IAA AAI SECTION COLLOQUIUM 2019

THE MODERN ACTUARY - CHALLENGE • INFLUENCE • LEAD

ASTIN • IAAHS • IAALS • IACA • PBSS

2 - 5 April
2019
Cape Town
South Africa
CTICC

www.colloquium2019.org.za

Hosted by

ACTUARIAL SOCIETY OF SOUTH AFRICA



A STOCHASTIC INVESTMENT MODEL FOR SOUTH AFRICAN USE

Şule Şahin* and Shaun Levitan**

***University of Liverpool, UK and Hacettepe University, Turkey**

**** Colourfield, SA**

Hosted by ACTUARIAL SOCIETY OF SOUTH AFRICA

SECTION COLLOQUIUM 2019

Outline

- **Introduction**
- **Data**
- **Structure of the Model**
- **Components of the Model**
  - Price Inflation
  - Share Dividend Yields
  - Share Dividends
  - Long-term Interest Rates
  - Short-term Interest Rates
    - Nominal Yield Curve
  - Index-linked Bonds
- **Parameter Stability**
- **Conclusions and Future Work**

Introduction

- There continues to be a global need for stochastic modelling in pension, life assurance, banking and investment industries
- Beyond the natural application, it is required by regulation in many instances
- The first published stochastic investment model was done by Wilkie in 1986 and updated and extended in 1995 and 2011
- Limited research has been done in South Africa
- The seminal paper was done by Thomson in 1996 (a model which by his own account had a number of practical limitations)

Introduction

- **The Wilkie Model** (1986, 1995, 2011, 2016, 2017 (4), 2018, 2019, *to be continued*)
  - first comprehensive stochastic model for actuarial applications
  - cascade structure – price inflation is the driving force
  - yearly data
  - long-term forecasting
- **Thomson** (1996, 2005, 2009, 2010)
  - SA data (price inflation, rental yields and growth rates, short-term and long- term interest rates, dividend yields and growth rates)
  - Wilkie type model
  - Equilibrium modelling – no arbitrage

Introduction

- We propose a stochastic investment model for South African use by modelling
  - Price inflation;
  - Short and long term interest rates;
  - Inflation-linked bond yields; and
  - Local equity returns

- Thomson's model used data from 1960-1993. Granger and Newbold (1977) express lack of confidence of successful model identification with much less than 45-50 observations

- We use data from **1960-2018**

Data

- Data from **1960-2018** has been used

- **Inflation Data**
  - South African Consumer Price Index (CPI) provided by Statistics South Africa

- **Equity Data**
  - Indices used are JSE-Actuaries All Share Index (CI101) by INET
  - Thomson proposed use of ADY – Dividend yield on the Index
  - Until June 2001
  - Data updated until 2001 by IRESS but data then used is J203 (ALSI Total Return Index) and J202 (ALSI Dividend Yield)

Data

- **Long-term interest bearing securities**

  - JSE-Actuaries Long Bond Yield (i.e. the 20 year Bond Yield) is used
  - Longer than the average outstanding term of quoted long-term interest-bearing securities
  - Quote is JAYC20 (Previously data provided by INET and updated by IRESS)

- **Short-term interest rates**

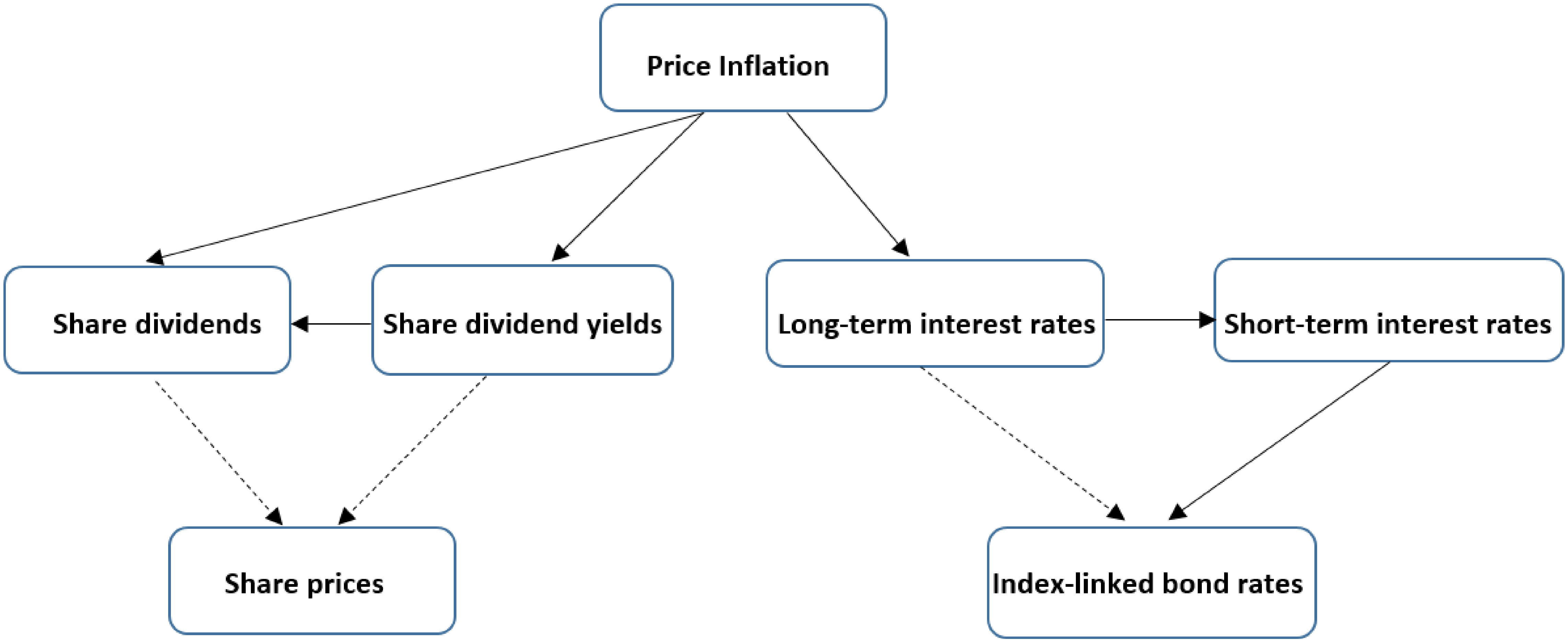
  - Thomson originally used data provided by Ginsburg Malan and Carsons Money-Market Index as listed under code GMC1.
  - This is now more commonly referred to as the Alexander Forbes Money-Market Index and was provided by IRESS.

Data

➢ **Index-linked bonds**

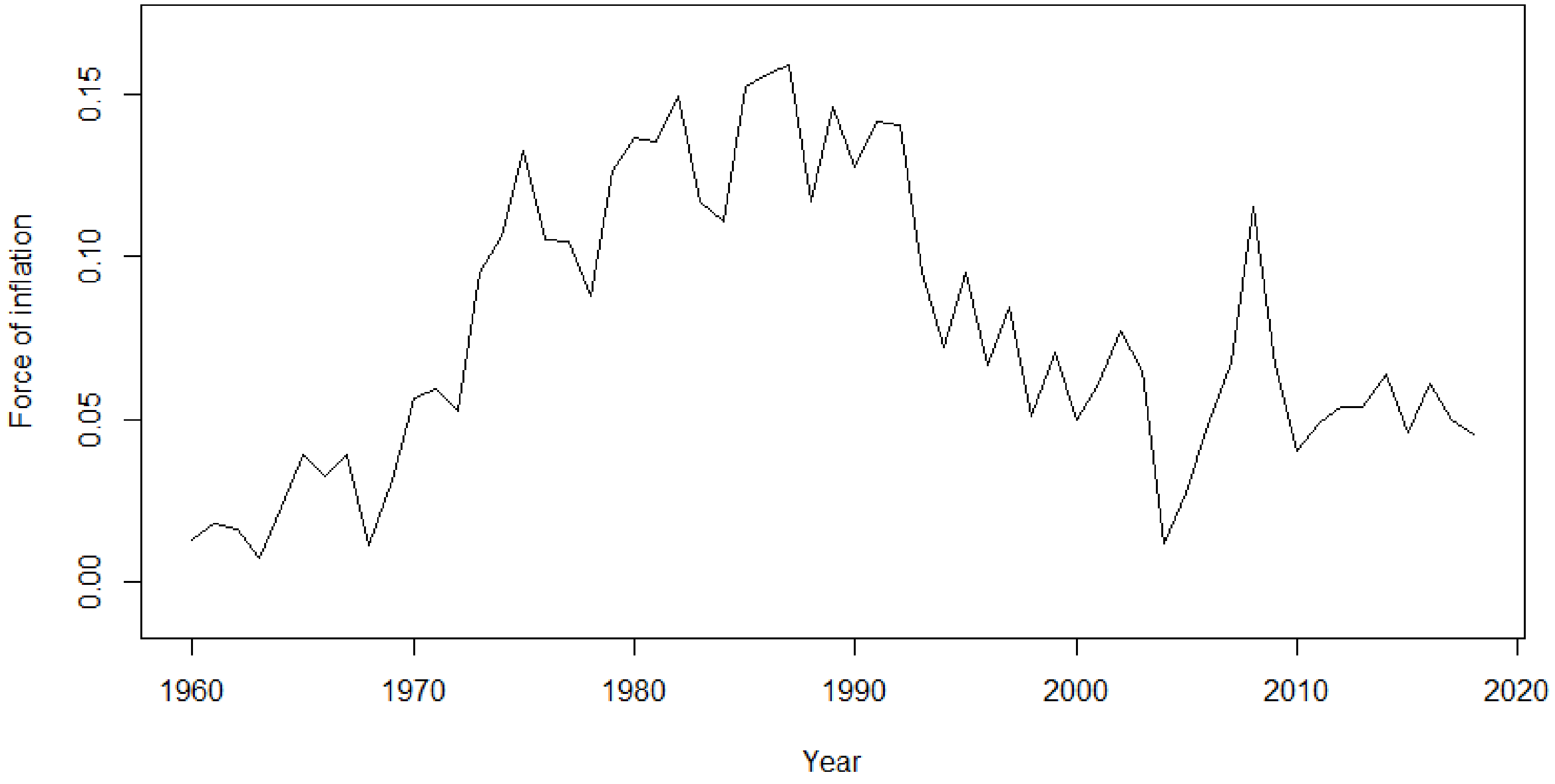
- The yields on SA Government bonds in issue since first issuance in March 2000 used
- Data provided by Bloomberg

Structure of the Model



Components of the Model

Price Inflation



Annual force of inflation based on CPI, 1960-2018

Price Inflation

The model for the Consumer Price Index (CPI), where Q_t is the value of CPI at time t, is:

$$
\begin{aligned}
Q_t &= Q_{t-1}.\exp\left(\delta_q(t)\right) \\
\delta_q(t) &= \mu_q + a_q.(\delta_q(t-1) - \mu_q) + \epsilon_q(t) \\
\epsilon_q(t) &= \sigma_q.z_q(t) \\
z_q(t) &\overset{\text{iid}}{\sim} N(0,1)
\end{aligned}
$$

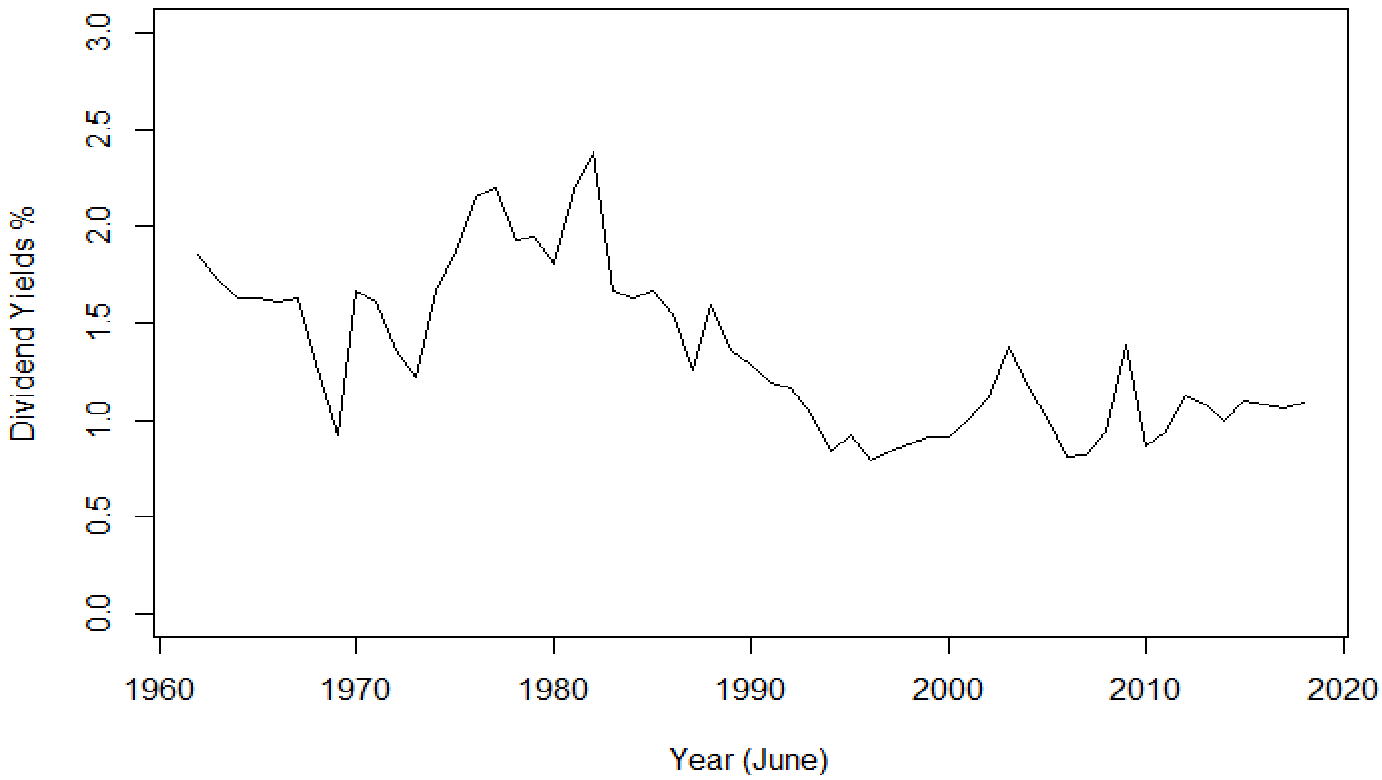
where μ_q is the long-run mean, a_q is the autoregressive parameter, σ_q is the standard deviation of the residuals and z_q is unit normal variables.

Price Inflation

$\delta_q(t)$	1960-2018
μ_q	0.0809 (0.0185)
a_q	0.8433 (0.0670)
σ_q	0.0220 (0.0020)
Log Likelihood	139.05
$r_z(1)$	-0.119
$r_{z^2}(1)$	-0.043
skewness $\sqrt{\beta_1}$	-0.1031
kurtosis β_2	2.7841
Jarque-Bera χ^2	0.2191
$p(\chi^2)$	0.8962

- All parameters are significant
- Residuals are independent and normally distributed
- No ARCH effect

Share Dividend Yields



Dividend Yields %, 1961-2018

Share Dividend Yields

$y(t)$ is the yield on the index at time t, $ym(t)$ is the moving average effect of inflation on dividend yields.

$$
\begin{aligned}
ym(t) &= d_y.\delta_q(t) + (1 - d_y).ym(t-1) \\
y_q(t) &= w_y.ym(t) + (1 - w_y).\delta_q(t) \\
ln\ y(t) &= y_q(t) + \mu_y + yn(t) \\
yn(t) &= a_y.yn(t-1) + \epsilon_y(t) \\
\epsilon_y(t) &= \sigma_y.z_y(t) \\
z_y(t) &\overset{\text{iid}}{\sim} N(0,1)
\end{aligned}
$$

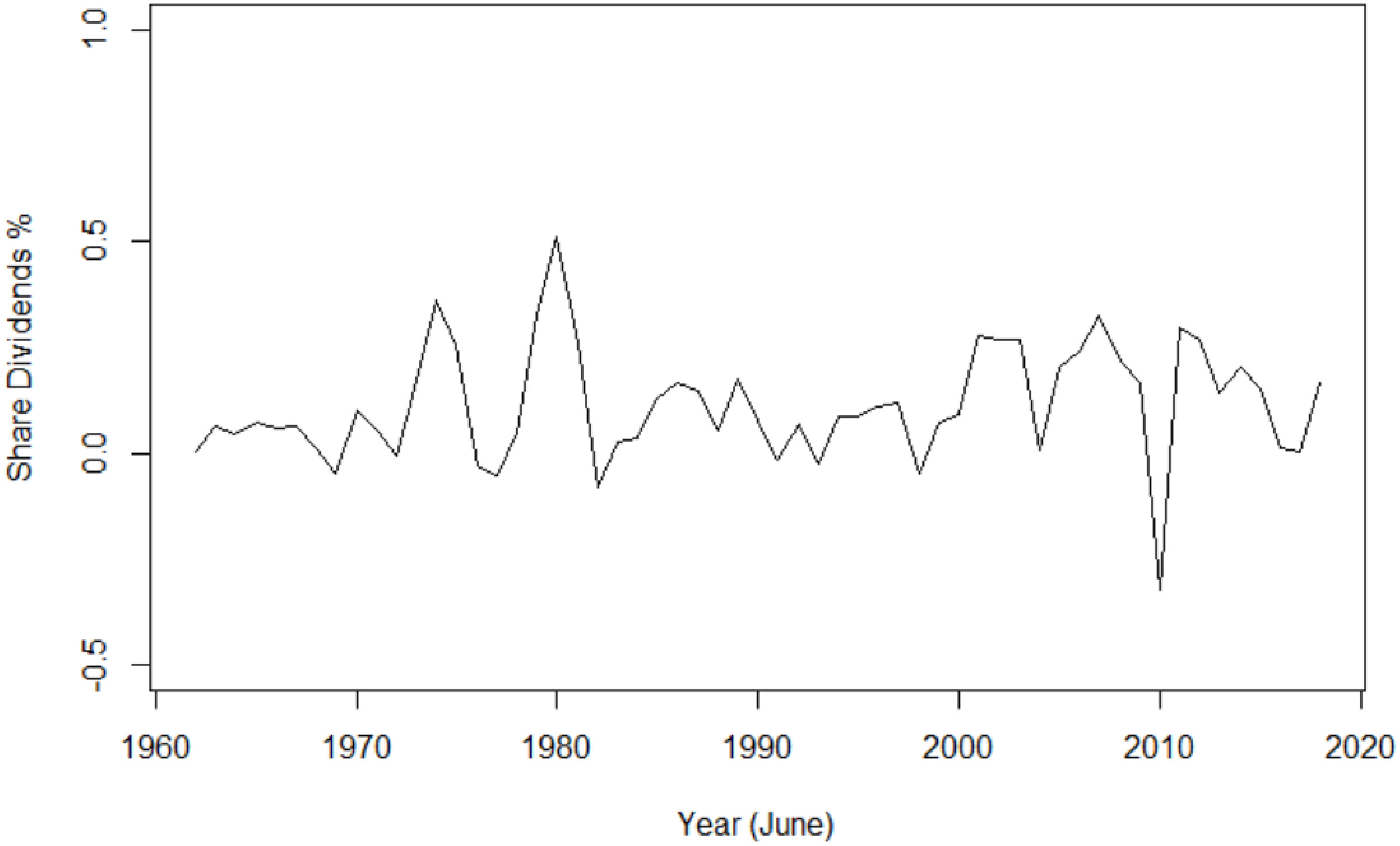
that is $z(t)$ is a series of independent, identically distributed unit normal variates.

Share Dividend Yields

1962-2018		
$\delta_y(t)$	AR(1)	MA Inflation effect
w_y		-4.0074 (1.2161)
d_y		0.1396 (0.0557)
μ_y	1.2695 (0.1774)	0.3781 (0.1152)
a_y	0.8266 (0.0727)	0.6318 (0.0890)
σ_y	0.2261 (0.0214)	0.1973 (0.0186)
Log Likelihood	3.81	11.42
$r_z(1)$	-0.049	-0.132
$r_{z^2}(1)$	0.180	-0.036
skewness $\sqrt{\beta_1}$	0.5254	0.2350
kurtosis β_2	4.0364	2.9365
Jarque-Bera χ^2	5.1731	0.5343
$p(\chi^2)$	0.0753	0.7656

- All parameters are significant
- Significant cross correlation between the price inflation and dividend yields
- Weighted moving average effect of inflation improves the model significantly
- Residuals are independent and normally distributed
- No ARCH effect

Share Dividends



Share Dividends

$$D_t = P_t . Y_t$$

where D_t is share dividends, P_t is share prices and Y_t is share dividend yields.

The logarithm of the dividend growth,

$$\delta_d(t) = lnD_t - lnD_{t-1}$$

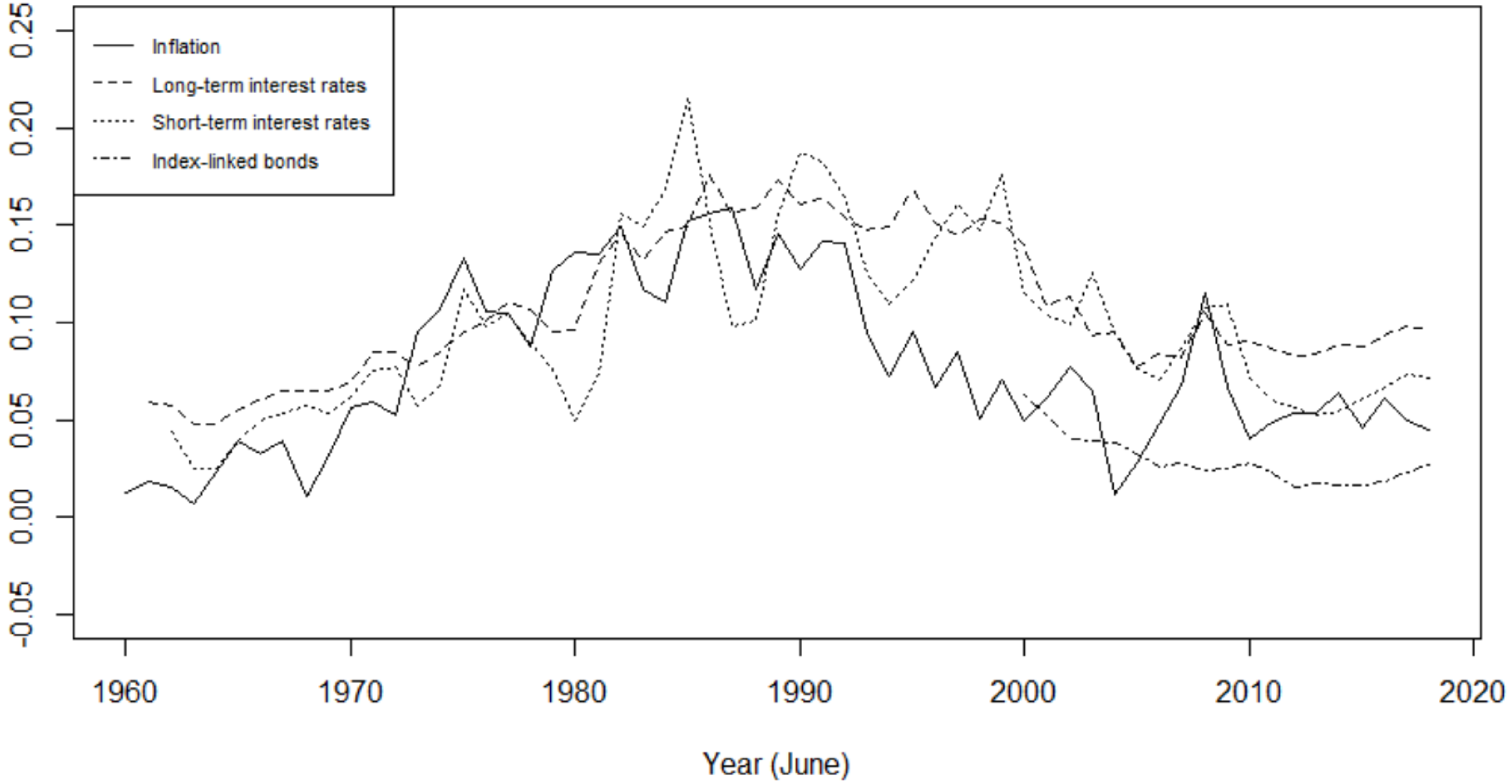
$$\begin{aligned}
dm(t) &= d_d.\delta_q(t) + (1 - d_d).dm(t-1) \\
d_q(t) &= w_d.dm(t) + (1 - w_d).\delta_q(t) \\
\delta_d(t) &= d_q(t) + \mu_d + y_d\epsilon_y(t-1) + k_d.\epsilon_d(t-1) + \epsilon_d(t) \\
\epsilon_d(t) &= \sigma_d.z_d(t) \\
z_d(t) &\overset{\text{iid}}{\sim} N(0,1)
\end{aligned}$$

Share Dividends

1962-2018			
$\delta_d(t)$	only δ_y effect	+ Inflation effect	+ MA Inflation effect
w_d			-4.7651 (2.9754)
d_d			0.6164 (0.2052)
q_d		0.8462 (0.4902)	
μ_d	0.1549 (0.0278)	0.0885(0.04690)	0.0784 (0.0255)
y_d	-0.1739 (0.0742)	-0.1773 (0.0712)	-0.1975 (0.0698)
k_d	0.3438 (0.1172)	0.3660 (0.1141)	0.3260 (01217)
σ_d	0.1222 (0.0115)	0.1190 (0.0112)	0.1116 (0.0105)
Log Likelihood	38.27	39.76	43.34
$r_z(1)$	0.009	0.031	0.038
$r_{z^2}(1)$	0.179	0.181	0.137
skewness $\sqrt{\beta_1}$	0.0187	-0.0293	0.0051
kurtosis β_2	3.9069	3.6078	3.1577
Jarque-Bera χ^2	1.9566	0.8855	0.0593
$p(\chi^2)$	0.3759	0.6423	0.9708

- All models fit the data well
- All parameters are significant?
- Residuals are independent and normally distributed
- No ARCH effect
- Moving average effect of inflation!

Long-term Interest Rates



SECTION COLLOQUIUM 2019

Long-term Interest Rates

For long-term interest rates $\delta_c(t)$, the JSE-Actuaries Long Bond Yield (i.e. JAYC20, the 20-year Bond Yield) is used as in Thomson (1996).

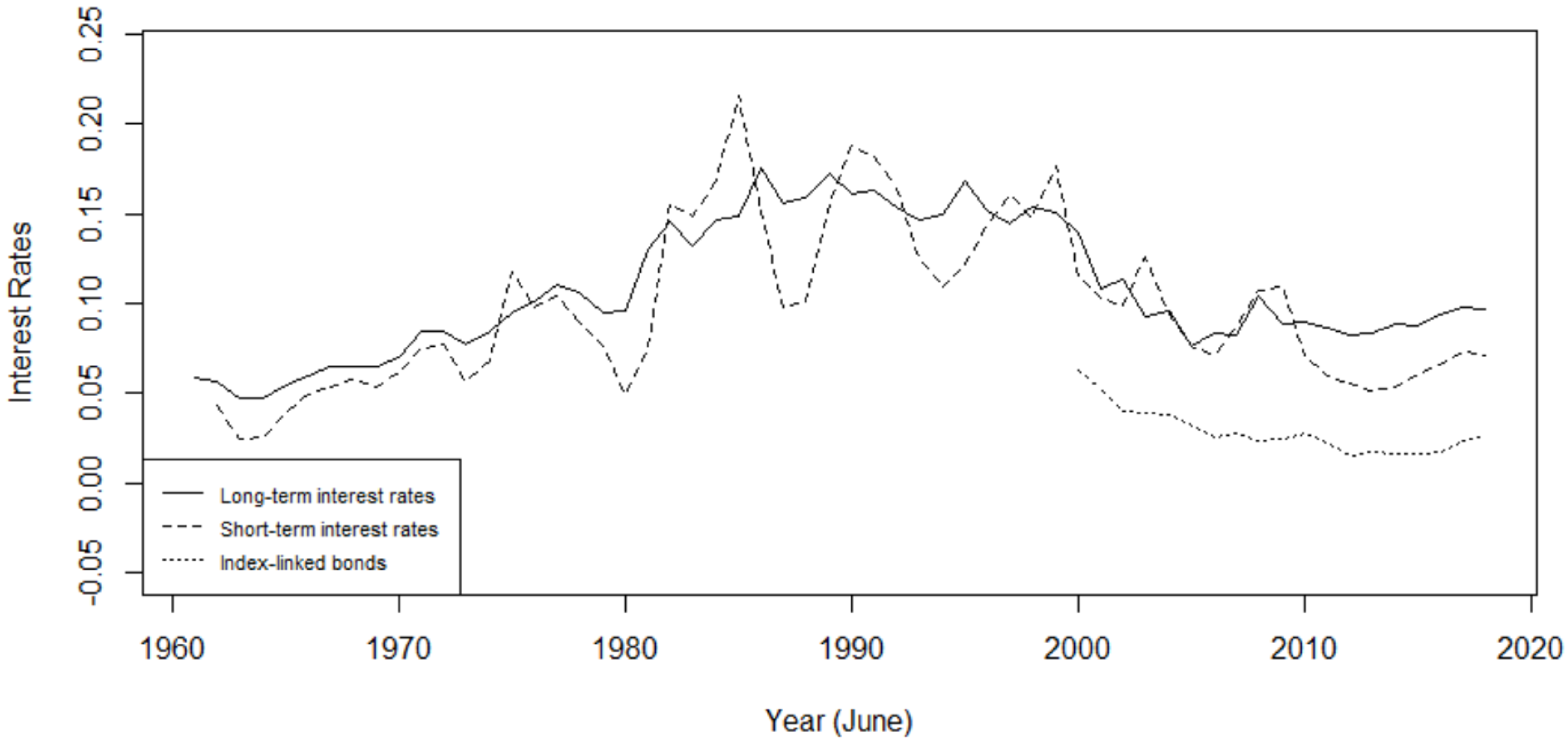
$$\begin{aligned}
cm(t) &= d_c.\delta_q(t) + (1 - d_c).cm(t-1) \\
cr(t) &= \delta_c(t) - w_c.cm(t) \\
ln\ cr(t) &= ln\ \mu_c + cn(t) \\
cn(t) &= a_c.cn(t-1) + \epsilon_c(t) \\
\epsilon_c(t) &= \sigma_c.z_c(t) \\
z_c(t) &\overset{\text{iid}}{\sim} N(0,1)
\end{aligned}$$

Long-term Interest Rates

$\delta_c(t)$	1961-2018
w_c	1.0
d_c	0.13
$ln\ \mu_c$	-3.3892 (3.37%) (0.1086)
a_c	0.5665 (0.1117)
σ_c	0.3610 (0.0341)
$r_z(1)$	-0.108
$r_{z^2}(1)$	0.109
skewness $\sqrt{\beta_1}$	-0.9368
kurtosis β_2	4.2522
Jarque-Bera χ^2	12.06
$p(\chi^2)$	0.0024

- **Fisher relation:** Nominal rates = Real interest + Implied Inflation
- Exponentially weighted moving average effect of inflation
- All parameters are significant
- Residuals are independent but not normally distributed
- No ARCH effect

Short-term Interest Rates



SECTION COLLOQUIUM 2019

Short-term Interest Rates

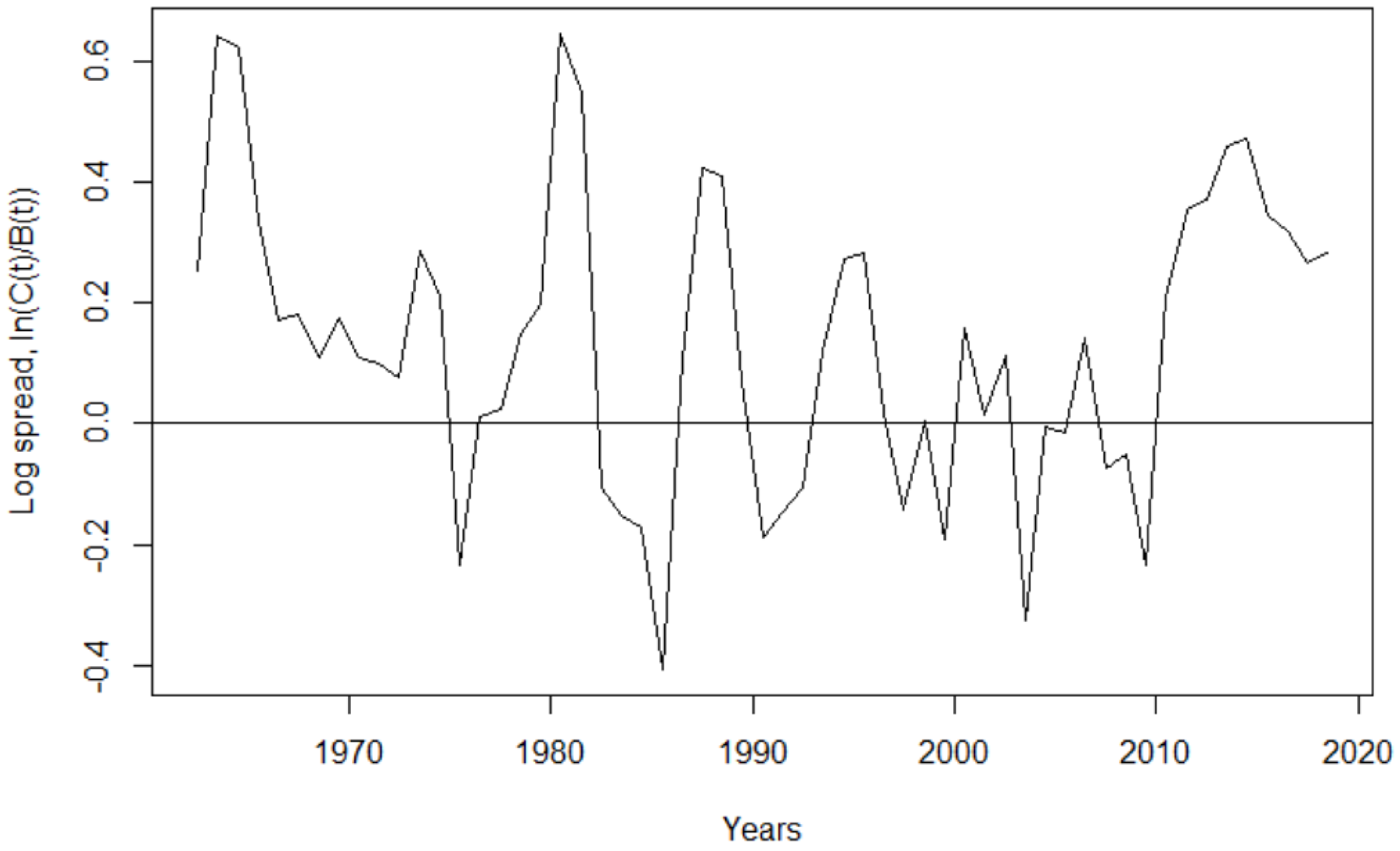
As in Thomson (1996) the Ginsburg Malan & Carsons Money-Market Index has been used which is under the code $GMC1$. The short-term interest rates, $\delta_b(t)$, is obtained as

$$\delta_b(t) = \ln \frac{GMC1_t}{GMC1_{t-1}}$$

$$\ln \delta_c(t) - \ln \delta_b(t) = -\ln(\delta_b(t)/\delta_c(t))$$

i.e. the logarithm of the ratio of the rates.

Short-term Interest Rates



Short-term Interest Rates

$$\delta_b(t) = \delta_c(t).\exp(-bd(t))$$

$$\begin{aligned} bd(t) &= \mu_b + a_b.(bd(t-1) - \mu_b) + \epsilon_b(t) \\ \epsilon_b(t) &= \sigma_b.z_b(t) \\ z_b(t) &\overset{\text{iid}}{\sim} N(0,1) \end{aligned}$$

Short-term Interest Rates

bd(t)	1962-2018
μ_b	0.1568 (0.0596)
a_b	0.5527 (0.1116)
σ_b	0.1996 (0.0189)
$r_z(1)$	-0.095
$r_{z^2}(1)$	0.098
skewness $\sqrt{\beta_1}$	-0.2012
kurtosis β_2	3.1408
Jarque-Bera χ^2	0.4318
$p(\chi^2)$	0.8058

- Log-ratio is modelled as AR(1)
- All parameters are significant
- Residuals are independent and normally distributed
- No ARCH effect

Nominal Yield Curve

$$Y(t,n) = \delta_c(t) + (\delta_b(t) - \delta_c(t))e^{-\beta n}$$

$Y(t,n)$: yield at time t for term n
$\delta_b(t)$: short-term interest rates
$\delta_c(t)$: long-term interest
β: a constant

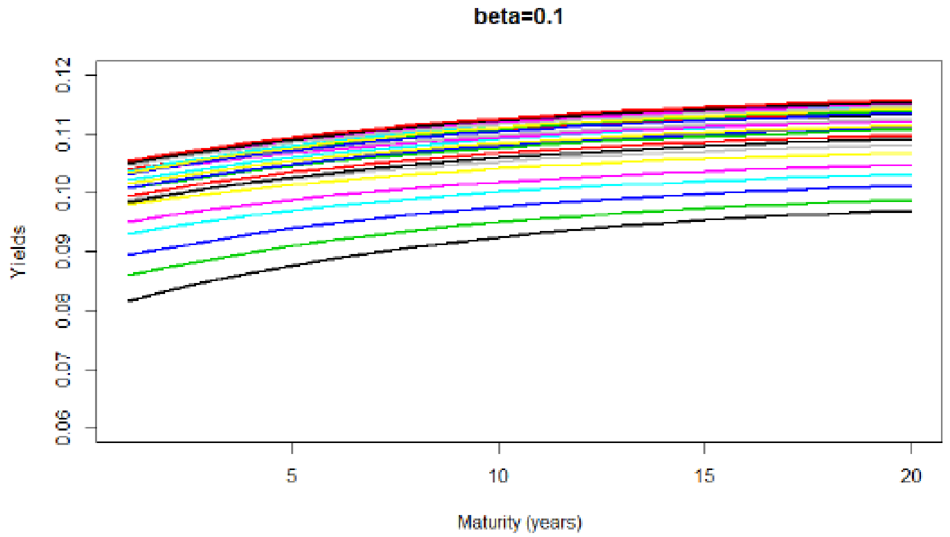


Figure 12: Nominal yield curve with $\beta = 0.1$

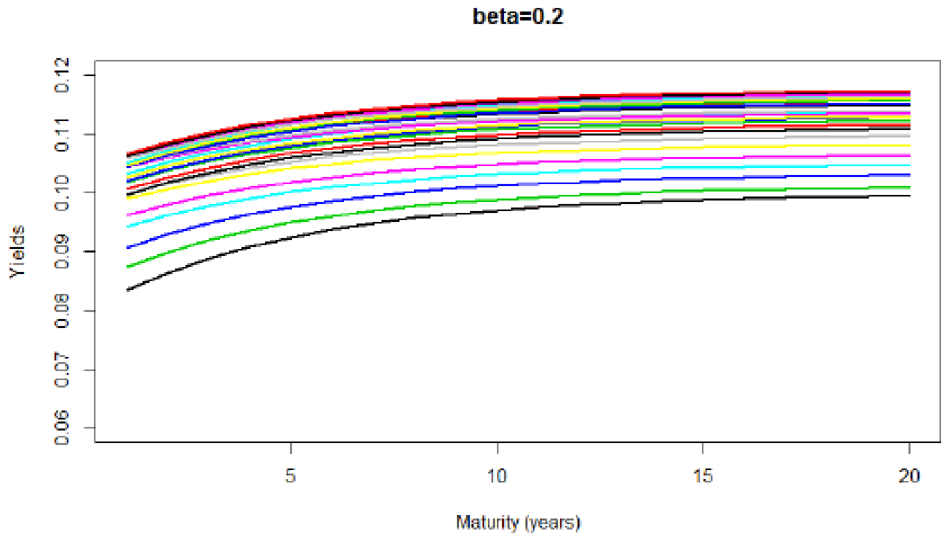


Figure 13: Nominal yield curve with $\beta = 0.2$

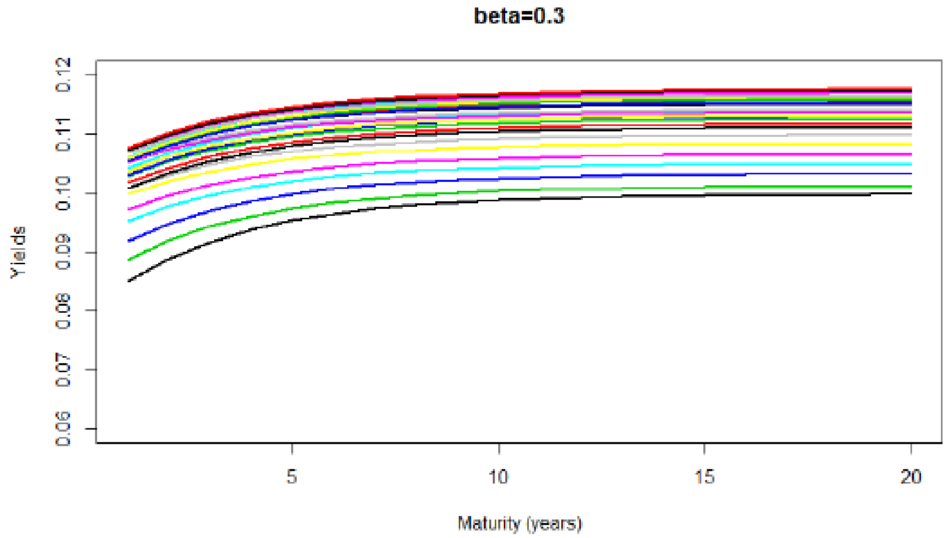


Figure 14: Nominal yield curve with $\beta = 0.3$

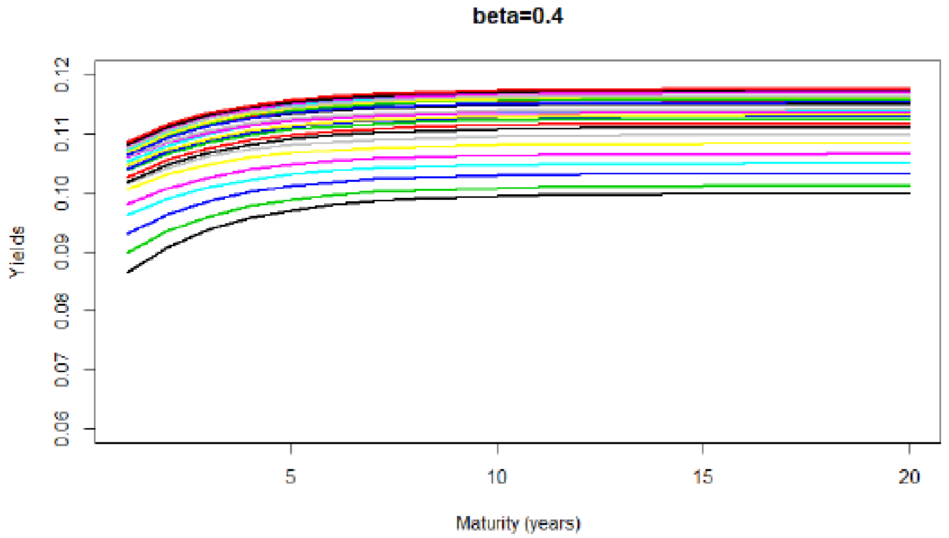
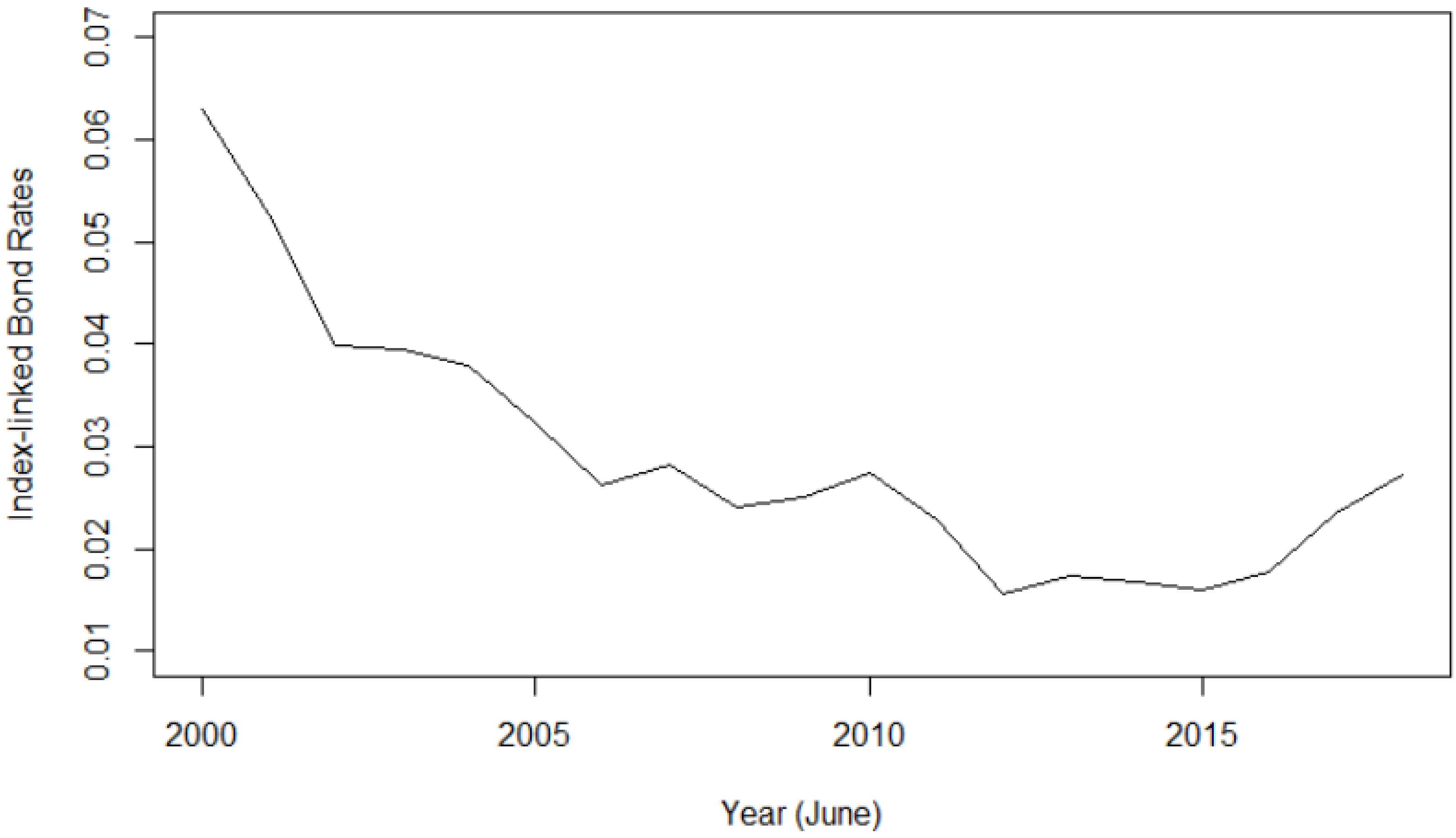


Figure 15: Nominal yield curve with $\beta = 0.4$

Index-Linked Bonds



Index-Linked Bonds

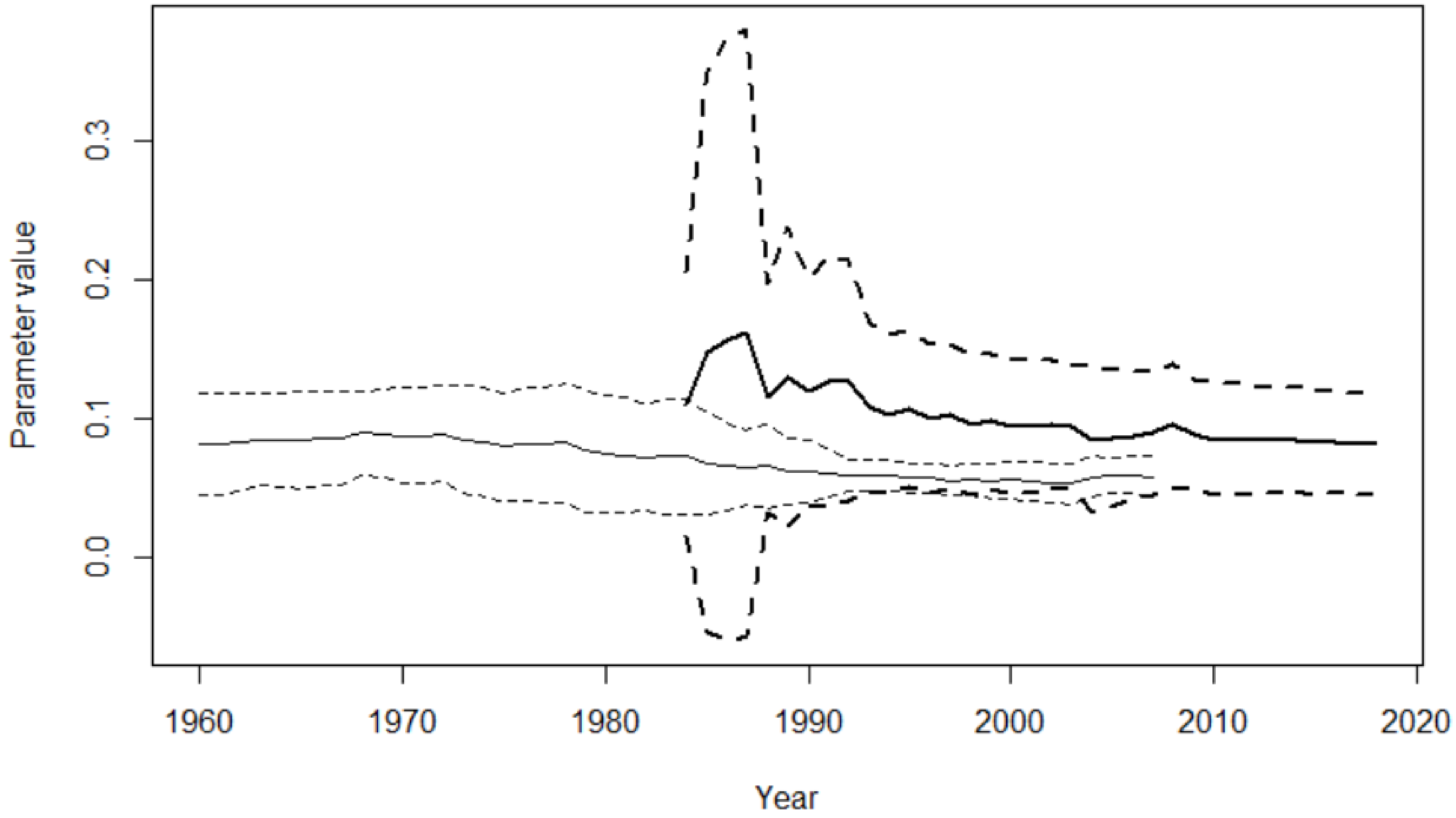
$$
\begin{aligned}
\delta_r(t) &= \mu_r + a_r.(\delta_r(t-1) - \mu_r) + c_r\delta_c(t) + b_r\delta_b(t) + \epsilon_r(t) \\
\epsilon_r(t) &= \sigma_r.z_r(t) \\
z_r(t) &\overset{\text{iid}}{\sim} N(0,1)
\end{aligned}
$$

$\delta_r(t)$: index-linked bond rates
$\delta_c(t)$: long-term interest rates
$\delta_b(t)$: short-term interest rates

Index-Linked Bonds

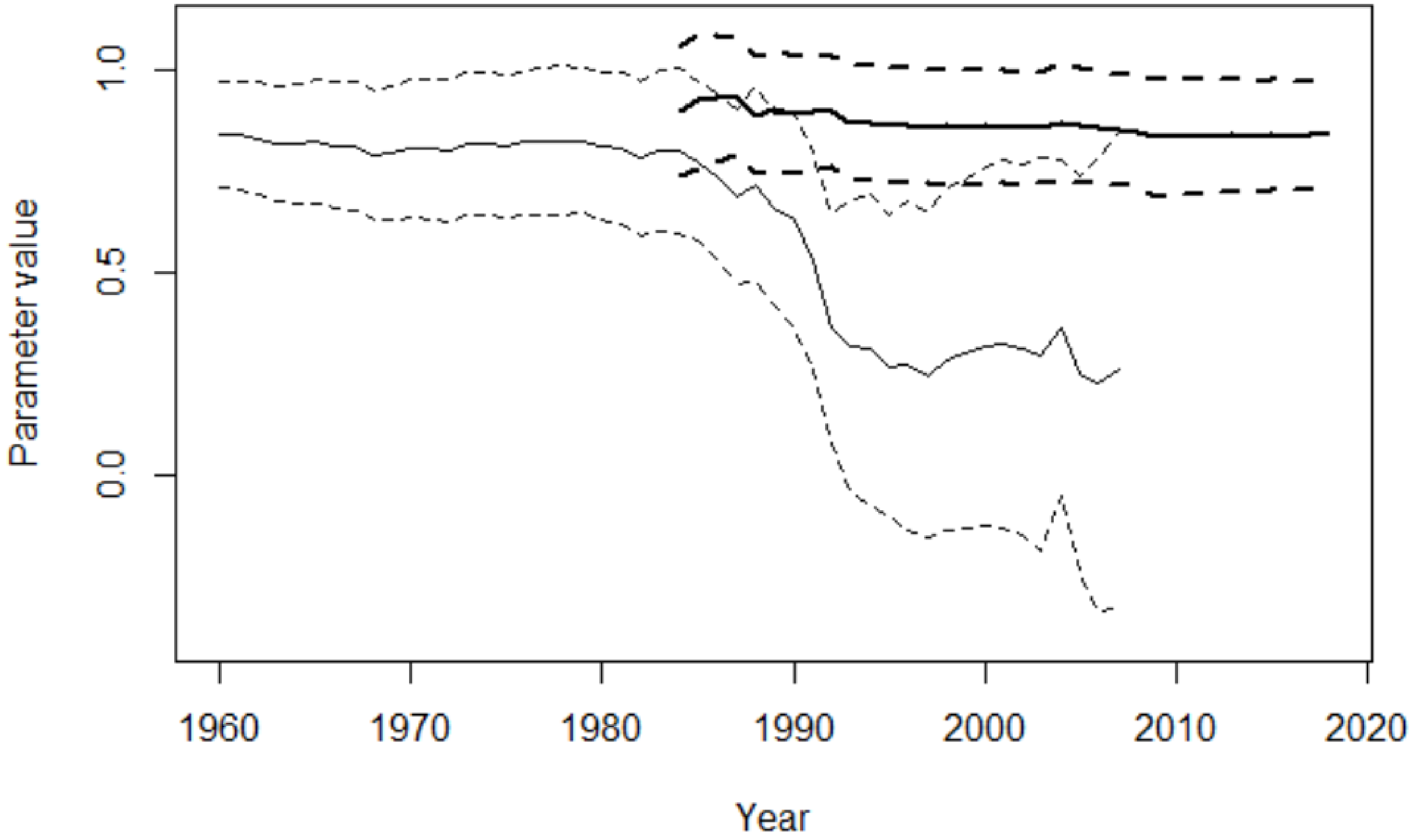
		2000-2018			
$\delta_r(t)$	AR(1)	$\delta_c(t)$ and $\delta_b(t)$	$\delta_c(t)$	$\delta_b(t)$	
				μ_r included	μ_r omitted
μ_r	0.0222 (0.0033)	0.0438 (0.0316)	-0.0118 (0.0266)	0.0038(0.0071)	
a_r	0.7194 (0.0618)	0.5942 (0.0957)	0.6877 (0.0745)	0.6174 (0.0698)	0.6165 (0.0703)
c_r		-0.2721 (0.1582)	0.1163 (0.0889)		
b_r		0.2142 (0.0727)		0.0973 (0.0419)	0.1144 (0.0272)
σ_r	0.0033 (0.0004)	0.0038 (0.0007)	0.0034 (0.0004)	0.0029 (0.0003)	0.0030 (0.0003)
Log Likelihood	77.10	74.61	76.95	79.46	79.32
$r_z(1)$	0.038			-0.068	-0.060
$r_{z^2}(1)$	-0.072			-0.230	-0.151
skewness $\sqrt{\beta_1}$	-0.2749			-0.3155	-0.3418
kurtosis β_2	2.2125			2.2151	2.3209
Jarque-Bera χ^2	0.7303			0.8030	0.7349
$p(\chi^2)$	0.6941			0.6693	0.6925

Parameter Stability



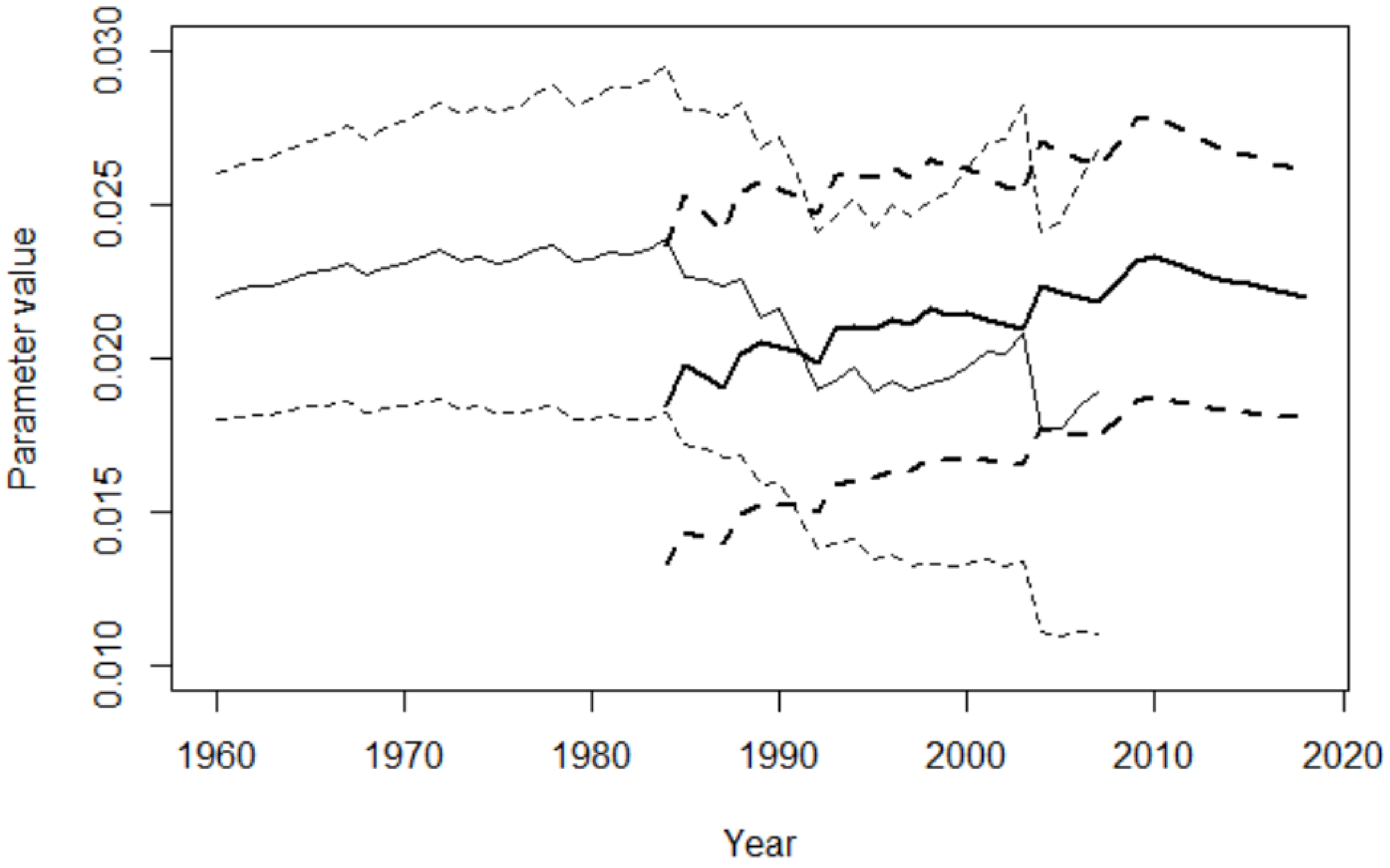
Estimates for parameter μ_q

Parameter Stability



Estimates for parameter a_q

Parameter Stability



Estimates for parameter σ_q

Conclusions

- Developed a stochastic investment model for SA actuarial applications based on integrated economic series

- Models fit well and the residuals are normally distributed (long-term interest rates!)

- Parameter stability should be analysed in detail

- Yield- curve models can be combined with the other economic series

Future Work

- **Real yield curve model**
  - daily index-linked bond rates for 2000-2018
  - maturities – vary from 1 to 20 years, even longer
    - a descriptive yield curve model to fill the missing values in the data
    - applying principal component analysis to obtain uncorrelated variables – level, slope, curvature
    - exploring bi-directional relation

References

- Granger, C.W.J & Newbold, P. (1977). *Forecasting economic time series.* Academic Press, New York.
- Thomson, R. J. (1996). Stochastic Investment Modelling: The Case of South Africa. British Actuarial Journal, Vol. 2, No. 3, pp. 765-801.
- Thomson, R. J. (2005). The pricing of liabilities in an incomplete market using dynamic mean–variance hedging Insurance: Mathematics and Economics (36)441–455.
- Thomson, R. J. and Gott, D. V. (2009). Stochastic models for actuarial use: the equilibrium modelling of local markets. ASTIN Bulletin (39)1, 339-370.
- Thomson, R. J. (2010). Modelling the market in a risk-averse world: the case of South Africa. South African Actuarial Journal (10).
- Wilkie, A. D. (1986). A Stochastic Investment Model For Actuarial Use. Transactions of the Faculty of Actuaries, 39, 341-403.
- Wilkie, A. D. (1995). More On A Stochastic Asset Model For Actuarial Use. British Actuarial Journal, 1, 777-964.
- Wilkie, A. D., Şahin, Ş., Cairns, A. J. G. & Kleinow, T. (2011). Yet More on a Stochastic Economic Model: Part 1: Updating and Retting, 1995 to 2009. Annals of Actuarial Science, 5, 53-99.

References

- Wilkie, A. D. and Şahin, Ş., 2016, "Yet more on a stochastic economic model: part 2: initial conditions, select periods and neutralising parameters", Annals of Actuarial Science.
- Wilkie, A. D. and Şahin, Ş., 2017, "Yet More on a Stochastic Economic Model: Part 3A: Stochastic Interpolation: Brownian and OU Bridges", Annals of Actuarial Science
- Wilkie, A. D. and Şahin, Ş., 2017, "Yet More on a Stochastic Economic Model: Part 3B: Stochastic Bridging for Retail Prices and Wages", Annals of Actuarial Science.
- Wilkie, A. D. and Şahin, Ş., 2017, "Yet More on a Stochastic Economic Model: Part 3C: Stochastic bridging for Share Yields and Dividends and Interest Rates", Annals of Actuarial Science.
- Wilkie, A. D. and Şahin, Ş., 2017, "Yet More on a Stochastic Economic Model: Part 4: A Model for Share Earnings, Dividends and Prices", Annals of Actuarial Science.
- Wilkie, A. D. and Şahin, Ş., 2018, "Yet More on a Stochastic Economic Model: Part 5: A Vector Autoregressive (VAR) Model for Retail Prices and Wages", Annals of Actuarial Science.
- Wilkie, A. D. and Şahin, Ş., 2019, "Yet More on a Stochastic Economic Model: Supplement to Part 4: A Model for Share Earnings, Dividends and Prices", Annals of Actuarial Science, accepted.