2023 INTERNATIONAL CONGRESS OF ACTUARIES

BRIDGE TO TOMORROW

28 MAY – 1 JUNE 2023 · SYDNEY

Actuaries Institute Australia

IAA AAI International Actuarial Association
Association Actuarielle Internationale

BRIDGE TO TOMORROW
2023 INTERNATIONAL CONGRESS OF ACTUARIES
28 MAY – 1 JUNE 2023 · SYDNEY

Actuaries Institute Australia

International Actuarial Association Association Actuarielle Internationale

Linking Annuity Benefits to Financial and Longevity Experience: A Joint Pricing Framework

Doreen Kabuche[1,2] and Annamaria Olivieri[1,3]

© 1. ARC Centre of Excellence in Population Ageing Research (CEPAR)
2. School of Risk and Actuarial Studies, UNSW Sydney
3. University of Parma, Italy

This presentation has been prepared for the 2023 International Congress of Actuaries.
The Actuaries Institute Australia Council wishes it to be understood that opinions put forward herein are not necessarily those of the Institute and the Council is not responsible for those opinions.

Background

- Low demand for longevity guarantees due to high costs
- Alternative solutions: longevity-linked products with flexible guarantees
- The benefit amount is updated to the mortality (longevity) experience
- Also common: financial-linked products, the benefit amount is updated to the realized investment experience
- Allow sharing of losses but also profits between the provider and annuitants

Motivation

❖ **Literature Gaps**

- Little previous work on linked annuity arrangements that include both financial and (partial) longevity participation

❖ **Research goals**

- Investigate the joint presence of financial and longevity participation
- The benefits are updated based on longevity and financial experience, including (partial) guarantees
- Devise the realistic price of the risk retained by the provider
- Explore trade-offs between the retained risk amount and the guarantee cost from the individual and provider perspectives

Methodology

- We follow the general linking structure proposed in [Olivieri and Pitacco, 2020]

- We estimate interest and mortality rates using an **affine term structure model**, the Arbitrage Free Nelson Siegel (AFNS) independent factor model: closed-form solutions and better forecast [Christensen et al., 2011, Huang et al., 2019]

- We use a periodic fee structure adopted in variable annuities [Bacinello et al., 2011, Olivieri, 2021]

Modelling Framework

- Financial linking framework

$$
\begin{aligned}
B_t &= B_{t-1} \times \begin{cases} 1 + \vartheta_t \tilde{r}_t, & \text{if } \vartheta_t \tilde{r}_t \geq r_{\min}, \\ 1 + r_{\min}, & \text{otherwise,} \end{cases} \qquad (1) \\
&= B_{t-1} \times \left(1 + \max\left(r_{\min}, \vartheta_t \tilde{r}_t\right)\right),
\end{aligned}
$$

where $\vartheta_t \in [0, 1]$ is the part of realized interest rate provided to the policyholder and $1 - \vartheta_t$ is the remaining part retained by the insurer, given a minimum guaranteed rate of $r_{\min}$

Modelling Framework

- Longevity linking framework

$$B_t = B_{t-1} \times \left(\frac{p^0_{x+t-1}}{\tilde{p}_{x+t-1}} \right), \text{ for } 1 - \psi_t \leq \frac{p^0_{x+t-1}}{\tilde{p}_{x+t-1}} \leq 1 + \psi_t \qquad (2)$$

$$B_t = B_{x_{\max}-x}, \text{ for } t > x_{\max} - x, \qquad (3)$$

where $\psi_t \in [0, 0.5]$ is part of realized longevity experience shared with the policyholder, the remaining amount $1 - \psi_t$ is retained by the insurer

Absolute bounds: $B_{\min}$ and $B_{\max}$ also the maximum age $x_{\max}$

Modelling Framework

- Financial and Longevity linking framework

$$B_t = B_{t-1} \times \max\left(1-\psi_t, \min\left(\frac{p^0_{x+t-1}}{\tilde{p}_{x+t-1}}, 1+\psi_t\right)\right) \times \left(1+\max(r_{\min}, \vartheta_t \tilde{r}_t)\right), \tag{4}$$

where $\psi_t \in [0, 0.5]$ is the longevity participation proportion and is the

$\vartheta_t \in [0, 1]$ financial participation proportion

Pricing Framework

- The metric used to determine the required fees is the business value for the provider
- Defined as the present value of future profits net of the cost of capital
- Examine the structure of periodic fees (the corresponding discount factor) under different assumptions
- The dynamics of the policy fund is described by the following equation:

$$A_t \cdot N_{x+t} = A_{t-1} \cdot N_{x+t-1} \cdot (1 - \zeta) \cdot (1 + \tilde{r}_t) - B_t \cdot N_{x+t}, \qquad (5)$$

- where ζ is the proportional premium loading (or fee) that is charged to each policy fund

Numerical Illustrations

- The proposed products are issued to Italian males aged 65 in 2021
- The maximum attainable age is assumed to be 100
- The annuity payments are made at the end of the year
- For financial linking: $\vartheta_t = 1$ and $r_{\min} = -0.1\%$
- For longevity linking: $\psi_t = 0.1$ so that $0.9 \leq \frac{p^0_{x+t-1}}{\tilde{p}_{x+t-1}} \leq 1.1$
- Other variables: $B_0 = 1$, $x_{\max} = 95$, $B_{\min} = 0.75$ and $B_{\max} = 2$
- Interest data: Italy Treasury bonds from 31/01/2005 to 31/12/2020
- Mortality data: Italy age-cohort mortality rates from 1856 to 1907

Main Results

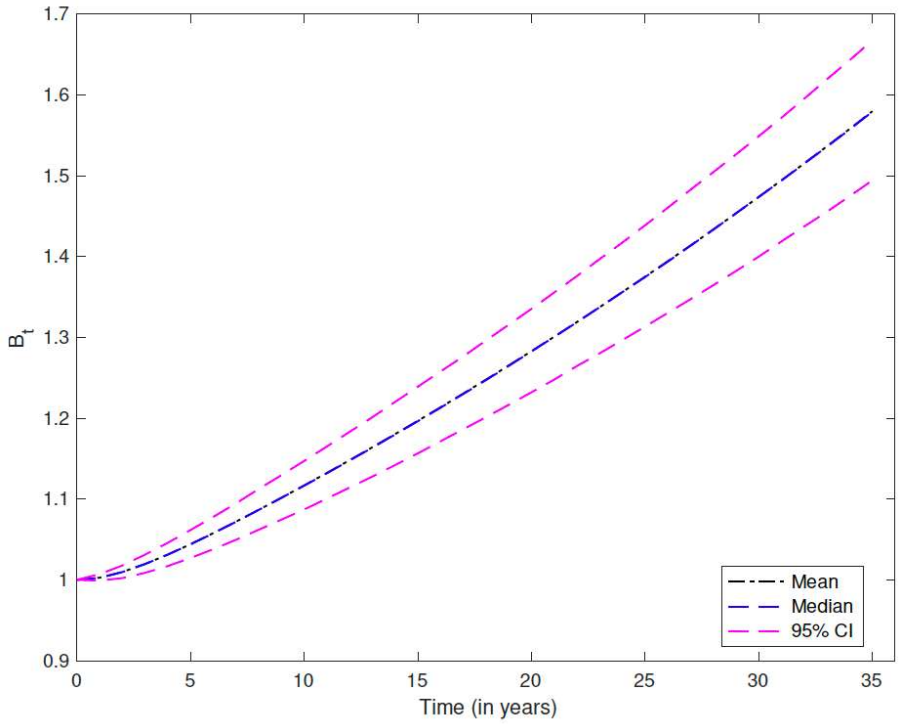
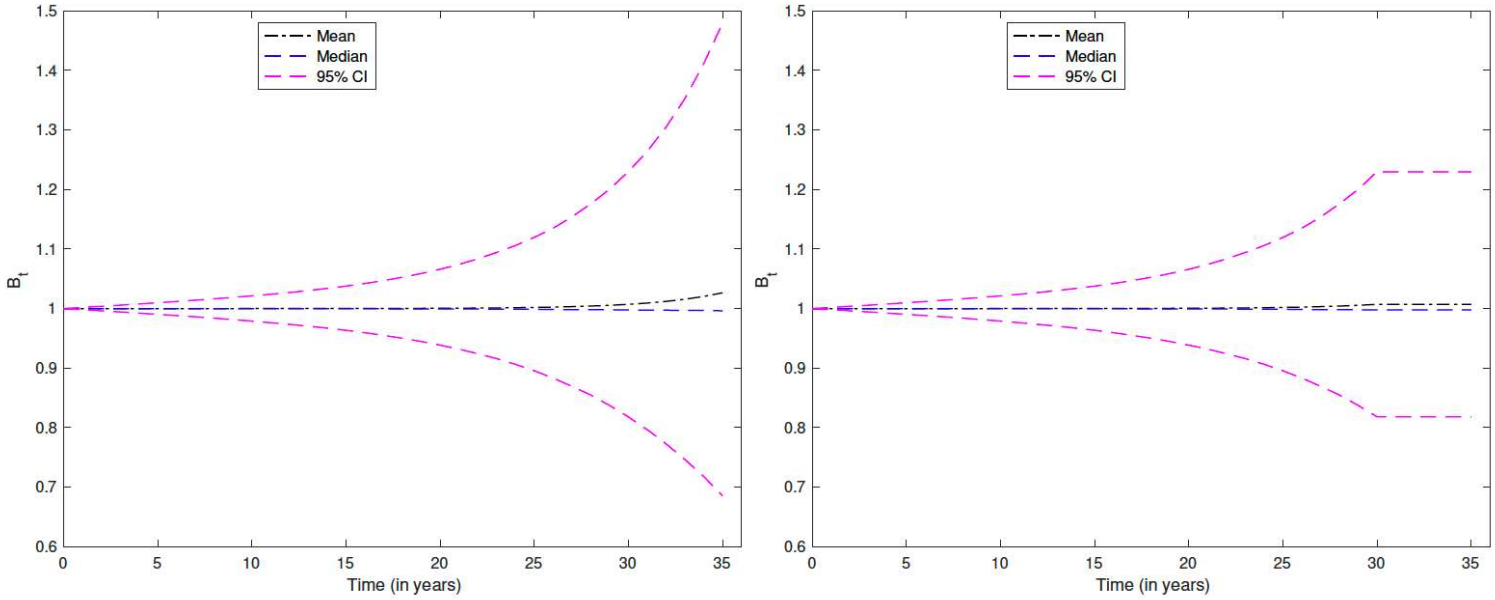


Figure 1: Financial-linked only annuity benefits with minimum guarantees

Main Results



(a) Longevity-linked benefits with bounds and no maximum age for participation.

(b) Longevity-linked benefits with bounds and maximum age for participation.

Figure 2: Longevity-linked only annuity benefits for an individual aged 65 to 100

Main Results

Table 1: The linked annuity benefits from financial linking, longevity linking, and financial and longevity linking-base case scenario

	Financial linking			Longevity linking			Financial and longevity linking		
	5%	Mean	95%	5%	Mean	95%	5%	Mean	95%
Age 75	1.0871	1.1168	1.1471	0.9789	1.0000	1.0213	1.0789	1.1168	1.1556
Age 80	1.1565	1.1967	1.2389	0.9635	1.0001	1.0377	1.1378	1.1969	1.2590
Age 96	1.4186	1.4942	1.5716	0.8181	1.0075	1.2299	1.1954	1.4837	1.8220
Age100	1.4941	1.5796	1.6674	0.8181	1.0075	1.2299	1.1954	1.4837	1.8220

Main Results

Now assume initial premium S=1000 in monetary units then the initial benefit B_0 =60.07

Table 2: Periodic fee to be charged each year to the policy fund value and the adjusted initial benefit amount given the initial premium of S = 1,000

	Periodic fee	Adjusted initial benefit
Financial linking	2.19%	45.80
Longevity linking	0.1%	59.39
Financial and longevity linking	1.99%	47.03

Conclusion

- The joint presence of financial and longevity participation has compensation effects as well as risk-return trade-offs for the provider and policyholders
- Financial and longevity-linked annuity benefits are slightly lower on average, but there is compensation in the form of a higher upside of the realized investment return
- Compensation effects from financial participation, reducing the participation fee or the retained cost of the guarantees
- Results are consistent with Bravo (2022) for participating longevity-linked life annuities

Appendix

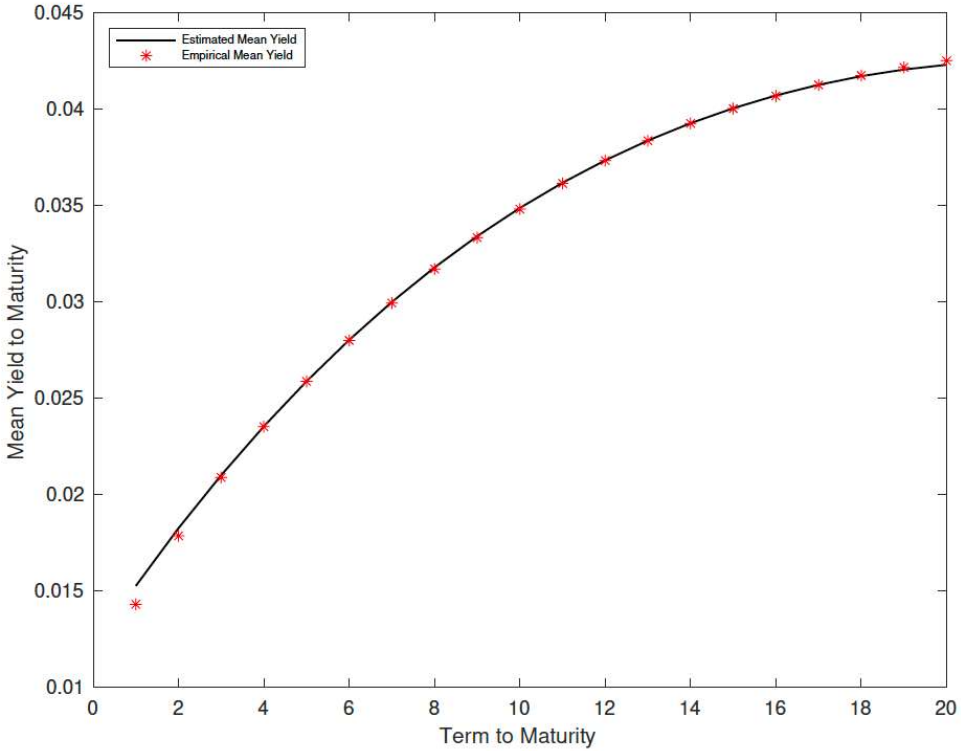
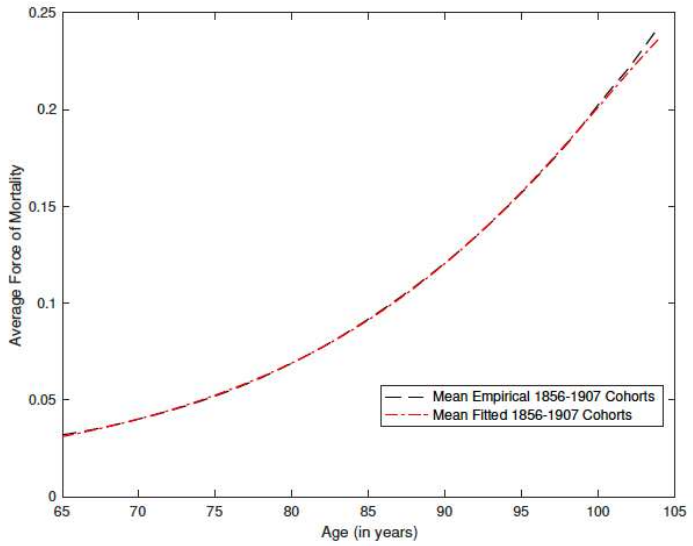
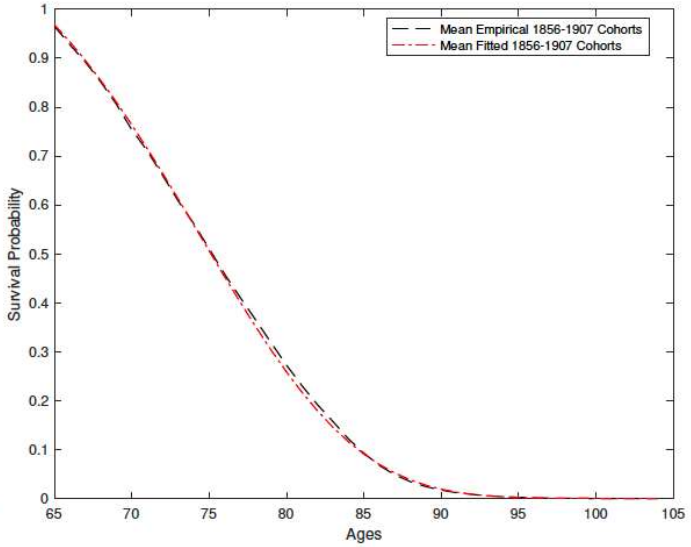


Figure A1: Mean of the estimated and empirical yield curve for 20 maturities

Appendix



(a) Mean empirical and fitted average force of mortality for the 1856-1907 cohorts.



(b) Mean survival probabilities of the empirical and fitted data for the 1856-1907 cohorts.

Figure A2: Mean empirical and fitted average force of mortality and survival probabilities for 1856-1907 Italian cohorts

Thank You

Contact: Doreen Kabuche
Early Career Academic Fellow
School of Risk and Actuarial Studies
University of New South Wales
Sydney, Australia
d.kabuche@unsw.edu.au

References

1. Bravo, J. M., 2022. Pricing participating longevity-linked life annuities: A bayesian model ensemble approach. European Actuarial Journal 12 (1), 125–159.

2. Bravo, J. M., de Freitas, N. E. M., 2018. Valuation of longevity-linked life annuities. Insurance: Mathematics and Economics 78, 212–229.

3. Chen, A., Guillen, M., Rach, M., 2021. Fees in tontines. Insurance: Mathematics and Economics 100, 89–106.

4. Denuit, M., Haberman, S., Renshaw, A., 2011. Longevity-indexed life annuities. North American Actuarial Journal 15 (1), 97–111.

5. Denuit, M., Haberman, S., Renshaw, A. E., 2015. Longevity-contingent deferred life annuities. Journal of Pension Economics & Finance 14 (3), 315–327.

6. Olivieri, A., Pitacco, E., 2019. Linking annuity benefits to the longevity experience: A general framework. Available at SSRN 3326672.

7. Olivieri, A., Pitacco, E., 2020a. Linking annuity benefits to the longevity experience: alternative solutions. Annals of Actuarial Science 14 (2), 316–337.

8. Piggott, J., Valdez, E. A., Detzel, B., 2005. The simple analytics of a pooled annuity fund. Journal of Risk and Insurance 72 (3), 497–520.