2023 INTERNATIONAL CONGRESS OF ACTUARIES

BRIDGE TO TOMORROW

28 MAY – 1 JUNE 2023 · SYDNEY

Actuaries Institute Australia

IAA AAI International Actuarial Association
Association Actuarielle Internationale

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Build Life insurance company investment portfolio based on the Black-Litterman model and a Hierarchical approach

Introduction - Difficulties in applying quantitative asset allocation in insurance companies

1. Cannot dynamically change allocation due to the large size of the assets

2. Need to consider long-term liabilities.

3. May face constraints from accounting and regulations

Introduction - Properties required for the model

- ✓ To quantitatively calculate a "realistic" portfolio in line with the management policy

1. Control the divergence from the current portfolio
2. Consider the liabilities' risk
3. Make the optimization objective function flexible
4. Consist with the company's financial environment outlook
5. Ensure robustness against small variations in assumptions

Related Works –Two types of portfolio

- ✓ Portfolio using expected returns
  1. Markowitz model
  2. Black-Litterman model
  3. Proposed model
- ✓ risk-based portfolio
  1. Minimum variance portfolio
  2. Risk Parity portfolio
  3. Inverse variance portfolio

Related Works – Markowitz model's summary and problems

Markowitz model

$$\text{Minimize} \quad \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} w_i \mu_i = \bar{r}, \quad \sum_{i=1}^{n} w_i = 1$$

μ_i: expected return of asset i
$\Sigma = (\sigma_{ij})$: Covariance matrix
$\bar{r}$: expected return of portfolio(w_1,… w_n)

Markowitz model's problem

- ✓ Difficulty in estimating expected return
- ✓ Lack of robustness derived from ""nearly ill-conditioned" covariance matrix

Proposed Model – To overcome Markowitz model's problem

- ✓ Difficulty in estimating expected return
  - ➢ A Robust Optimization Approach (Ben-Tal and Nemirovski 1998)
  - ➢ Black-Litterman model (Black and Litterman, 1992)

- ✓ Lack of robustness derived from ""nearly ill-conditioned" covariance matrix
  - ➢ Risk diversified portfolios using principal component analysis (Meucci 2009)
  - ➢ Hierarchical Risk Parity (HRP) approach (López de Prado [2016])

Proposed Model - To overcome Difficulty in estimating expected return

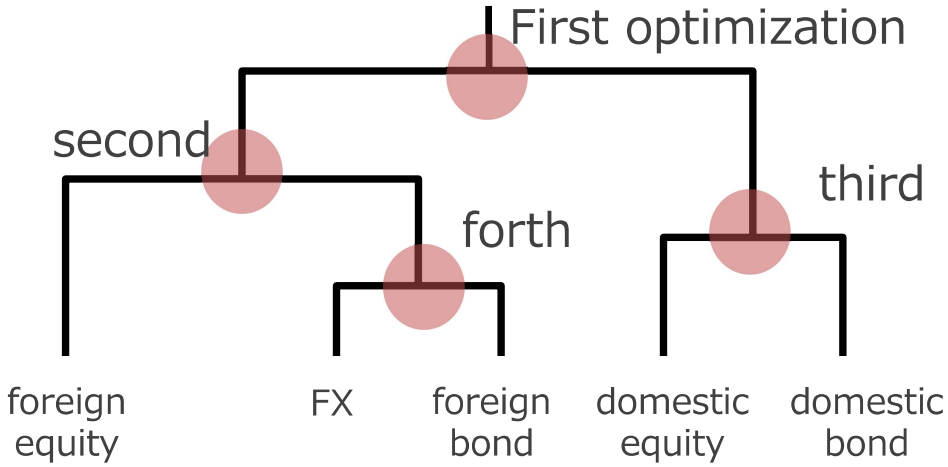
- ✓ Calculate the expected return using Black-Litterman model

    1. Calculate the implied expected return from the covariance matrix and the exposure of risk factors
    2. Calculate the return based on the outlook for the financial environment
    3. Combine 1 and 2, and calculate the expected return

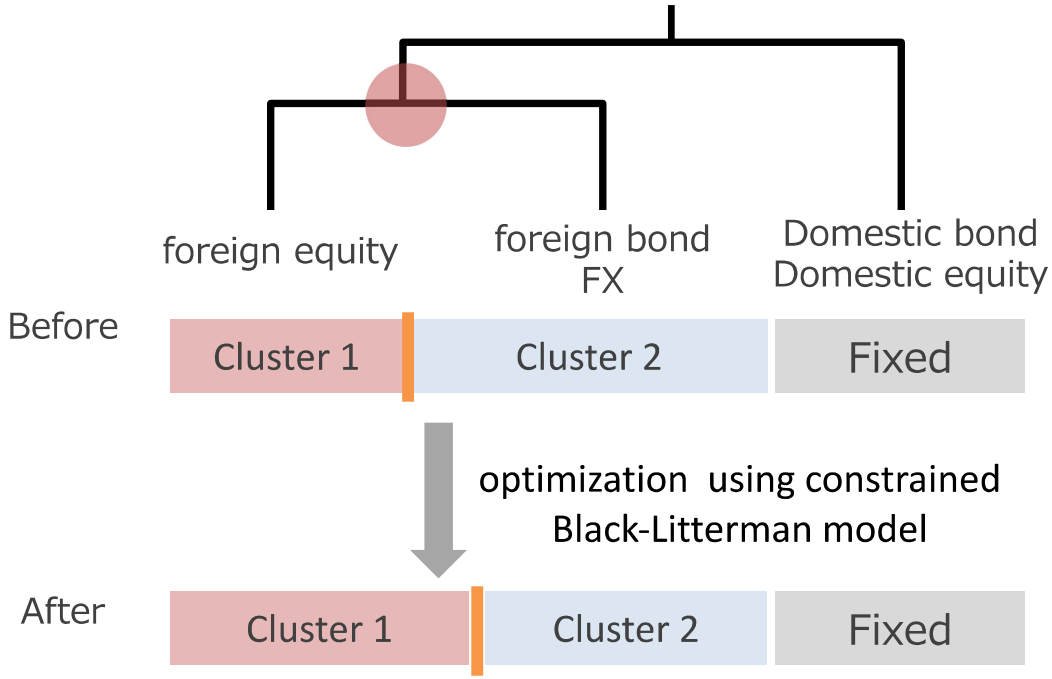
Proposed Model - To overcome Lack of robustness derived from ”nearly ill-conditioned” Σ

- ✓ “Hierarchical” optimization using cluster analysis

Optimization orders



Second optimization‘s image



Experiment - Monte Carlo simulation (10,000 times)

- ✓ 10 series of random Gaussian returns were generated, which have 520 observations

✓ Used methods

	Model Feature		
	Return	debt risk	condition number
IVP	-	-	-
MVP	-	✓	-
MV	✓	✓	-
BL	✓	✓	-
HRP	-	-	✓
HBL	✓	✓	✓

✓ Initial weight

- ➢ Asset

$$X_0 = 0.4$$
$$\left\{X_i = \frac{0.6}{8}\right\}_{\mathrm{i}=1,\ldots,8}$$
$$X_9 = 0$$

- ➢ Debt

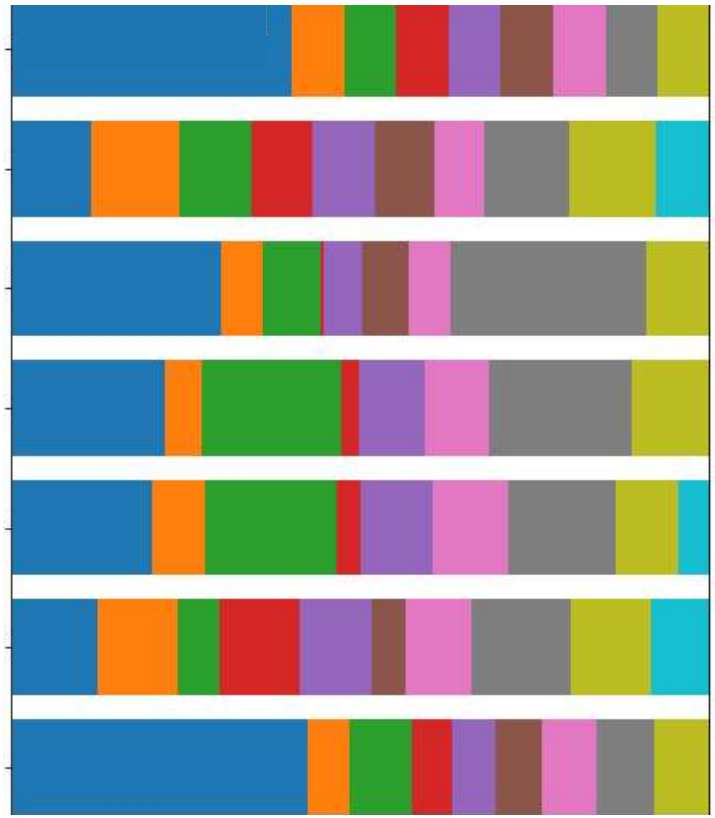
$$X_0 = 0.5$$

Experiment Result – One Monte Carlo simulation's result

Asset	In-sample return
0	19%
1	2%
2	18%
3	-49%
4	-17%
5	17%
6	-13%
7	2%
8	-8%
9	-49%

Optimizing Portfolio

	Return	debt risk
INIT	-	-
IVP	-	-
MVP	-	✓
MV	✓	✓
BL	✓	✓
HRP	-	-
HBL	✓	✓



Experiment Result - Out-of-sample performances in 10,000 times Monte Carlo simulations

	Experiment Result			Model Feature	
	Return mean	Return std	Allocation change	Return	debt risk
IVP	-2.01%	16.72%	69%	-	-
MVP	-0.01%	**1.13%**	68%	-	✓
MV	**1.79%**	**4.20%**	86%	✓	✓
BL	**2.24%**	**4.55%**	91%	✓	✓
HRP	-2.01%	16.71%	76%	-	-
HBL	**1.36%**	**6.85%**	**10%**	✓	✓

Conclusion

✓ Advantages

1. Control the divergence from the current portfolio
2. Consider the liabilities' risk, and expected return (the company's financial outlook)
3. Easy to analyze and allowing for it to be stopped midway if needed

✓ Problems

1. Difficult to apply constraints to individual weights
2. Difficult to tune many parameters
3. Not consider fat-tail or serial correlation