Long-Term Insurance Products and Volatility under the Solvency II Framework

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Abstract

Solvency II will become the new regulatory framework for insurance companies in Europe. Solvency II aims to determine the capital requirements in function of the risk exposure of the insurance undertaking.

We argue in this paper that, unless the specific characteristics of the long-term nature of certain insurance contracts are taken into account in Solvency II, the framework will not contribute to a more accurate risk measurement for said contracts. Moreover, it will inadvertently lead to a fundamental change in the financial nature of long-term insurance products.

First, we will extend the deflator formalism for defaultable bonds developed by M. V. Wüthrich[10] to include a time-variable spread process. Using the extended formalism, we will show that Solvency II will induce large capital flows that were up to now not necessary in the long-term insurance business.

We then show that the volatility of the capital flows depends on the definition of the market value of the non-tradable liabilities and on the frequency at which the capital is computed. These capital flows and the volatility associated will render products such as pensions much more expensive.

We estimate this price increase by considering the (exotic) option an insurer could decide to buy, to eliminate the extra volatility due to the Solvency II framework. This exotic option is constructed by considering the non-tradable insurance contract linked to a unique policyholder, versus a risk-free bond, while imposing the condition that the payout to the policyholder remains equal in both cases.

Keywords: Solvency II, Long-Term Insurance Products, volatility, defaultable bonds, spread process, matching adjustment.

1 Introduction

The key difficulty in establishing a market consistent framework for the insurance business is that there is no market for insurance liabilities. While the market value of assets can easily be determined using a mark-to-market approach, for liabilities one has to use the mark-to-model approach. The difficulty thus lies in valuing liabilities that are not traded, with a model in such a way that would still be consistent with the market.

Under the draft implementing measures[5] of the Solvency II Directive[4], liability cash flows are discounted at the risk-free rate to determine the market consistent value of these liabilities. We assume here, very generally, that the counter cyclical premium is zero at time of calculation and that the product is not ring fenced, such that the matching premium as defined in [5] is not applied. This valuation method thus implies that the liabilities are valued as if they were non-defaultable bonds. However, insurance liabilities have characteristic that deviate significantly from non-defaultable bonds. The principal distinction being that insurance policies are linked with a unique policyholder. A policyholder cannot sell the policy to someone else. We argue that it would not be market consistent to value both as if it were the same financial instrument.

The paper is organized as follows. Section 2 reviews the balance sheet model developed in [10]. We extend the model to include a time-variable spread insurance products with a longer maturity.

In section 3, we show that the market consistent solvency analysis as presented in section 2.3 of [10] leads to volatility in the consumption stream. Insurers will be entirely exposed to the daily gyrations of the spread risk of the bond market. This effect is even more pronounced for long-term insurance products.

In section 4 we derive a model which uses a different definition for the market value of an insurance liability. We show that this model still satisfies the no arbitrage condition. The consumption stream of this model is less volatile.

Section 5 focuses on the downside risk of having a negative consumption stream. We analyze the option that an insurance companies could buy to insure themselves against the downside risk. We conclude by estimating the behaviour of the option price for insurance products with a longer maturities.

2 Market-Consistent Solvency Analysis

In this section we set up the theoretical framework which we will use to analyze the questions at hand. We start from the example presented in [10]. We proceed to extend this model to include a time dependent illiquidity spread. This extension will allow us to investigate the impact of a variable credit spread. We then extend the model to a multi-period model. This is necessary to study the effect of having long-term insurance products.

We note that throughout this paper work with deflators in the real world measure $\mathbb{E}[\ldots]$. One could translate the assumptions and calculations in an equivalent risk neutral measure $\tilde{\mathbb{E}}[\ldots]$. However, for the derivations of the volatility and probability of negative consumption in the sections below, we have to use the real world measure since we are interested in real world volatilities and probabilities.

2.1 Insurance Balance Sheet Model

In this section we review in detail the example presented in [10].

2.1.1 State price deflator and pricing of zero-coupon bonds

The example is constructed in a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{F})$ with finite time horizon $n \geq 2$ and discrete time filtration $\mathbb{F} = (\mathcal{F})_{t=0,\ldots,n}$ with $\mathcal{F}_0 = \{\emptyset, \Omega\}$ We construct the state price deflator $\boldsymbol{\varphi} = (\varphi_t)_{t=0,\ldots,n}$ using two strictly positive and $\mathbb{F}$-adapted stochastic processes $\boldsymbol{\psi} = (\psi_t)_{t=0,\ldots,n}$ and $\boldsymbol{\chi} = (\chi_t)_{t=0,\ldots,n}$. We take $\psi_0 = 1$ and assume that χ_t is independent from $\sigma\{\mathcal{F}_{t-1}, \boldsymbol{\psi}\}$ with $\mathbb{E}[\chi_t] = 1$ for all t:

$$\varphi_t = \psi_t \prod_{u=0}^{t} \chi_u \,. \tag{2.1}$$

The decoupling property of the state price deflator $\boldsymbol{\varphi}$ into a "global market state price deflator", $\boldsymbol{\psi}$, and "idiosyncratic distortions", $\boldsymbol{\chi}$, will be crucial.

The price of a default-free zero-coupon bond is given by

$$P(t,m) = \frac{1}{\varphi_t}\,\mathbb{E}[\varphi_m|\mathcal{F}_t] = \frac{1}{\psi_t}\,\mathbb{E}[\psi_m|\mathcal{F}_t]\,. \tag{2.2}$$

The proof of the second equation is given in [10] and makes use of the independence χ_t from $\sigma\{\mathcal{F}_{t-1}, \boldsymbol{\psi}\}$ and the condition $\mathbb{E}[\chi_t] = 1$.

To compute the price of defaultable zero-coupon bonds we introduce a default process $\boldsymbol{\Gamma} = (\Gamma_t)_{t=0,\ldots,n}$, where $\boldsymbol{\Gamma}$ is a a non-increasing process with $\Gamma_0 = 1$ and $\Gamma_t = 0$ if the bond has defaulted in $[0,t]$ otherwise $\Gamma_t = 1$. We assume the process $\boldsymbol{\Gamma}$ to be $\mathbb{F}$-adapted and independent of $\boldsymbol{\psi}$. In addition, we assume for all $t > 0$

$$\begin{aligned} \mathbb{E}[\Gamma_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p)\,\Gamma_{t-1} && (2.3)\\ \mathbb{E}[\chi_t\,\Gamma_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p)\,(1-s)\,\Gamma_{t-1} && (2.4) \end{aligned}$$

for fixed $p \in (0,1)$ parametrizing the real world probabilty of default and $s \in [0,1)$ which represents the illiquidity spread. We can now compute the price of a defaultable zero-coupon bond:

$$B(t,m) = \frac{1}{\varphi_t}\,\mathbb{E}[\varphi_m \Gamma_m | \mathcal{F}_t] = P(t,m)\,(1-p)^{m-t}\,(1-s)^{m-t}\,\Gamma_t\,. \tag{2.5}$$

The proof of the second equation is given in [10]. The price of a defaultable zero-coupon bond conditional on the fact that the bond survived at least up to time k (with $k \leq t \leq m$) is given by

$$B^{(k)}(t,m) = \frac{1}{\varphi_t}\,\mathbb{E}[\varphi_m \Gamma_m^{(k)} | \mathcal{F}_t] = P(t,m)\,(1-p)^{m-t}\,(1-s)^{m-t}\,\Gamma_t^{(k)}\,. \tag{2.6}$$

with $\Gamma_t^{(k)}$ the default process conditional on the fact the $\Gamma_k = 1$.

2.1.2 Solvency analysis of a simple insurance liability

Following [10] we now study the balance sheet of an insurance company. We consider an insurance liability given by a default-free cash flow of size $M = 1$ at time $m = 2$. Strictly speaking this is not an insurance contract since we consider a completely predictable liability cash flow. The time series of the market-consistent value of the liability of the balance sheet is given by

$$L_0 = P(0,2) \tag{2.7}$$

$$L_1 = P(1,2) \tag{2.8}$$

$$L_2 = P(2,2) = 1 \tag{2.9}$$

$$L_{t\geq 3} = 0 \tag{2.10}$$

We briefly review the analysis in [10].

Time $t = 0$. The value of the insurance contract is $L_0 = P(0,2)$. The policyholder is thus charged a (single pure risk) premium of $\pi = L_0$. We assume the insurer chooses to invest the premium income in a defaultable zero-coupon bond of the same maturity as the liability:

$$A_0 = \pi = (1-p)^{-2}\,(1-s)^{-2}\,B(0,2) \tag{2.11}$$

For this operation, the insurer does not need extra available capital, nor does it generate available capital. The capital consumption is

$$C_0 = A_0 - \pi = 0\,. \tag{2.12}$$

Time $t = 1$. The asset and liability values have evolved to:

$$\begin{aligned} A_0 \;&\rightarrow\; A_1^- = (1-p)^{-2}\,(1-s)^{-2}\,B(1,2) = (1-p)^{-1}\,(1-s)^{-1}\,\Gamma_1\,P(1,2) && (2.13)\\ L_0 \;&\rightarrow\; L_1 = P(1,2) && (2.14) \end{aligned}$$

To reinstate $A_1 = L_1 = P(1,2)$, the insurer will either need to inject capital in case the asset defaulted or the asset will have created capital which becomes available. As in [10], we assume that the injection of new capital is done through new (defaultable) bonds that have not defaulted yet. The capital consumption is:

$$C_1 = A_1^- - A_1 = \left[(1-p)^{-1}\,(1-s)^{-1}\,\Gamma_1 - 1\right]\,P(1,2)\,. \tag{2.15}$$

Time $t = 2$. Assets and liabilities are now:

$$A_1 \;\rightarrow\; A_2^- = (1-p)^{-1}\,(1-s)^{-1}\,B^{(1)}(2,2) = (1-p)^{-1}\,(1-s)^{-1}\,\Gamma_2^{(1)}\,P(2,2) \tag{2.16}$$

$$L_1 \;\rightarrow\; L_2 = 1 \tag{2.17}$$

To pay out the policyholder $A_2 = L_2 = 1$, the insurer faces the following capital consumption:

$$C_2 = A_2^- - A_2 = \left[(1-p)^{-1}\,(1-s)^{-1}\,\Gamma_2^{(1)} - 1\right] \tag{2.18}$$

The consumption stream is thus given by

$$C_0 = 0 \tag{2.19}$$

$$C_1 = \left[(1-p)^{-1}\,(1-s)^{-1}\,\Gamma_1 - 1\right]\,P(1,2) \tag{2.20}$$

$$C_2 = \left[(1-p)^{-1}\,(1-s)^{-1}\,\Gamma_2^{(1)} - 1\right] \tag{2.21}$$

One can show that the value $Q_0[\boldsymbol{C}]$ of the consumption stream at time 0 is given by

$$Q_0[\boldsymbol{C}] = \sum_{u=0}^{2} \mathbb{E}[\varphi_u\,C_u] = 0 \tag{2.22}$$

which implies that there exists no arbitrage strategy for this insurance product since the pure risk premium was correctly chosen.

2.2 Extension of the model with a time dependent spread

2.2.1 Deflator and zero-coupon bonds with time dependent spread

We will extend the example introduced in section (2.1.2) with a time-dependent correlation between $\boldsymbol{\chi}$ and $\boldsymbol{\Gamma}$. We therefore replace the time-invariant assumptions (2.3)-(2.4) with the conditions that for all $t > 0$,

$$\begin{aligned}
\mathbb{E}[\Gamma_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p)\,\Gamma_{t-1} && (2.23)\\
\mathbb{E}[\chi_t\,\Gamma_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p)\,(1-\tilde{s}_{t-1})\,\Gamma_{t-1} && (2.24)\\
\mathbb{E}[(1-\tilde{s}_t)^k\chi_t\,\Gamma_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p)\,(1-\tilde{s}_{t-1})^{k+1}\,\Gamma_{t-1}, \quad \forall k : 0 \leq k \leq n-t && (2.25)
\end{aligned}$$

where the illiquidity spread $\tilde{\boldsymbol{s}} = (\tilde{s}_t)_{t=0,\ldots,n}$ with $\tilde{s}_t \in [0,1), \forall t$ is now a $\mathbb{F}$-adapted stochastic process independent of $\boldsymbol{\psi}$. Note that since $\tilde{s}_t$ is $\mathcal{F}_t$-measurable, but not $\mathcal{F}_{t-1}$-measurable, we cannot take the term $(1-\tilde{s}_t)^k$ outside the expectation $\mathbb{E}[\ldots]$ in (2.25). Given these assumptions, defaultable bonds have a price at time t with $t \leq m$ given by

$$\begin{aligned}
B(t,m) &= \frac{1}{\varphi_t}\,\mathbb{E}[\varphi_m\,\Gamma_m|\mathcal{F}_t] = \frac{1}{\psi_t}\,\mathbb{E}\left[\psi_m \prod_{u=t+1}^{m} \chi_u\,\Gamma_m \middle| \mathcal{F}_t\right] \\
&= \frac{1}{\psi_t}\,\mathbb{E}\left[\psi_m \prod_{u=t+1}^{m-1} \chi_u\,\mathbb{E}[\chi_m\,\Gamma_m|\mathcal{F}_{m-1},\boldsymbol{\psi}] \middle| \mathcal{F}_t\right] \\
&= (1-p)\frac{1}{\psi_t}\,\mathbb{E}\left[\psi_m \prod_{u=t+1}^{m-2} \chi_u\,\chi_{m-1}\,(1-\tilde{s}_{m-1})\,\Gamma_{m-1} \middle| \mathcal{F}_t\right] \\
&= (1-p)\frac{1}{\psi_t}\,\mathbb{E}\left[\psi_m \prod_{u=t+1}^{m-2} \chi_u\,\mathbb{E}[(1-\tilde{s}_{m-1})\,\chi_{m-1}\,\Gamma_{m-1}|\mathcal{F}_{m-2},\boldsymbol{\psi}] \middle| \mathcal{F}_t\right] \\
&= (1-p)^2\frac{1}{\psi_t}\,\mathbb{E}\left[\psi_m \prod_{u=t+1}^{m-3} \chi_u\,\chi_{m-2}\,(1-\tilde{s}_{m-2})^2\,\Gamma_{m-2} \middle| \mathcal{F}_t\right] \\
&= \ldots = (1-p)^{m-t}\,(1-\,\tilde{s}_t)^{m-t}\,\Gamma_t\,P(t,m)\,. \qquad (2.26)
\end{aligned}$$

We see that with the new assumptions (2.23)-(2.25), our asset universe now includes a set of bonds with different maturities $0 \leq m \leq n$, with a constant default probability per time step p, and with a variable illiquidity spread $\tilde{s}_t$, which under our assumptions is independent from the maturity m of the bond. For example, taking $n = 3$, we have a set of 4 bonds with different maturities with the following market values $B(t,m)$ (where $q = 1 - p$):

	$t=0$	$t=1$	$t=2$	$t=3$
$m=0$	1			
$m=1$	$q\,(1-\,\tilde{s}_0)\,P(0,1)$	$\Gamma_1\,P(1,1)$		
$m=2$	$q^2\,(1-\,\tilde{s}_0)^2\,P(0,2)$	$q\,(1-\,\tilde{s}_1)\,\Gamma_1\,P(1,2)$	$\Gamma_2\,P(2,2)$	
$m=3$	$q^3\,(1-\,\tilde{s}_0)^3\,P(0,3)$	$q^2\,(1-\,\tilde{s}_1)^2\,\Gamma_1\,P(1,3)$	$q\,(1-\,\tilde{s}_2)\,\Gamma_2\,P(2,3)$	$\Gamma_3\,P(3,3)$

If we would want to add another set of defaultable bonds to our asset universe, we would introduce another default process, $\mathbf{\Gamma'}$, that characterizes these bonds. Using this new default process, one can compute the associated default probability p' and illiquidity spread $\tilde{s}'_t$:

$$\begin{aligned} \mathbb{E}[\Gamma'_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p')\,\Gamma'_{t-1} && (2.27) \\ \mathbb{E}[\chi_t\,\Gamma'_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p')\,(1-\tilde{s}'_{t-1})\,\Gamma'_{t-1} && (2.28) \end{aligned}$$

Note that it is, for example, possible to have two sets of bonds with the same default probability $p' = p$, but another spread process $\tilde{s}'_t \neq \tilde{s}_t$.

For the rest of the paper, we will redefine the spread process as

$$1 - \tilde{s}_t = e^{-s_t} \in (0,1]\,, \qquad (2.29)$$

with $s_t \in [0, \infty)$. The assumptions (2.23)-(2.25) are thus rewritten as

$$\begin{aligned} \mathbb{E}[\Gamma_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p)\,\Gamma_{t-1} && (2.30) \\ \mathbb{E}[e^{-k\,s_t}\chi_t\,\Gamma_t|\mathcal{F}_{t-1}, \boldsymbol{\psi}] &= (1-p)\,e^{-(k+1)\,s_{t-1}}\,\Gamma_{t-1}, \quad \forall k : 0 \leq k \leq n-t && (2.31) \end{aligned}$$

and we have

$$B(t,m) = (1-p)^{m-t}\,e^{-(m-t)\,s_t}\,\Gamma_t\,P(t,m)\,. \qquad (2.32)$$

2.2.2 Solvency Analysis with varying spread

We now revisit the insurance balance sheet from section 2.1.2 and work out the consumption stream.

Time $t = 0$. The insurance company invests the premium $\pi = L_0 = P(0,2)$ in a defaultable zero-coupon bond with the same maturity.

$$A_0 = \pi = (1-p)^{-2}\, e^{2s_0}\, B(0,2) \tag{2.33}$$

The consumption is

$$C_0 = A_0 - \pi = 0\,. \tag{2.34}$$

Time $t = 1$. We now have:

$$\begin{aligned} A_0 &\;\rightarrow\; A_1^- = (1-p)^{-2}\, e^{2s_0}\, B(1,2) = (1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1\, P(1,2) && (2.35)\\ L_0 &\;\rightarrow\; L_1 = P(1,2) && (2.36) \end{aligned}$$

and

$$C_1 = A_1^- - A_1 = \left[(1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1 - 1\right]\, P(1,2) \tag{2.37}$$

such that $A_1 = L_1$. Notice that if the bond has defaulted ($\Gamma_1 = 0$) then $C_1 < 0$. This is similar to the constant spread model. However, unlike in the constant spread model, we can have $C_1 < 0$ without default ($\Gamma_1 = 1$):

$$e^{-s_1} < (1-p)\, e^{-2s_0}\,. \tag{2.38}$$

Time $t = 2$. We get

$$\begin{aligned} A_1 &\rightarrow A_2^- = (1-p)^{-1}\, e^{s_1}\, B^{(1)}(2,2) = (1-p)^{-1}\, e^{s_1}\, \Gamma_2^{(1)}\, P(2,2) && (2.39) \\ L_1 &\rightarrow L_2 = 1 && (2.40) \end{aligned}$$

with consumption

$$C_2 = A_2^- - A_2 = \left[(1-p)^{-1}\, e^{s_1}\, \Gamma_2^{(1)} - 1\right] . \tag{2.41}$$

The total consumption stream is:

$$\begin{aligned} C_0 &= 0 && (2.42) \\ C_1 &= \left[(1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1 - 1\right]\, P(1,2) && (2.43) \\ C_2 &= \left[(1-p)^{-1}\, e^{s_1}\, \Gamma_2^{(1)} - 1\right] && (2.44) \end{aligned}$$

and the no arbitrage condition is still satisfied:

$$Q_0[\boldsymbol{C}] = \sum_{u=0}^{2} \mathbb{E}[\varphi_u\, C_u] = 0\,. \tag{2.45}$$

2.3 Extension of the model to long-term insurance products

We now derive the consumption stream for an insurance contract with a larger maturity m.

Time t-1. After consumption we have

$$A_{t-1} = L_{t-1} \tag{2.46}$$

or

$$A_{t-1} = P(t-1,m) = (1-p)^{-(m-t+1)}\, e^{(m-t+1)\, s_{t-1}}\, B^{(t-1)}(t-1,m)\,. \tag{2.47}$$

Time t. We have

$$\begin{aligned} A_{t-1} \quad &\rightarrow \quad A_t^- = (1-p)^{-(m-t+1)}\, e^{(m-t+1)\, s_{t-1}}\, B^{(t-1)}(t,m) && (2.48)\\ &\qquad\quad = (1-p)^{-1}\, e^{(m-t+1)\, s_{t-1}-(m-t)\, s_t}\, \Gamma_t^{(t-1)}\, P(t,m) && (2.49)\\ L_{t-1} \quad &\rightarrow \quad L_t = P(t,m) && (2.50) \end{aligned}$$

giving a consumption of

$$C_t = A_t^- - A_t = \Big[(1-p)^{-1}\, e^{(m-t+1)\, s_{t-1}-(m-t)\, s_t}\, \Gamma_t^{(t-1)} - 1\Big]\ P(t,m) \tag{2.51}$$

The entire consumption stream is given by

$$\begin{aligned} C_0 \quad &= \quad 0 && (2.52)\\ C_t \quad &= \quad \Big[(1-p)^{-1}\, e^{-(m-t)\,(s_t-s_{t-1})}\, e^{s_{t-1}}\, \Gamma_t^{(t-1)} - 1\Big]\ P(t,m) && (2.53) \end{aligned}$$

with $1 \leq t \leq m$. For $t > 0$, we can compute

$$\begin{aligned}
&\mathbb{E}[\varphi_t \, C_t] \\
&= \mathbb{E}\Big[\psi_t \, \chi_0 \ldots \chi_{t-1} \, \chi_t \, (1-p)^{-1} \, e^{-(m-t)\, s_t} \, e^{(m-t+1)\, s_{t-1}} \, \Gamma_t^{(t-1)} \, P(t,m) - \varphi_t P(t,m)\Big] \\
&= \mathbb{E}\Big[\psi_t \, \chi_0 \ldots \chi_{t-1} \, (1-p)^{-1} \, e^{(m-t+1)\, s_{t-1}} \, P(t,m) \, \mathbb{E}\big[\chi_t \, e^{-(m-t)\, s_t} \, \Gamma_t^{(t-1)} \big| \mathcal{F}_{t-1}, \boldsymbol{\psi}\big]\Big] \\
&\quad - \mathbb{E}\Big[\varphi_t P(t,m)\Big] \\
&= 0 \qquad (2.54)
\end{aligned}$$

and thus the no arbitrage condition holds:

$$Q_0[\boldsymbol{C}] = \sum_{u=0}^{m} \mathbb{E}[\varphi_u \, C_u] = 0 \, . \qquad (2.55)$$

3 Analysis of the consumption stream volatility

As we noticed in section 2.2.2, there are two possible reasons for a negative consumption C_t. If the assets defaults, then $C_t < 0$. If the asset does not default, then having

$$e^{-(m-t)\,(s_t - s_{t-1})} < (1-p) e^{-s_{t-1}} \qquad (3.1)$$

also leads to a negative consumption. Given that $m - t$ can easily be of the order of 30 (or higher) for long-term insurance products, this condition can be satisfied for relatively smaller spread increases.

Let us estimate the volatility of the consumption C_1 at time 1. We remind ourselves that we compute the volatility using the real world measure $\mathbb{E}[\ldots]$, since we are interested in real world volatilities. We compute

$$\begin{aligned}
\mathbb{E}[C_1] &= \mathbb{E}\Big[(1-p)^{-1}\, e^{-(m-1)\,(s_1-s_0)}\, e^{s_0}\, \Gamma_1\, P(1,m) - P(1,m)\Big] \\
&= (1-p)^{-1}\, e^{s_0}\, \mathbb{E}\Big[\mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\, \Gamma_1|\boldsymbol{\psi}\big]\, P(1,m)\Big] - \mathbb{E}\Big[P(1,m)\Big] \\
&= (1-p)^{-1}\, e^{s_0}\, \mathbb{E}\Big[\mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\big](1-p)\, P(1,m)\Big] - \mathbb{E}\Big[P(1,m)\Big] \\
&= e^{s_0}\, \mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\big]\, \mathbb{E}\big[P(1,m)\big] - \mathbb{E}\big[P(1,m)\big]\,, \qquad (3.2) \\
\mathbb{E}[C_1^2] &= \mathbb{E}\Big[(1-p)^{-2}\, e^{-2(m-1)\,(s_1-s_0)}\, e^{2s_0}\, \Gamma_1\, P^2(1,m) + P^2(1,m) \\
&\qquad - 2\,(1-p)^{-1}\, e^{-(m-1)\,(s_1-s_0)}\, e^{s_0}\, \Gamma_1\, P^2(1,m)\Big] \\
&= (1-p)^{-1}\, e^{2s_0}\, \mathbb{E}\big[e^{-2(m-1)\,(s_1-s_0)}\big]\, \mathbb{E}\big[P^2(1,m)\big] + \mathbb{E}\big[P^2(1,m)\big] \\
&\quad - 2\, e^{s_0}\, \mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\big]\, \mathbb{E}\big[P^2(1,m)\big]\,. \qquad (3.3)
\end{aligned}$$

In this derivation, we have used the additional assumption of independence of the spread and the stochastic process $\boldsymbol{\Gamma}$:

$$\mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\, \Gamma_1|\boldsymbol{\psi}\big] = \mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\big]\mathbb{E}\big[\Gamma_1|\boldsymbol{\psi}\big] \qquad (3.4)$$

This additional assumption is compatible with the assumptions (2.30)-(2.31)) and simplifies the computation. Without this condition the variance of C_1 (see derivation below) would contain cross-terms between $\mathbb{V}\mathrm{ar}\big[e^{-(m-1)\,(s_1-s_0)}\big]$ and $\mathbb{V}\mathrm{ar}[\Gamma_1]$. Using the simplifying assumption we find that

$$
\begin{aligned}
&\mathbb{V}\mathrm{ar}[C_1] \\
&= (1-p)^{-1}\, e^{2s_0}\, \mathbb{E}\big[e^{-2(m-1)\,(s_1-s_0)}\big]\, \mathbb{E}\big[P^2(1,m)\big] \\
&\quad - 2\, e^{s_0}\, \mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\big]\, \mathbb{E}\big[P^2(1,m)\big] + \mathbb{E}\big[P^2(1,m)\big] \\
&\quad - e^{2s_0}\, \mathbb{E}^2\big[e^{-(m-1)\,(s_1-s_0)}\big]\, \mathbb{E}^2\big[P(1,m)\big] \\
&\quad + 2\, e^{s_0}\, \mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\big]\, \mathbb{E}^2\big[P(1,m)\big] - \mathbb{E}^2\big[P(1,m)\big] \\
&= \ldots \\
&= e^{2s_0}\, \frac{\mathbb{E}[P^2(1,m)]}{\mathbb{E}[\Gamma_1^2]} \mathbb{V}\mathrm{ar}\big[e^{-(m-1)\,(s_1-s_0)}\big] \\
&\quad + e^{2s_0}\, \frac{\mathbb{E}^2[P(1,m)]}{\mathbb{E}^2[\Gamma_1]} \mathbb{E}^2\big[e^{-(m-1)\,(s_1-s_0)}\big] \mathbb{V}\mathrm{ar}[\Gamma_1] \\
&\quad + \left(1 - 2\, e^{s_0}\, \mathbb{E}\big[e^{-(m-1)\,(s_1-s_0)}\big] + e^{2s_0}\, \frac{1}{\mathbb{E}[\Gamma_1^2]} \mathbb{E}^2\big[e^{-(m-1)\,(s_1-s_0)}\big]\right) \mathbb{V}\mathrm{ar}\big[P(1,m)\big] \qquad (3.5)
\end{aligned}
$$

The first term is due to the volatility in the spread, the second term due to the volatility in the default process and the last term is linked with the volatility of the term structure which determines the price of non-defaultable zero-coupon bonds.

If we consider a bond with maturity m and approximate the associated spread process $(s_1 - s_0)$ with a normal distribution $\mathcal{N}\big(\mu = 0; \sigma(m)\big)$ where we parametrize $\sigma(m)$ with an empirical formula which exhibits a decaying volatility with maturity $(m \gg 1)$,

$$
\sigma^2(m) = \sigma_0^2 + \sigma_1^2 \frac{1}{(m-1)^2}\,, \qquad (3.6)
$$

then we obtain:

$$\begin{aligned}\mathbb{V}\mathrm{ar}\Big[e^{-(m-1)\,(s_1-s_0)}\Big] &= \Big(e^{(m-1)^2\,\sigma_0^2+\sigma_1^2}-1\Big)e^{(m-1)^2\,\sigma_0^2+\sigma_1^2} \qquad (3.7)\\ \mathbb{E}\Big[e^{-(m-1)\,(s_1-s_0)}\Big] &= e^{\frac{1}{2}(m-1)^2\,\sigma_0^2+\frac{1}{2}\sigma_1^2}\,. \qquad (3.8)\end{aligned}$$

We conclude that the volatility of the consumption stream, $\mathbb{V}\mathrm{ar}[C_1]$, increases very quickly with the maturity of the insurance product. Moreover, this rapid increase is entirely due to time-dependent illiquidity spread. Indeed, if we assume a constant illiquidity spread s, then we see that the first term in (3.5) disappears and the m-dependence of the second and third term becomes implicit through $P(1, m)$.

If we assume a sufficiency capitalized product then the consumption stream is owned by the shareholders. Shareholders will demand a high risk premium to compensate for this high volatility. Or, the Sharpe ratio of the investment will be small which will dissuade insurance companies from offering long-term products.

4 Impact of the definition of the market value of non-tradable liabilities on the capital flow volatility

4.1 Reduced time grid

Let us now reconsider the example from section 2.2.2. We will alter the time grid by leaving out the odd times. The consumption stream is now given by

$$\widetilde{C}'_0 = 0 \tag{4.1}$$

$$\widetilde{C}'_2 = \left[(1-p)^{-2}\, e^{2s_0}\, \Gamma_2 - 1\right] \tag{4.2}$$

In this example there still is no over-consumption of capital which would violate market consistency and the no arbitrage condition holds:

$$Q_0[\widetilde{\boldsymbol{C}}'] = \sum_{u=0}^{1} \mathbb{E}[\varphi_{2u}\, \widetilde{C}'_{2u}] = 0\,. \tag{4.3}$$

The consumption stream now only depends on the default not on spread movements. Indeed, $\widetilde{C}'_2 < 0$ implies that $\Gamma_2 = 0$. The volatility of the consumption stream is thus smaller than the case from section 2.2.2.

We could extend this example again to larger maturities m. This would lead us to the same conclusion. Indeed, since we only have consumption streams on even times, we will find that the volatility of this consumption stream is lower than a situation with a consumption stream every time step as in section 2.3. The spread term in the volatility (see (3.5)) will be less dominant over the volatility term from the default process. The number of consumption streams over the lifetime of an insurance contract affects the volatility of the product from the point of view of the shareholder.

There is another equivalent point of view to consider this reduced time grid example. Recall that given the absence of insurance contract trading, liabilities have to be valued using a mark-to-model approach respecting market consistency. Leaving out the odd time step in the above example is equivalent to imposing $\widetilde{C}_1' = 0$ or, *defining* the market value of the liability in the model to be equal to $L_1 = A_1^- = A_1$.

4.2 Reduced consumption stream

Let us first point out that the policyholder in the reduced time grid example of section 4.1 does receive exactly the same payout as in the example of section 2.2.2. The difference between both examples is the definition of the market value of the liability.

However, the reduced time grid example is merely theoretical example, since it contains a serious flaw from a practical point of view. If the bond defaults at time 1 ($\Gamma_1 = 0$ and thus $A_1^- = A_1 = 0$), no capital is injected at $t = 1$. The policyholder relies entirely on the consumption stream at $t = 2$ to back the liability. In this section, we will develop the market consistent model with a reduced consumption stream at $t = 1$ with $\widetilde{C}_1 = 0$ unless a default happened.

Time $t = 0$. Just as in the previous examples, the insurance company invests the premium $\pi = L_0 = P(0,2)$ in a defaultable zero-coupon bond with the same maturity.

$$A_0 = \pi = (1-p)^{-2}\, e^{2s_0}\, B(0,2) \tag{4.4}$$

The consumption is

$$\widetilde{C}_0 = A_0 - \pi = 0\,. \tag{4.5}$$

Time $t = 1$. The value of the bond has evolved to:

$$A_0 \;\;\rightarrow\;\; A_1^- = (1-p)^{-2}\, e^{2s_0}\, B(1,2) = (1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1\, P(1,2)\,. \tag{4.6}$$

We now impose a consumption stream which injects cash only if a default occured:

$$\widetilde{C}_1 = A_1^- - A_1 = -(1-\Gamma_1)P(1,2)\,. \tag{4.7}$$

We thus, implicitly, *defined* the market value of the liability as

$$\begin{aligned} L_1 = A_1 &= \left[(1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1 - \Gamma_1 + 1\right]\, P(1,2) \\ &= \left[(1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1 - \Gamma_1 + 1\right]\, B^{(1)}(1,2)\,(1-p)^{-1}\, e^{s_1}\,. \end{aligned} \tag{4.8}$$

On the other hand, when a default happens we inject capital through the consumption stream (see equation (4.7)), such that we restore $L_1 = A_1 = P(1,2)$.

Time $t = 2$. We get

$$\begin{aligned} A_1 \to A_2^- &= \left[(1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1 - \Gamma_1 + 1\right]\; B^{(1)}(2,2)\,(1-p)^{-1}\, e^{s_1} \\ &= \left[(1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1 - \Gamma_1 + 1\right]\; \Gamma_2^{(1)}\,(1-p)^{-1}\, e^{s_1} \\ &= (1-p)^{-2}\, e^{2s_0}\, \Gamma_2 + (1-p)^{-1}\, e^{s_1}\,(1-\Gamma_1)\,\Gamma_2^{(1)} \qquad (4.9) \end{aligned}$$

where we used that $\Gamma_1 \Gamma_2^{(1)} = \Gamma_2$. The consumption is thus:

$$\widetilde{C}_2 = A_2^- - A_2 = (1-p)^{-2}\, e^{2s_0}\, \Gamma_2 + (1-p)^{-1}\, e^{s_1}\,(1-\Gamma_1)\,\Gamma_2^{(1)} - 1\,, \qquad (4.10)$$

with $A_2 = L_2 = 1$.

The total consumption stream is:

$$\begin{aligned} \widetilde{C}_0 &= 0 && (4.11) \\ \widetilde{C}_1 &= -(1-\Gamma_1)P(1,2) && (4.12) \\ \widetilde{C}_2 &= (1-p)^{-2}\, e^{2s_0}\, \Gamma_2 + (1-p)^{-1}\, e^{s_1}\,(1-\Gamma_1)\,\Gamma_2^{(1)} - 1 && (4.13) \end{aligned}$$

We now confirm that this model is still market consistent by checking that the no arbitrage condition is still satisfied:

$$\begin{aligned} Q_0[\widetilde{\boldsymbol{C}}] &= \mathbb{E}[\varphi_1\,(\Gamma_1 - 1)P(1,2)] + \mathbb{E}[\varphi_2\,(1-p)^{-2}\,e^{2s_0}\,\Gamma_2] \\ &\quad + \mathbb{E}[\varphi_2\,(1-p)^{-1}\,e^{s_1}\,(1-\Gamma_1)\,\Gamma_2^{(1)}] - \mathbb{E}[\varphi_2] \\ &= \mathbb{E}[\psi_2\chi_0\chi_1\,(\Gamma_1 - 1)] + \mathbb{E}[\varphi_2] + \mathbb{E}[\psi_2\chi_0\chi_1\,(1-\Gamma_1)] - \mathbb{E}[\varphi_2] \\ &= 0\,. \end{aligned} \tag{4.14}$$

A further discussion of the reduced consumption stream model can be found below in section 6.

5 Downside risk and exotic options as protection

5.1 Probability of a negative consumption

Following [3], we will analyze the probability of a negative consumption. Let us first consider the example from 2.2.2.

$$
\begin{aligned}
\mathbb{P}\mathrm{rob}[C_1 < 0] &= \mathbb{P}\mathrm{rob}\Big[(1-p)^{-1}\, e^{2s_0 - s_1}\, \Gamma_1 - 1 < 0\Big] \\
&= \mathbb{P}\mathrm{rob}\Big[e^{-s_1}\, \Gamma_1 < (1-p)\, e^{-2s_0}\Big] \\
&= p + \mathbb{P}\mathrm{rob}\Big[e^{-s_1} < (1-p)\, e^{-2s_0} \Big| \Gamma_1 = 1\Big] \\
&\geq p \qquad (5.1) \\
\mathbb{P}\mathrm{rob}[C_2 < 0] &= \mathbb{P}\mathrm{rob}\Big[(1-p)^{-1}\, e^{s_1}\, \Gamma_2^{(1)} - 1 < 0\Big] \\
&= \mathbb{P}\mathrm{rob}\Big[\Gamma_2^{(1)} < (1-p)\, e^{-s_1}\Big] \\
&= p\,. \qquad (5.2)
\end{aligned}
$$

Let us now do the same exercise for the reduced consumption stream example:

$$
\begin{aligned}
\mathbb{P}\mathrm{rob}[\widetilde{C}_1 < 0] &= \mathbb{P}\mathrm{rob}\Big[-(1-\Gamma_1) < 0\Big] \\
&= p \qquad (5.3) \\
\mathbb{P}\mathrm{rob}[\widetilde{C}_2 < 0] &= \mathbb{P}\mathrm{rob}\Big[(1-p)^{-2}\, e^{2s_0}\, \Gamma_2 + (1-p)^{-1}\, e^{s_1}\, (1-\Gamma_1)\, \Gamma_2^{(1)} - 1 < 0\Big] \\
&= \mathbb{P}\mathrm{rob}\Big[\big((1-p)^{-2}\, e^{2s_0}\, \Gamma_1 + (1-p)^{-1}\, e^{s_1}\, (1-\Gamma_1)\big)\, \Gamma_2^{(1)} < 1\Big] \\
&= p\,. \qquad (5.4)
\end{aligned}
$$

The total probability to have a negative consumption in the original example is bigger than $2p$, while it is exactly equal to $2p$ with the reduced consumption stream. Indeed, in the original example it might happen that at time 1 the spreads have risen such that the asset value went down enough to necessitate a capital injection ($C_1 < 0$ with $\Gamma_1 = 1$). However, at time 2 the bond did not default ($\Gamma_2 = 1$) so that the claim can be paid out and the remaining asset value, including the capital injected at $t = 1$, is released via a positive consumption stream ($C_2 > 0$). In the reduced consumption stream case, such scenario would not provoke a negative consumption at time 1.

5.2 An option as protection against the downside spread risk

Let us now consider an insurance company of which the shareholders are willing to support the reduced consumption stream (see equations (4.11)-(4.13)), but where the condition $A_1 \geq P(1,2)$ has to be respected, as in the example in section 2.2.2. The reduced consumption stream at $t = 1$ is non-zero if a default happened, but if the illiquidity spread led to a bond value less than $P(1,2)$, an additional cash injection will be needed to comply with the condition $A_1 \geq P(1,2)$.

The insurer could decide to buy an option which guarantees exactly this additional cash flow, to protect against this downside risk. Another solution suggested in [1] is for the insurer to hold an additional reserve. This additional reserve would be used to supply the additional cash flow. Both solutions come at a cost, either the cost of buying the option, or the cost to remunerate the capital in the additional reserve. The insurance premium increase necessary to cover the cost of renumerating the additional buffer was estimated at 10% to 15% in [8].

In this section, we consider the option that would generate the additional cash flow. The price of this option is exactly the price to comply with the condition $A_1 \geq P(1,2)$, while operating with a reduced consumption stream. In the example in section 2.2.2, we obtained the consumption stream C_1 by requiring $A_1 = P(1,2)$. To safeguard $A_1 \geq P(1,2)$, we thus need a cash flow injection of $(-C_1)^+$, since a cash injection equals a negative consumption stream. Let us assume we operate with the reduced consumption stream. The payoff from the option $\Delta_1 (\geq 0)$ necessary to preserve $A_1 \geq P(1,2)$ is then given by:

$$(-C_1)^+ \leq -\widetilde{C}_1 + \Delta_1\,, \tag{5.5}$$

or,

$$\begin{aligned}
\Delta_1 &= \left(\widetilde{C}_1 + (-C_1)^+\right)^+ && (5.6)\\
&= \begin{cases} \widetilde{C}_1 - C_1 & \text{if } C_1 < \widetilde{C}_1 < 0\,; \\ 0 & \text{otherwise.} \end{cases} && (5.7)\\
&= \left[1 - (1-p)^{-1}\, e^{2s_0 - s_1}\right]^+ \Gamma_1 P(1,2)\,. && (5.8)
\end{aligned}$$

We see that the payout is indeed zero if the bond has defaulted at $t = 1$, since in that case the reduced consumption stream $\widetilde{C}_1$ is used as capital injection. Notice that no additional option is needed at maturity ($t = 2$) since the reduced consumption stream already ensures that $L_2 = P(2,2) = 1$.

For insurance contracts with a long-term character, an option at each time step will be required to cover the downside risks from illiquidity spread. Extending the formalism to larger m, one would find the dependency of the option price on m, using the Black-Scholes formula. Given equation (2.53), we will have an exponential behaviour in m, rendering these options expensive for long-term insurance contracts.

6 Conclusions

In section 2, we extended the model for defaultable bonds developed in [10] to longer maturities and to include a time-varying illiquidity spread process. This paper does not deal with the question on how to measure the real world default probability p and the illiquidity spread s_t.

In section 3, we applied the liability valuation rules of Solvency II to a long-term contract. We showed that it will induce volatile capital streams, as it will expose insurance companies to immediate swings of the bond market credit spreads. This volatility of the consumption stream is composed of three contributions: a term linked with the volatility of the term structure, another term originating from the default process and a final term coming from the illiquidity spread. The illiquidity spread term grows quickly with the maturity of the insurance products, making it the dominant volatility source for long-term insurance products. With Solvency II, these more volatility capital streams will lead to a lower Sharpe ratio and will, as such, render long-term products more expensive.

In section 4, we set up the reduced consumption stream example, where the insurer assumes the risk of default of the bond at times $t = 1$ and $t = 2$, but not the illiquidity spread risk. We proved that the model still satisfies the no arbitrage condition in equation

(4.14). In addition, we note that an arbitrage by the policyholder is impossible, since she cannot trade or transfer the contract (liability) at $t = 1$.

Since liabilities are not traded, they have to be valued using a mark-to-model approach. The definition of the market value of the liability in the reduced consumption stream model differs from the Solvency II liability valuation. The liability value at time 1 in the reduced consumption stream model is given by equation (4.8). If the backing bond defaulted ($\Gamma_1 = 0$), then we have $L_1 = P(1, 2)$. If the bond did not default, then the market value is given by $L_1 = (1 - p)^{-1}\, e^{2s_0 - s_1}\, P(1, 2)$. In the latter case, the liability value is asset dependent. Indeed, since the liability is non-tradable at $t = 1$ for the policyholder, the market consistent price can vary with the backing assets. This phenomena reflects the reality that two policyholders with identical contracts purchased with different insurers can end up with a different payout in case the policyholder is allowed to terminate the contract before maturity through an market value adjustment clause. The possible loss incured by the insurer due to the early sale of backing assets is then borne by the policyholder. We note that the factor

$$(1 - p)^{-1}\, e^{2s_0 - s_1} = \frac{1}{P(1,2)}\, B^{(1)}(1, 2)\, \frac{P(0, 2)}{B(0, 2)} \tag{6.1}$$

can both be bigger, as well as smaller than 1. If the illiquidity spread s_1 has not increased significantly compared to s_0, then the factor will be bigger than 1. This would, in reality, often allow insurers to give profit sharing to the policyholder. Even if $s_0 = s_1 (\neq 0)$, then we find that both $(1 - p)^{-1} > 1$ and $e^{2s_0 - s_1} > 1$. If, on the other hand, the illiquidity spread s_1 increased significantly, though without a default, then we have $L_1 < P(1, 2)$.

Notice that there is never any upstreaming of capital at time 1 in our model, even if the factor is bigger than 1, since we always have $\widetilde{C}_1 \leq 0$.

The policyholder is not at risk since we impose in the model that $L_2 = P(2,2) = 1$. This is achieved via the consumption stream backing the liability. However, in providing this backing, the shareholder does not run an illiquidity spread risk. Indeed, both $\widetilde{C}_1$ and $\widetilde{C}_2$ are only negative in case a default is realized in the respective time step. We also notice, contrary to time step 1, that there is upstreaming of capital at time 2 ($\widetilde{C}_2 > 0$), as long as no default is realized at $t = 2$.

In reality, the illiquidity spread process e^{-s_t} and the default process Γ_t are often positively correlated. Indeed, as the spreads on certain government bonds have increased significantly during the European sovereign-debt crisis, it was the shareholders, who are liable for the consumption stream, that grew increasingly worried, with insurers' ratings decreasing, forcing insurers to reorient their asset portfolios. This would correspond in our model to replacing the bond A at time 1 with another bond A' where $A_1 < A'_1$. This necessitates a cash injection $A'_1 - A_1$ at time 1 to maintain the liability backing ($L'_1 = A'_1$). Just as in the case of default (see equation (4.7)), we impose $A'_1 = P(1,2)$. Indeed, by executing this transaction, the insurer assumes the full spread risk. Shareholder would tend to prefer such (known) cash injection over the possibly bigger and unknown consumption stream needed to back the liability in case of default at a later time. On the other hand, when economic conditions are good, the opposite transaction where $A_1 > A'_1 = P(1,2)$ could be executed. In this case capital is released which could be upstreamed, allocated as profit sharing or used to supply an additional countercyclical reserve as suggested in [1]. Another regulatory approach could be the restrict (full) consumption when $A_1 > P(1,2)$. In that case, we would have $L_1 > P(1,2)$, as if a countercyclical reserve is built into the liability valuation.

The reduced consumption stream model can be extended to larger maturities m. The volatility of the reduced consumption stream is lower than the volatility computed in section 3. Indeed, the volatility in section 3 was dominated by the spread. Due to its intrinsic construction, the volatility of the reduced consumption stream will be dominated by the default process. The liability market value definition in the reduced consumption stream model thus leads to a financial product which, from the point of view of the investor, has a lower Sharpe ratio.

In section 5, we verified that the probability of needing a cash injection was higher under the Solvency II valuation rules than in the reduced consumption model. The additional cash injection at time 1 under the Solvency II valuation rules is due when the illliquidity spread increased without having a realized default. An insurer could either opt to constitute a reserve from which this additional cash injection could be sourced, or it could opt to buy an option which would cover this risk. The price of such option tells us then the exact additional cost linked with the Solvency II liability valuation method. We find that the price of this option increases rapidly with the maturity of the insurance contract.

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