2023 INTERNATIONAL CONGRESS OF ACTUARIES

BRIDGE TO TOMORROW

28 MAY – 1 JUNE 2023 · SYDNEY

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A hybrid variable annuity contract embedded with living and death benefit riders

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Variable Annuities

Variable Annuities (VAs) were first introduced in the early 1950s and various 'GMxBs' have become available:

- Guaranteed Minimum Death Benefit introduced in 1980s.
- Guaranteed Minimum Living Benefits introduced in late 1990s.
    - GMAB - Accumulation,
    - GMIB - Income,
    - GMWB - Withdrawal,
    - (GLWB - Lifelong form of GMWB).

GMAB rider is the building block of all other GMxB rider and hence the focus of this research.

VA market over the world

- VA market in the US is huge: excess of US$2 trillion in total assets.
- In Australia and Europe, the market is very thin:
  - In Australia, there are only a few notable players.[1]
  - In Europe, the VAs' market was worth 188 billion in 2010 (EIOPA 2011). However, after the Global Financial Crisis, their popularity decreased and various life insurers stopped their VA offering.
- In Japan, the VA market grew from a market of less than $1 billion in 2000 to over $50 billion, subsequent to a period of financial deregulation in the late 90s (Zhang 2006).

[1] e.g. AMP Financial Services, BT Financial Group and MLC. (Vassallo et al. 2016)

Surrender behavior

- We consider a hybrid VA embedded with GMAB and GMDB riders which promise the return of the premium payment, or a higher stepped up value at the end of the accumulation period of the contract or death.
- Typically, the valuation frameworks study the effect of the underlying fund distribution (GBM, Levy, etc) on the fee.
- Recently, the surrender behavior is studied more closely in the literature (Bernard et al. 2014; Kang and Ziveyi 2018) as underpricing lapse risk has resulted in significant losses for insurers (Moody's Investor Service 2013).
- **Here:** the contract can be surrendered any time prior to maturity, and the payments are liable for **taxes** (policyholder perspective) in the event of surrendering before the preservation age.
- We combine optimal stopping theory with uncertainty due to the event of death, in line with how such contracts are structured in practice.

Importance of Incorporating Tax

- One of the main attractive features of VAs is their tax-advantaged investing (Milevsky and Panyagometh 2001; Brown and Poterba 2006).
- Incorporating taxation in riders such as GMWB reconciles empirically observed fees with the theory (Moenig and Bauer 2015).
- The financial planning literature has long looked at ways of providing rules to follow so as to maximise post-tax returns (Sumutka et al. 2012; Horan and Robinson 2008).
- **In this study:** we examine the impact of taxation upon surrender before the preservation age when losses can offset gains, reflective of the Australian setting.

Two account setting

- The bulk of existing literature consider VA valuation where guarantee fees are deducted from the investment account.
- In practice, some products are structured on a two account basis where the policyholder opens an investment account and a cash account as part of the VA contract.
- **Here:** we study this approach and consider management and transaction fees being deducted from the **investment account** indexed to the underlying fund.
- Guarantee fees funding the GMAB and GMDB riders are deducted from the **cash account**.

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The two accounts

- Let (S_ν) be the underlying asset which evolves according to the standard geometric Brownian motion (GBM) under the risk-neutral measure such that

$$dS_\nu = rS_\nu d\nu + \sigma S_\nu dW_\nu. \tag{1}$$

- The **investment component** of the VA (x_ν) can be expressed as $x_\nu = e^{-q_m\nu} S_\nu$ where q_m is the management fee rate.
- Applying Ito's Lemma to the process (x_ν) yields:

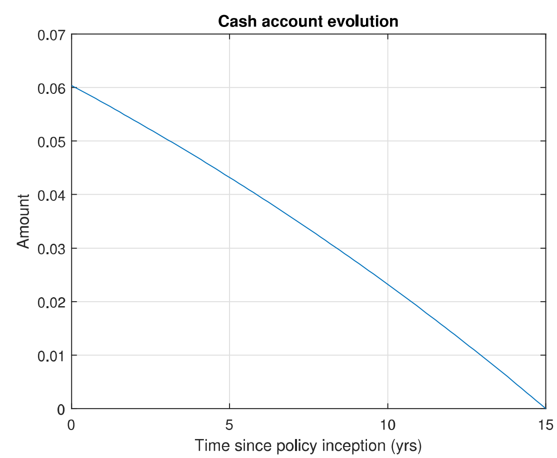
$$dx_\nu = (r - q_m)(x_\nu)d\nu + \sigma(x_\nu)dW_\nu. \tag{2}$$

- The **cash account** evolution is described as

$$c(\nu) = e^{r\nu}\int_\nu^{T_m} q_g G e^{(\delta-r)s} ds = q_g G e^{\delta\nu}\frac{e^{(\delta-r)(T_m-\nu)} - 1}{\delta - r}. \tag{3}$$

Toy example

- Management fees depend on the underlying, which is stochastic. Guarantee fees are deducted from a cash account, which is deterministic.
- This cash account decreases at a rate of $q_g G$ while also earning the risk-free interest at rate r. At maturity, the cash account should be depleted, $c(T_m) = 0$.
- Evolution of the cash account:



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Item	Held until maturity	Early surrender at t=7.5
Fund value at commencement (a)	100	100
Fund value at termination (b)	120	110
Surrender penalty (c)	0	5*
Cash account payout (d)	0	1.20[2]
Capital gain on inv acc [(b) +(d) - (c) - (a) = (e)]	20	6.20
Capital loss on cash acc [(d) - c(0) = (f)]	4.32	3.12
Total capital gain [(e) - (f) = (g)]	15.68	3.09
Tax paid [0.30*(g) = (h)]	4.70	0.93
Inv account payout [(b) - (c) - (h) = (i)]	115.30	107.07

Table: Illustrative example with cashflows associated with the cash account. A VA contract matures in 10 years, $\delta = 0$, guarantee fee rate 0.005, $r = 3\%$, $c(0) = 4.32$ corresponding to the initial investment in the cash account.
*An arbitrary surrender penalty is used for illustrative purposes.

[2] This corresponds to $c(7.5)$.

The hybrid VA product

- T_d time at death, T_m maturity, α age at inception

$$V_\alpha(x, T_m) = \operatorname*{ess\,sup}_{\nu \le \epsilon^* \le T_m} \mathbb{E}^{\mathbb{Q}} \Bigg[\overbrace{\mathbb{1}_{\{T_d > T_m\}}}^{\text{Survival}} \overbrace{\left(e^{-\int_\nu^{\epsilon^*} r(s)ds} g(\epsilon^*, x_{\epsilon^*})\right)}^{\text{PV GMAB payout}}$$

$$+ \overbrace{\mathbb{1}_{\{T_d < T_m\}}}^{\text{Death before maturity}} \overbrace{e^{-\int_\nu^{T_d} r(s)ds} h(\nu, x_\nu)}^{\text{PV of GMDB payout}} \,|\mathcal{F}_\nu \Bigg]$$

$$= {}_{T_m} p_\alpha \overbrace{\left[\operatorname*{ess\,sup}_{\nu \le \epsilon^* \le T_m} \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_\nu^{\epsilon^*} r(s)ds} g(\epsilon^*, x_{\epsilon^*}) \right] \right]}^{\text{Only living benefits to optimise}}$$

$$+ \mathbb{E}^{\mathbb{Q}} \left[\mathbb{1}_{\{T_d < T_m\}} e^{-\int_\nu^{T_d} r(s)ds} h(\nu, x_\nu) \right] |\mathcal{F}_\nu,$$

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Financial base case parameters

Parameter	Value	Parameter	Value
x_0	1	T	5,10,15
σ	0.20	κ	0.001
r	0.03	τ	0.225
δ	0.015		

Table: Base case parameters for numerical illustrations

- Weekly time discretisation.
- Maximum possible fund value and total fees set to $4 \cdot G$.

Interplay of management and guarantee fees

- Intuitively, $q_m \nearrow$, the policyholder's fair guarantee fee $q_g \searrow$. Indeed, they are paying more for the product in the form of administrative fees deducted from the investment account.
- However, if management fees q_m high $\rightarrow$ investment account $\searrow$ and guarantee becomes more likely.
- Thus, it is of interest to examine the fair fee charging structures and the trade-off between the guarantee and the investment management fees.

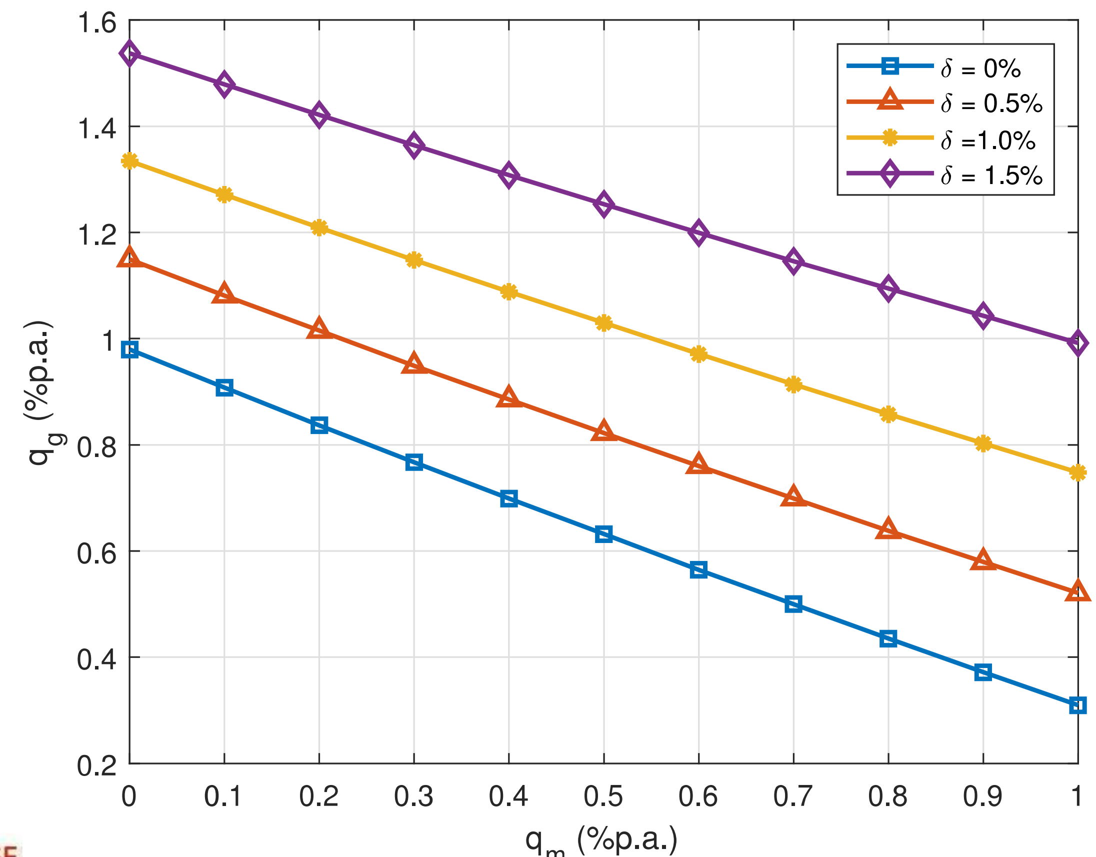
Interplay of management and guarantee fees C'td...

qm\delta	**0.0%**	**0.5%**	**1.0%**	**1.5%**
0.0%	**0.98%**	1.15%	1.33%	1.54%
0.1%	0.91%	1.08%	1.27%	1.48%
0.2%	0.84%	1.02%	1.21%	1.42%
0.3%	0.77%	0.95%	1.15%	1.36%
0.4%	0.70%	0.89%	1.09%	1.31%
0.5%	0.63%	0.82%	1.03%	1.25%
0.6%	0.56%	0.76%	0.97%	1.20%
0.7%	0.50%	0.70%	0.91%	1.15%
0.8%	0.44%	0.64%	0.86%	1.09%
0.9%	0.37%	0.58%	0.80%	1.04%
1.0%	0.31%	0.52%	0.75%	**0.99%**

- Return-on-premium ($\delta = 0\%$) with no management fees have a comparable guarantee fee to $\delta = 1.5\%$ with 1% management fees.
- Guarantee fees decrease with management fees & increase with roll-up.

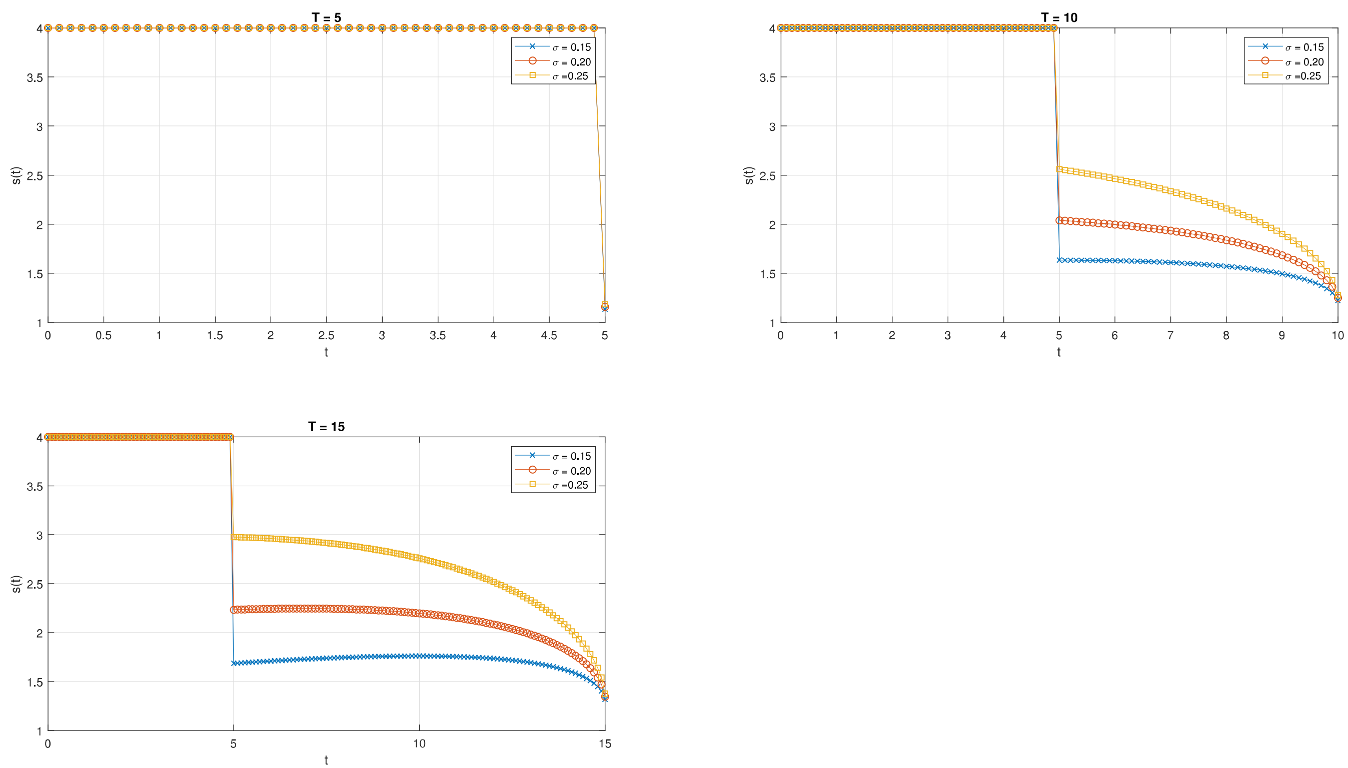
Varying roll up rates

Figure: Fair fee curves for various guarantee growth rates



Early surrender boundaries increases with volatility

Figure: Impact of varying volatility



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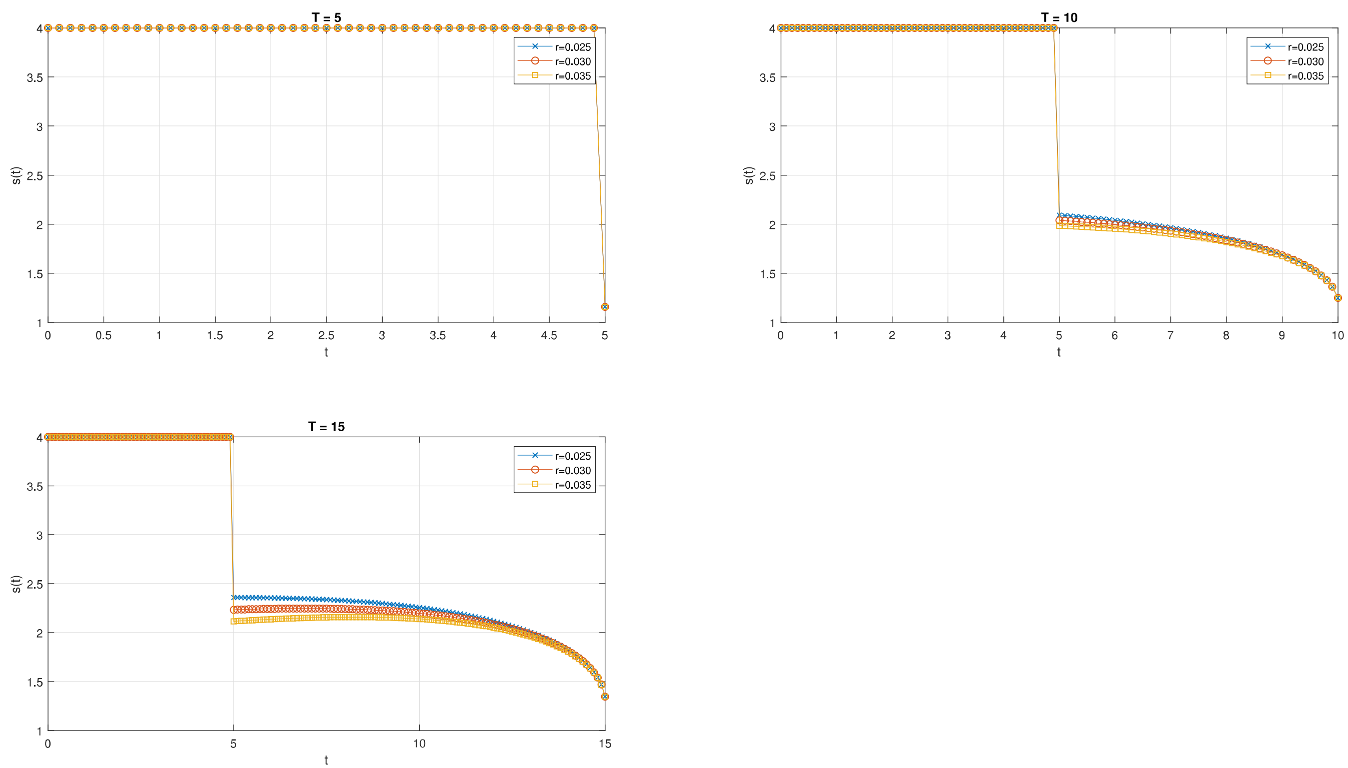
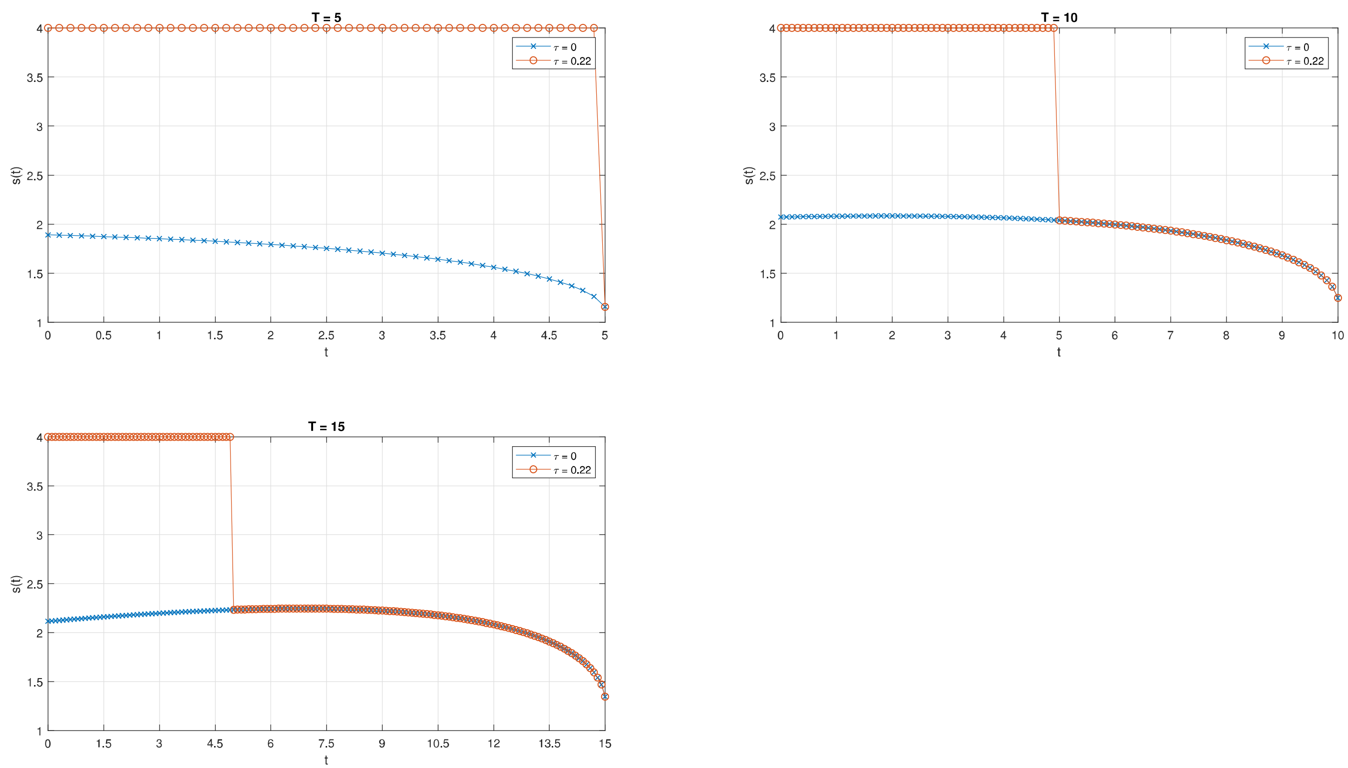
Early surrender boundaries (C'td)

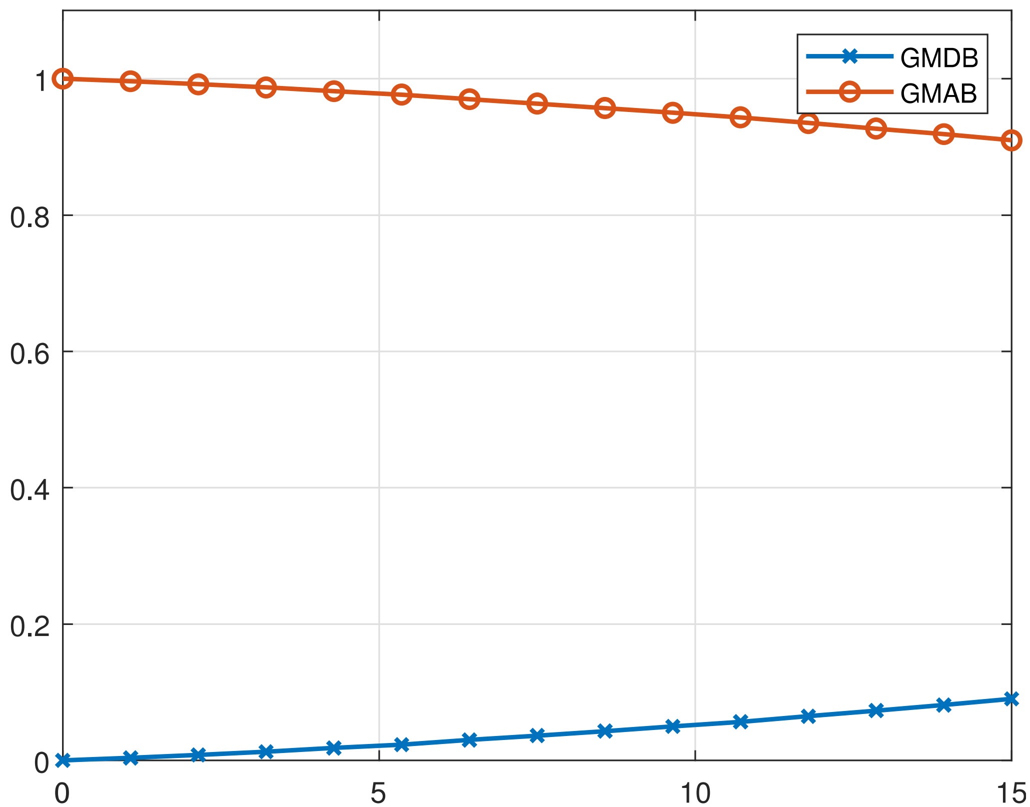
Figure: Impact of varying risk free rate



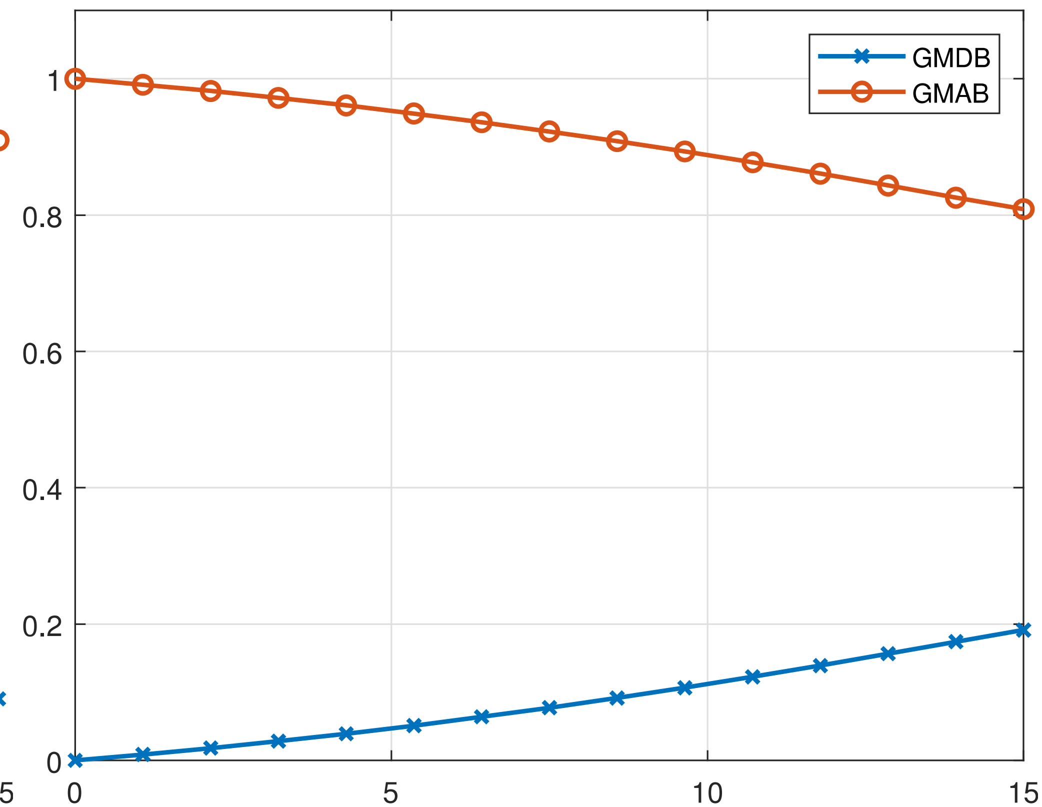
Early surrender boundaries (C'td)

Figure: Impact of varying taxation





(a) Entry age 55



(b) Entry age 65.

Figure: GMAB and GMDB components

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Conclusion

- Presented a hybrid framework for VAs embedded with GMAB and GMDB riders as is often the case in practice.
- A novel two account structure mimicking some product offerings in the Australian market.
- Incorporating taxation in the event of contract exercised before the preservation age in Australia.
- Sensitivity analyses have been performed for varying parameter settings, providing insights on the interplay of management fees and guarantee fees.

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Thanks

Thank you for your attention

Questions?

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