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Build Life insurance company investment portfolio based on the Black-Litterman model and a Hierarchical approach

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Abstract

The objective of this paper is to create a "realistic" and "quantitative" model for determining asset allocation in insurance companies. Applying a quantitative model to insurance companies is often difficult due to the following three characteristics: 1) the large number of assets and the difficulty in maneuvering them, 2) the need to consider liability risk, and 3) the frequent influence of accounting and regulatory constraints. In this paper, the author has constructed a new model that overcomes these characteristics. In constructing the model, the hierarchical risk parity approach and the Black-Litterman method were referenced. The main features of the model are as follows: 1) clustering and hierarchically optimizing asset allocation to ensure robustness, 2) using the Black-Litterman method to calculate expected return rates, 3) adding constraints for asset fluctuations, 4) evaluating risk in consideration of liabilities, and 5) using a risk-adjusted return index in the objective function during optimization. Furthermore, a Monte Carlo simulation was conducted to compare the proposed method with five existing models. The results showed that the proposed method optimizes risk-return while minimizing changes in asset allocation when compared to other models, and its usefulness was confirmed.

1. Introduction

It is often difficult to apply a quantitative model such as "efficient frontier" (Markowitz, 1952) in practice. The purpose of this paper is to create a "realistic" and "quantitative" model for determining an insurance company's asset allocation. It can also be applied to non-insurance companies with huge balance sheets. Following three features make it difficult to apply a quantitative model.

- The assets of insurance companies are enormous, often exceeding hundreds of billions of dollars, and it is difficult to reorganize it frequently.
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- insurance companies are often subject to accounting and regulatory constraints.

An insurance company's portfolio optimization model should meet following five requests.

- To limit the variation from the current asset allocation to some extent
- To optimize the model based on the risk characteristics of very long-term liabilities
- To provide flexibility to choose which indicators, such as accounting profit, surplus, ESR, etc., are to be maximized;
- To allow the company to look ahead to the financial environment scenarios within the company
- To be robust

The most famous asset allocation optimization method would be Markowitz's the Critical Line Algorithm (CLA) [1]. However, several problems have been identified with this model, which can be divided into two main categories.

The first is the difficulty in estimating the expected rate of return. Michaud [4] showed that it has been pointed out that a slight difference in the expected rate of return can result in the formation of completely different portfolios (Michaud, 1998).

The second is the issue of the variance-covariance matrix. López [3] pointed out that as the number of factors increases, the "condition number" of the variance-covariance matrix (the ratio of the maximum and minimum eigenvalues of the matrix) increases, resulting in a biased optimized portfolio [5].

De Miguel et al. [6] pointed that these two issues point out that the performance is inferior to the equal-weighted portfolio in the out-of-sample.

2. Material and Method

2.1 Assignment setting

Consider the asset portfolio optimization problem in a life insurance company. The company's assets are enormous, the risk of liabilities is taken into account, and the company has a reasonable outlook on the market.

2.2 Proposed model: HBL

2.2.1 Overview

To overcome two main weaknesses of CLA, The proposed model is devised with reference to two previous studies. This paper introduced the

Hierarchical Black-Litterman model (HBL).

First, this model used the Black-Litterman model [2] in estimating the expected rate of return. In this paper, the Black-Litterman market portfolio utilizes the pre-optimized portfolio. Specifically, it assumes the pre-optimized portfolio to be efficient, and then sets the prior distribution of expected returns. The optimized portfolio, on the other hand, represents a rebalanced portfolio taking into account the financial environment outlook.

Second, the Hierarchical Risk Parity (HRP) approach [3] was applied to this model to resolve issues arising from the "condition number" of the variance-covariance matrix.

In addition to these, HBL had there features. First is the addition of constraints on asset changes. The second is that the optimization is limited to assets only and incorporates liability information only for the risk penalty. The third point is the use of a risk-adjusted return measure as the objective function during optimization.

2.2.2 Details

Consider N assets $\{X_i\}_{\mathrm{i}=1,\ldots,\mathrm{N}}$, and the correlation of them is computed with entries $\{\rho_{i,j}\}_{\mathrm{i,j}=1,\ldots,\mathrm{N}}$. The first step is to calculate the expected rate of return using the Black-Litterman method. Next, following HRP, tree clustering is performed. The distance measure is defined as $d_{i,j} = \sqrt{\frac{1}{2}(\mathbf{1} - \rho_{i,j})}$.

Then, define clusters using agglomerative clustering whose method is single by the distance measure.

Next, recursively replace for each node of the cluster with its respective component. Finally, Adjust the portfolio as in the following algorithm.

Algorithm

Inputs / Definitions:

w: all assets $weights\ vector$

w_d: Debt exposure

Σ: variance-covariance matrix

$\{L_i\}_{\mathrm{i}=1,\ldots,\mathrm{N}}$: each node of the cluster, L_0 covers all assets

$w_i\,(\mathrm{i} = 1, \ldots \mathrm{N})$: Weights of assets in the node L_i

Settings:

w_{max} ($\in(0,1]$) : Parameter for what percentage of total assets to allow to change in one optimization

σ_{max} ($\in[1,\infty]$) : Parameter that defines how many times the increase in volatility is allowed in one optimization.

$f\ (w(\text{asset } weights) \rightarrow \Re)$: Maximizing utility function.

ex) $f(w) = w'\mu - 2.57\sqrt{w'\Sigma \mathrm{w}}$

output: w_n: optimized portfolio

function HBL

  $\mathrm{w} = initial\ portfolio$

  for $\mathrm{i} = 1, \ldots N$ **do**

    bisect L_i into two subsets, $L_i^1 \cup L_i^2 = L_i$, where $\left(\left|L_i^1 = int[\frac{1}{2}|L_i|]\right|\right)$

    $w_i(\alpha) := w_n + w_d + \alpha\frac{w_i^1}{|w_i^1|} - \alpha\frac{w_i^2}{|w_i^2|}$

    $\Omega_\mathrm{i} := \{\alpha_i \in [-w_{max}, w_{max}] \mid w(\alpha)'\Sigma w(\alpha) < \sigma_{max} w(0)'\Sigma w(0)\}$

    $\alpha_i^* = \arg\max_{\alpha \in \Omega_\mathrm{i}} f(w(\alpha))$

    $\mathrm{re-scale\ allocations}$

$$w = w + \alpha_i^*\frac{w_i^1}{|w_i^1|} - \alpha_i^*\frac{w_i^2}{|w_i^2|}$$

  end for

  return w

end function

2.3 Monte Carlo simulation

Assume there were 10 asset classes ($\{X_i\}_{\mathrm{i}=1,\ldots,10}$), and one of them ($X_0$) has about 50% debt to total assets. The initial portfolio was following.

$$X_0 = 0.4$$
$$\left\{X_i = \frac{0.6}{8}\right\}_{\mathrm{i}=1,\ldots,8}$$
$$X_9 = 0$$

First, 10 series of random Gaussian returns were generated, which have 520 observations, equivalent to 2 years of daily history. Randomly select 3 out of 10 assets that have an annualized rate of 10% and 3 out of 10 assets that have an annualized rate of -10%.

All assets' arbitrary standard deviations were 10%. As Prado (2016) did, to resemble real prices, random shocks and a random correlation structure were added to generated data.

Second, six portfolio optimization methods were computed by looking back at 260 observations (a year of daily history). These portfolios are re-estimated and rebalanced every 22 observations (equivalent to a monthly frequency).

Third, the out-of-sample returns were calculated. This procedure is repeated 10,000 times.

2.4 Benchmark models

For benchmarking the HBL, five models are chosen.
$w(\{w_i\}_{i=0,\dots,9})$: all assets $weights\ vector$
w_d: Debt exposure
$\Sigma(\{\sigma_{i,i}\}_{i,i=0,\dots,9})$: variance-covariance matrix

- *the Inverse-Variance Portfolio (IVP)*

$$w_i = \frac{1}{\sigma_{i,i}} \div \sum_{k=0}^{9} \frac{1}{\sigma_{k,k}}$$

- *the Minimum-Variance Portfolio (MVP)*

$$w = \arg\min_{w} (w' + w_d{}') \, \Sigma (w + w_d)$$

$$s.t. \sum_{k=0}^{9} w_i = 1, w_i \geq 0 (i = 0, \dots 9)$$

- *the Critical Line Algorithm (CLA)*

Let μ be the average annual return for each asset in the in-sample.

$$w = \arg\min_{w} (w' + w_d{}') \, \Sigma (w + w_d)$$

$$s.t. \sum_{k=0}^{9} w_i = 1, w_i \geq 0 (i = 0, \dots 9)$$

$$\sum_{k=0}^{9} w_i{}' \mu \geq 0.03$$

- *the Black-Litterman model (BL)*

Using the average annual return for each asset in the in-sample and the initial portfolio weights, calculate the expected rate of annual return μ^*.

$$w = \arg\min_{w} (w' + w_d{}') \, \Sigma (w + w_d)$$

$$s.t. \sum_{k=0}^{9} w_i = 1, w_i \geq 0 (i = 0, \dots 9)$$

$$\sum_{k=0}^{9} w_i{}' \mu^* \geq 0.03$$

- *the Hierarchical Risk Parity (HRP)*

Following Prado (2016), Clustering is performed by defining a distance matrix from the correlation of each asset. After initially giving equal weights to all assets, hierarchical optimization was performed by IVP.

2.5 Evaluation

Using one simulation as an example, the portfolios of each of the asset portfolio optimization methods were analyzed.

We calculated average out-of-sample annual rate of return, volatility of annual rate of return, and difference between pre- and post-optimized portfolios over 10,000 Monte Carlo simulations.

3. Results

3.1 Portfolio status of one simulation

Table 1

Annual Returns in-sample for 10 assets in a Monte Carlo simulation. Assets 0, 2, and 5 had positive returns, while assets 3 and 9 have large negative returns due to stressed

Asset	0	1	2	3	4
In-sample return	19%	2%	18%	-49%	-17%
Asset	5	6	7	8	9
In-sample return	17%	-13%	2%	-8%	-49%

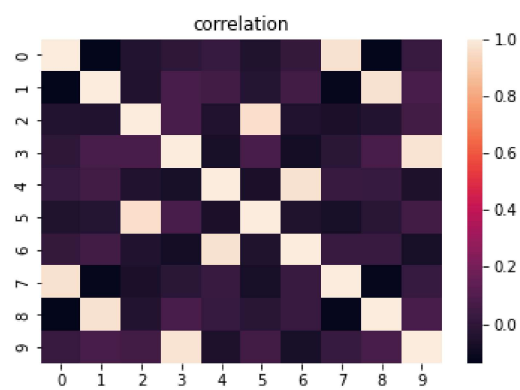

Figure 1

As Prado (2016) did, this correlation matrix was computed on random series $X = \{X_i\}_{i=0,\dots,9}$ drawn as follows. First, 5 random vectors from a standard Normal distribution were drawn, $X \left\{X_j = z\right\}_{j=0,\dots,4}$. Second, 5 random integer numbers from a uniform distribution were drawn, with replacement, $\vartheta = \{\vartheta_k\}_{k=0,\dots,4}$. Third, $X_{5+k} = X_{\vartheta_k} + \frac{1}{4}z$, $\forall\, k = 1, \dots, 5$ was computed. This forced the 5 last columns to be partially correlated to some of the first 5 series.

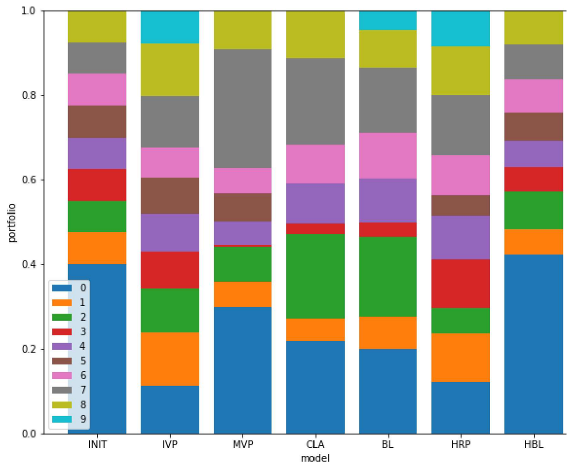

Figure 2
Each assets' weights of the initial portfolio and six methods' ones. IVP, HRP were equal to the weight of any asset. The other models had a large weighting of asset 0. HBL has a very similar portfolio to the initial model (INIT).

3.2 Results of 10,000 Monte Carlo simulations

The out-of-sample annual returns, their volatilities, and their differences from the averaged initial portfolio were shown in Table 2. The three models with positive returns were CLA, BL, and HBL, with BL having the highest returns. The models with relatively low volatility were the MVP, CLA, BL, and HBL models, with the MVP model having the lowest volatility. Initially, the HBL model had an outstandingly low variance from the portfolio.

Table 2 Results of 10,000 Monte Carlo simulations

	Return mean	Return std	Allocation change
IVP	-2.01%	16.72%	68.10%
MVP	-0.01%	**1.13%**	68.44%
CLA	**1.79%**	**4.20%**	86.07%
BL	**2.24%**	**4.55%**	91.08%
HRP	-2.01%	16.71%	75.85%
HBL	**1.36%**	**6.85%**	**10.01%**

4. Discussion

Six models were evaluated from three perspectives: first, whether the model considered expected returns; second, whether the model considered liability risk; and third, whether the model addressed "condition number" issues as Table 3 shows.

Table 3 Characteristic of 6 models

	Return	debt risk	condition number
IVP	-	-	-
MVP	-	✓	-
CLA	✓	✓	-
BL	✓	✓	-
HRP	-	-	✓
HBL	✓	✓	✓

Figure 1 shows that the MVP and CLA models, which considered debt risk, had a relatively high ratio of asset 0 with debt, while the IVP and HRP models did not have a high ratio of asset 0.

In Table 1, the assets with the highest in-sample returns were 0, 2, and 5, and these assets were overweighted in the return-conscious models BL, CLA, and HBL. As a result, Table 1 shows that these models showed high performance in 10,000 Monte Carlo simulations.

Figure 1 shows that the MVP, CLA, BL, and HBL models, which considered debt risk, had a relatively high ratio of asset 0 with debt, while the IVP and HRP models did not have a high ratio of asset 0. As a result, Table 2 shows that the standard deviations of IVP and HRP were relatively large for 10,000 Monte Carlo simulations.

Figure 2 and Table 2 show that The HBL model had a volatility constraint, which kept the volatility of assets as low as one-sixth to one-ninth that of other models.

5. Conclusions

The HBL model proposed in this study showed high performance on Monte Carlo simulation while controlling the volatility of the assets.

On the other hand, there are some disadvantages, such as the large number of parameters, which are difficult to adjust, and the inability to apply weighting constraints to individual assets.

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