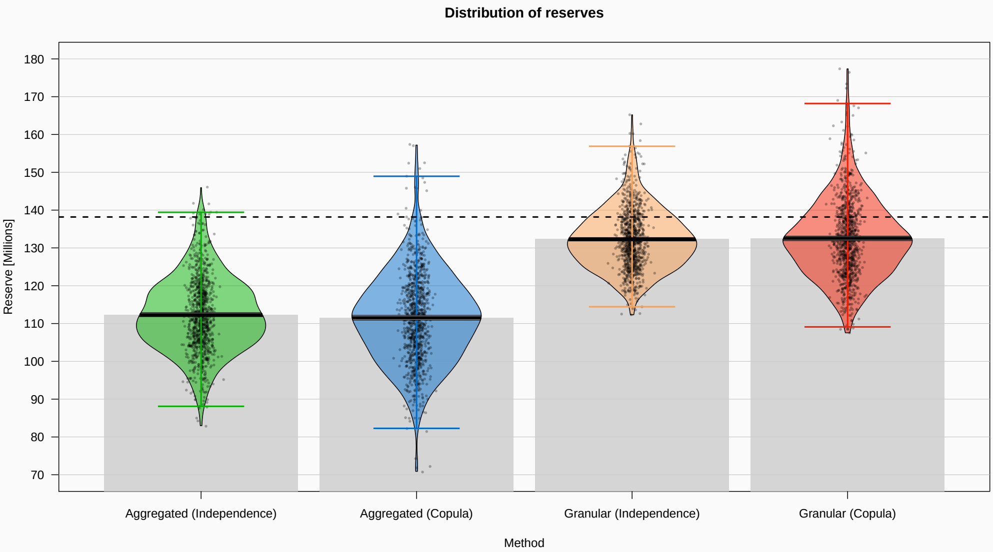


Bodily

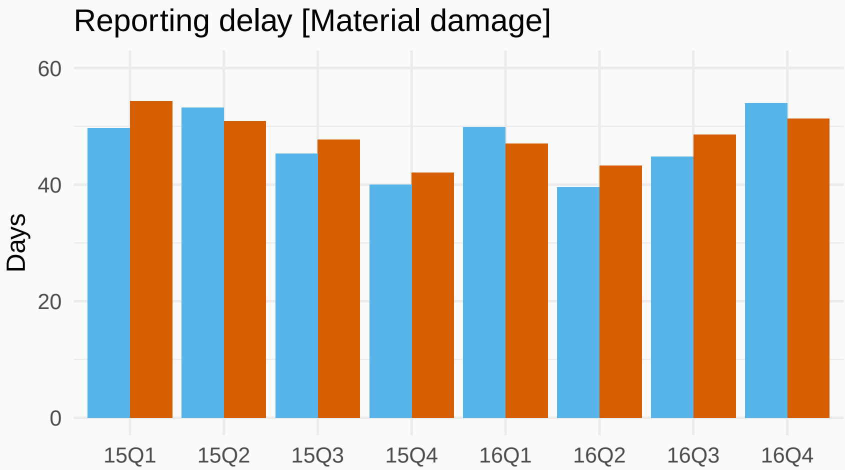


Distribution of reserves
Reserve [Millions]
90
80
70
60
50
40
30
20
Aggregated
Granular
Method

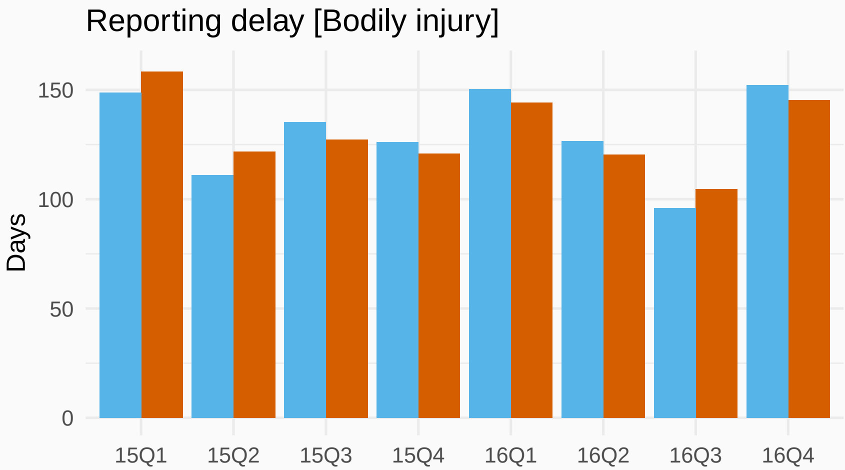
Total


Distribution of reserves
Reserve [Millions]
180
170
160
150
140
130
120
110
100
90
80
70
Aggregated (Independence)
Aggregated (Copula)
Granular (Independence)
Granular (Copula)
Method

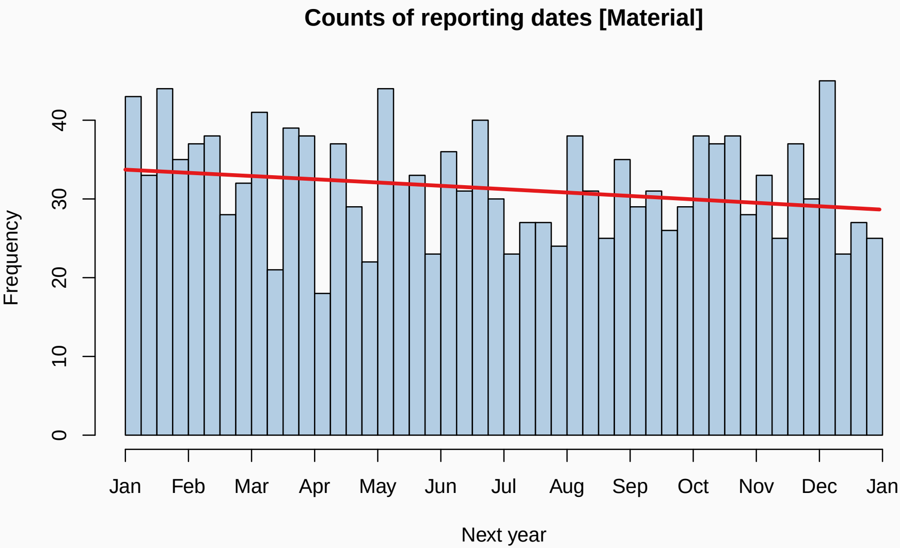
Waiting Period (Material)



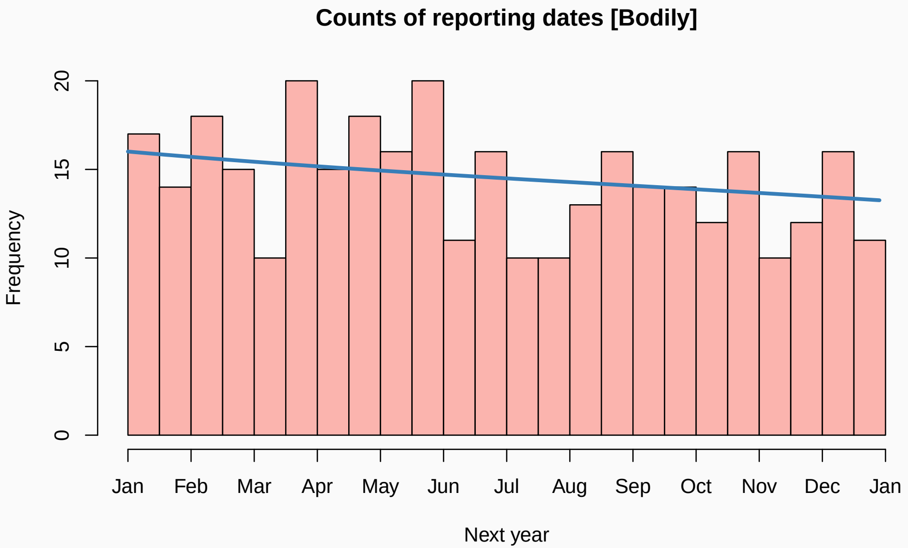
Waiting Period (Bodily)



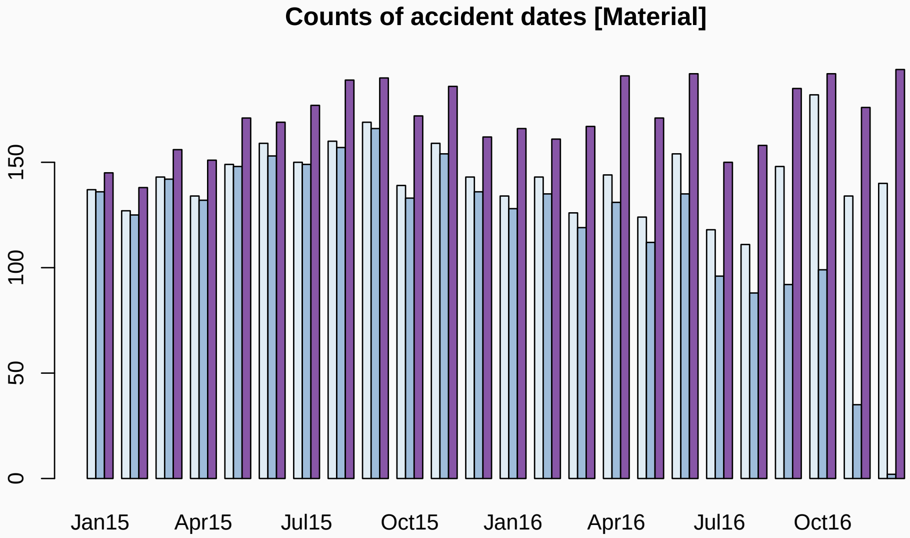
Predicted Future Reportings



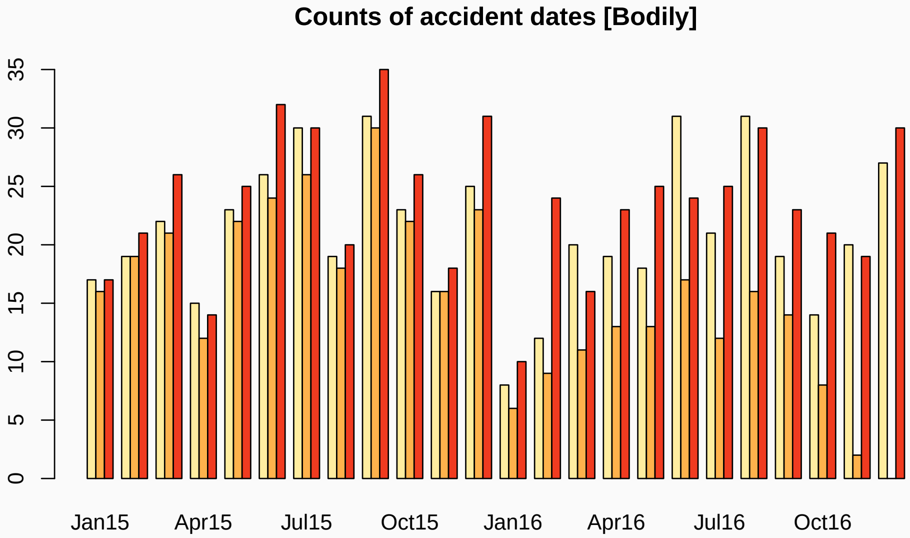
Predicted Future Reportings



Back-fitted Recent Accidents



Back-fitted Recent Accidents



Conclusions

Summary

- Focus on three synergic research areas:

1. Inventing stochastic methods for loss reserving based on claim-by-claim data
2. Using dynamic copulae for modeling dependencies among types of claims
3. Deriving appropriate statistical inference for these approaches

Questions?

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References

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- Maciak, M., Okhrin, O., and Pešta, M. (2018): Micro and Dynamic Claims Reserving Embracing Dependencies, Submitted.
- Norberg, R. (1993): Prediction of outstanding liabilities in non-life insurance, ASTIN Bull. 23(1), 95–115