International Actuarial Association
Association Actuarielle Internationale

Climate Change: Can Extreme Value Theory predict the “end of times” ?

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Welcome to the Webinar
We will begin shortly

A few housekeeping items before we begin:

1. Questions & Comments – Verbal capabilities have been turned off however questions or comments can be submitted by clicking on the Q&A icon at the bottom of your screen. All questions will be answered during the Q&A session at the end of the presentations.
2. Recording and presentation – A YouTube recording along with the presentation will be made available on the IAA website within the next day or so. For these and other past events, visit the IAA website at **www.actuaries.org** and click on the drop-down title **News&Events/Past Events Library**.

Welcome and thank you for joining today's webinar.

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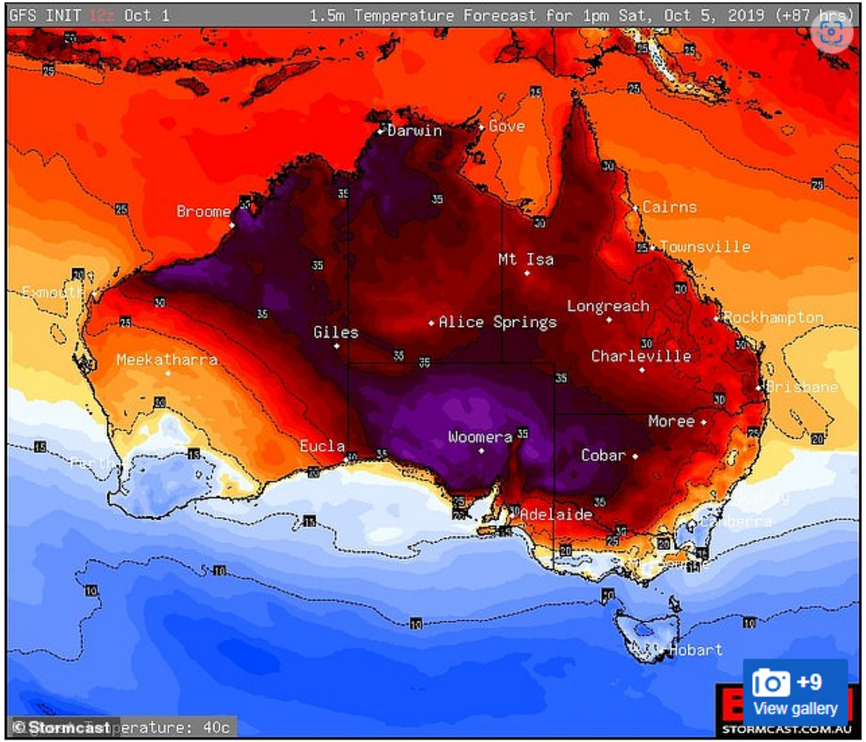
NatCat – 2021 – Heat Dome Canada

Photo via The Weather Network

NatCat – 2025 – Los Angeles Wildfire

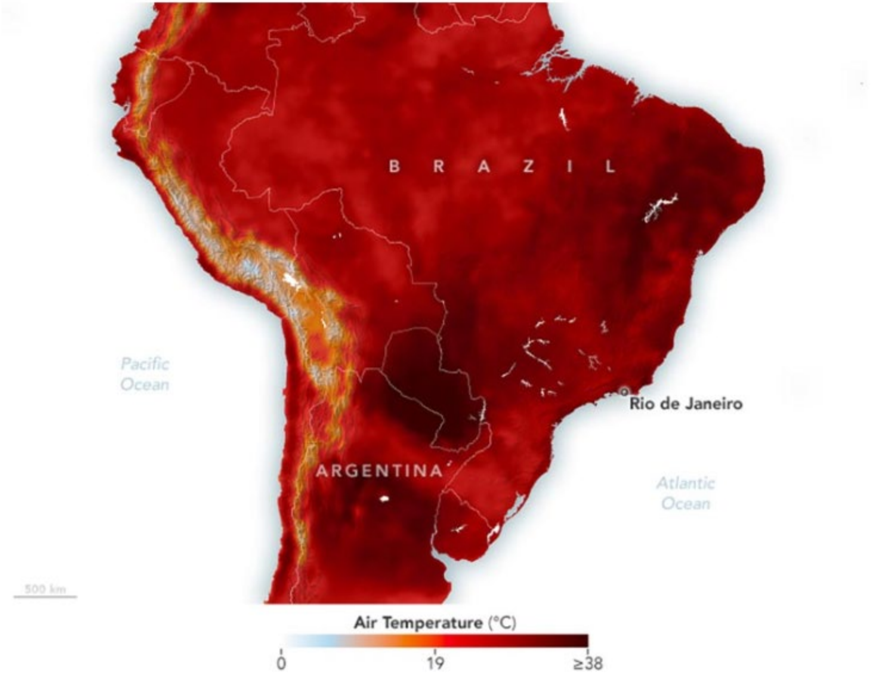
January 10, 2025 - California wildfire updates | CNN

NatCat – 2019 – Australia heatwave



A frightening weather map (pictured) shows how Australia is set to scorch through five day heatwaves and record temperatures during a blistering spring, with the purple indicating temperatures of 38C-42C

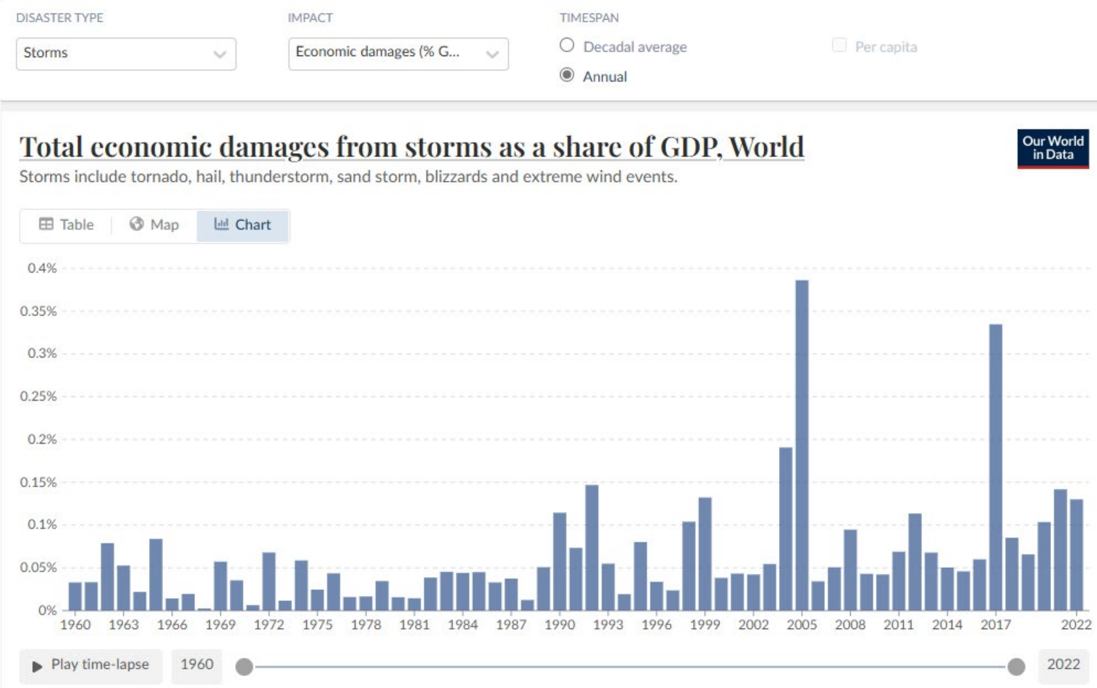
NatCat – 2025 – South America heatwave



February 2025 was a scorcher in South America, with record temperatures in parts of Brazil and notable heat in Argentina.

In February 2025, an unrelenting high-pressure system sent temperatures soaring across South America.

NatCat history – Some trends ?



Source: https://ourworldindata.org/natural-disasters

2005 : Hurricanes Katrina and Wilma

2017: Hurricanes Harvey, Maria, Irma

A few things to keep in mind

- Tail risk reassessment: example of return period of Lothar & Martin (1999, France)
- Clusters of events vs classical Extreme value theory approach
- Climate cycles
- As-if claims
- Man-made or « God/nature »-made? Nature made but badly handled?
- Trade-off beween parameters of Generalized Pareto Distribution
- What does infinite mean mean? Is it even possible? Does it make insurance or risk sharing totally impossible? Prevention, management actions!
- Uncertainty, uncertainty, uncertainty!
- Earth as a system (see feedback from Rade Musulin at GI forum after COP)

End of times in actuarial and green finance literature

- New infinite mean risks appear (with real-valued r.v.'s)
- New infinite mean risks appear (with positive probability to have infinite claim amounts)
- Finite mean claim amounts get closer to infinite mean, or become infinite mean
- Tipping points affect global climate

Worsening risks and infinite mean in actuarial research (Müller (2025)'s super-Cauchy distributions, worsening risks by Kortschak, Loisel & Ribereau (2015)

by several authors. Hofert and Wüthrich (2012) consider nuclear power accidents and estimate the parameters of a Pareto tail to have infinite mean. Eling and Wirfs (2019) consider cyber risk and also find parameter values that lead to infinite mean models. Similar observations have already been found for operational risks in Nešlehová et al. (2006).

In recent years several new papers have appeared that study this phenomenon under the assumption of Pareto distributions and more general. Chen et al. (2024b) have shown that i.i.d. Pareto distributions with infinite mean fulfill the inequality

$$X_1 \leq_{st} \sum_{i=1}^{n} \theta_i X_i, \tag{1}$$

for all $\theta_i \geq 0$, $i = 1, \ldots, n$, with $\sum_{i=1}^{n} \theta_i = 1$. Notice that we can assume without loss of generality that the weights fulfill $\theta_i > 0$, as n is arbitrary. We will sometimes use this fact in proofs. This condition means that no rational decision maker will agree to any version of linear risk sharing, see Chen et al. (2024a) for a detailed study of this case.

Extract from Müller(2025):

Nuclear power accidents
Cyber
Operational

Soon: nat cat?

Two models with worsening risks

Concretely let N_t be a Poisson process with intensity λ and X_t are independent Pareto distributed random variables with distribution $\overline{F}_t = (1+x/d_t)^{-\alpha_t}$ where the change of the distribution over time is characterized by

$$\mathbb{E}\left[X_t\right] = \frac{d}{\alpha_0 - 1}\left(1 + c_\alpha t\right). \tag{1}$$

We consider the risk process ($\rho > 0$)

$$R_t = u + \int_0^t (1+\rho)\lambda \mathbb{E}\left[X_t\right] \mathrm{d}t - \sum_{i=1}^{N_t} X_{\mathcal{T}_i} = u + \frac{(1+\rho)\lambda d(1+c_\alpha t)^2}{2c_\alpha(\alpha_0 - 1)} - \sum_{i=1}^{N_t} X_{\mathcal{T}_i},$$

where $\mathcal{T}_i$ is the time of the i-th jump. Further we will denote with $S_t = \sum_{i=1}^{N_t} X_{\mathcal{T}_i}$ and

$$p(t) = \frac{(1+\rho)\lambda d(1+c_\alpha t)^2}{2c_\alpha(\alpha_0 - 1)}.$$

Two models with worsening risks

We will now study two sets of parameters that assure that (1) holds. In the first model we change the parameter α which means that the distribution of X_t gets more and more heavy tailed. In this case we have that $\overline{F}_t^{(1)}(x) = (1 + x/d)^{-\alpha_t}$ where

$$\alpha_t = \frac{\alpha_0 - 1}{1 + c_\alpha t} + 1.$$

we will call this variant model 1. An obvious alternative to this model 2 where we only change the other parameter. i.e. we choose $\overline{F}_t^{(2)}(x) = (1 + x/d_t)^{-\alpha_0}$ where

$$d_t = d(1 + c_\alpha t).$$

Two models with worsening risks

We now want to compare these two models. An obvious method therefore is to consider the ruin probability $\psi^{(i)}(u) = \mathbb{P}(\inf_{t>0} R_t^{(i)} \leq 0)$. Denote with $\mu = \mathbb{E}[X_0] = \frac{d}{\alpha_0 - 1}$. We get

$$\psi^{(1)}(u) \sim \lambda\sqrt{\frac{2u(\alpha_0-1)}{(1+\rho)\lambda c_\alpha}}(1+u/d)^{-1}\int_0^\infty (1+t^2)^{-1}\mathrm{d}t \sim \frac{\pi d u^{-0.5}}{2}\sqrt{\frac{2(\alpha_0-1)}{(1+\rho)\lambda c_\alpha}}.$$

$$\begin{aligned}\psi^{(2)}(u) &\sim \frac{\lambda}{c_\alpha}\sqrt{u}\left(1+\frac{\sqrt{u}}{d}\right)^{-\alpha_0}\int_0^\infty \left(\frac{1}{t}+\frac{\rho\lambda\mu}{2c_\alpha}t\right)^{-\alpha_0}\mathrm{d}t \\ &\sim \frac{\lambda}{c_\alpha}d^{\alpha_0}u^{-\frac{\alpha_0-1}{2}}\int_0^\infty \left(\frac{1}{t}+\frac{\rho\lambda\mu}{2c_\alpha}t\right)^{-\alpha_0}\mathrm{d}t.\end{aligned}$$

Tipping points in actuarial and green finance literature (Green paradox, see De Angelis & Tankov (2025) and Albrecher and Loisel (2025))

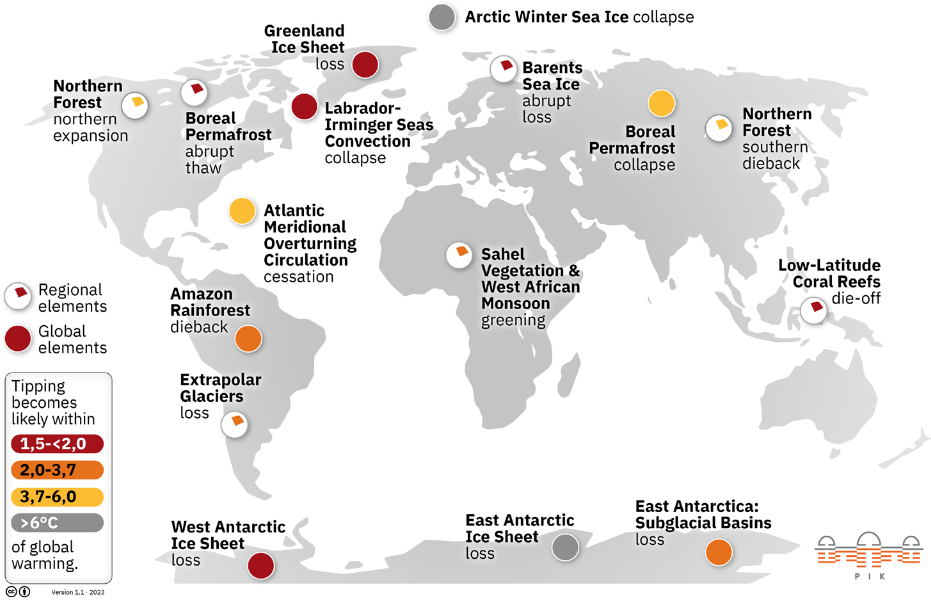
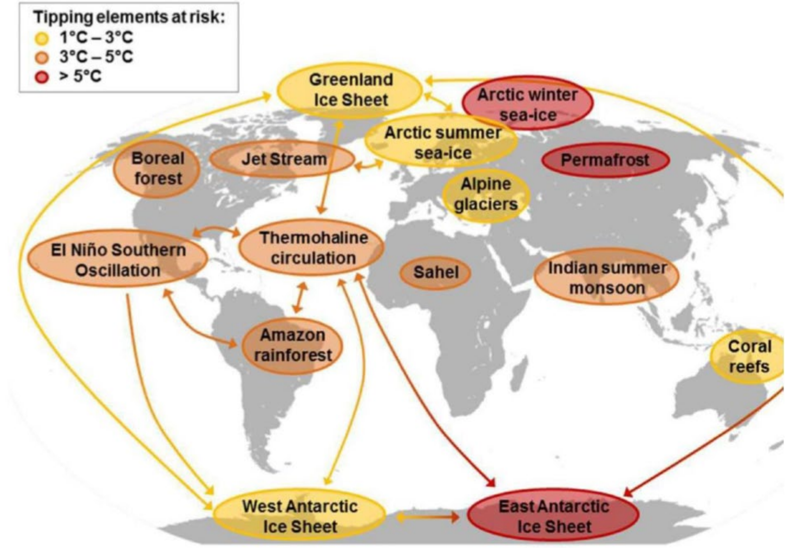


Figure designed at PIK (under cc-by licence), based on Armstrong McKay et al., Science (2022).



Steffen et al. 2018, 8252-8259

Agenda

1. Database of historical natural catastrophes
2. Fitting GPD parameters
3. Projecting GPD parameters
4. Potential applications

Database of historical natural catastrophes

EM-DAT - The international disaster database (emdat.be). Data can be freely downloaded. As indicated on their website, "the main objective of the database is to serve the purposes of humanitarian action at national and international levels. The initiative aims to rationalize decision-making for disaster preparedness and disaster risk reduction strategies, as well as provide an objective base for vulnerability assessment and priority setting".

"EM-DAT globally records, at the country level, human and economic losses for disasters with at least one of the criteria:

- 10 fatalities;
- 100 affected people;
- a declaration of state of emergency;
- a call for international assistance."

All the catastrophes were downloaded from this source and the following filtering was done:

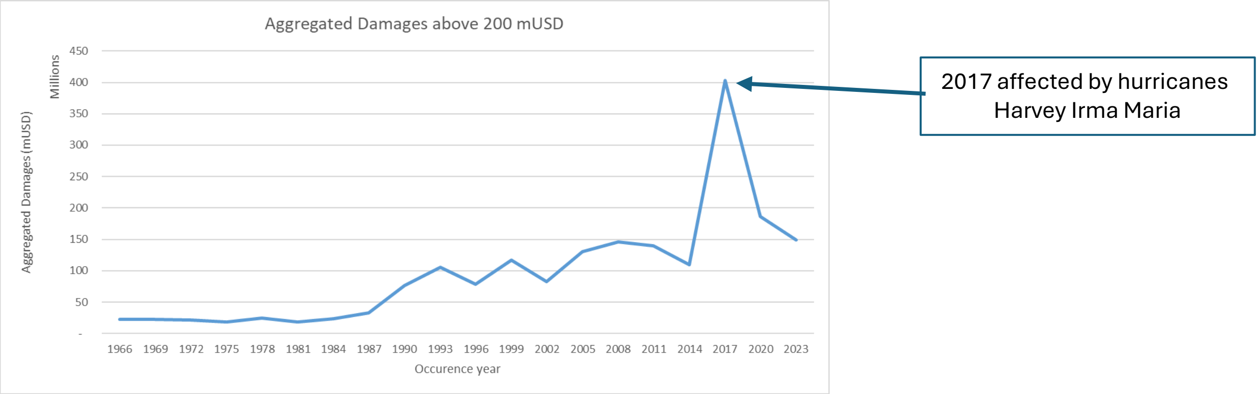
- The following natural disaster types were selected: Drought, Extreme temperature, Flood, Glacial lake outburst flood, Mass movement, Storm, Wildfire;
- Starting year for the event was selected as post 1945;
- Only events with a total damage adjusted of more than 0 and non blank were selected;
- The following three unusual events were not taken: Hurricane Katrina (2005), hurricane Andrew (1992) and the Yang Tsé flooding (1998 – 0165 – CHN).

In the database, all data starting from 1946 is used. In addition, the total damage, adjusted (USD) is used for modelling the GPD distribution.

Caveat: The provider of the data warns that data prior to occurrence year 2000 is not as trustworthy as the data post 2000.

Database of historical natural catastrophes

Based on this selection, for losses above 200 000 USD (Total damage, adjusted), the following graph can be constructed:



Extract of the database

End year (occurrence)	Total Damage, Adjusted ('000 USD)
2017	200'135
2014	200'787
2014	200'787
2001	201'206
…	…

Database contains then 2 049 observations on which we will fit GPD parameters to the Total Damage observations.

Fitting GPD parameters

Pickands–Balkema–De Haan theorem (see Pickands (1975) and Balkema et al. (1974))
Let's assume an unknown distribution function F of a random variable X. The conditional distribution function F_u of the variable X above a certain threshold u is called the conditional excess distribution function and is defined as:

$$F_u(y) = P(X - u \leq y | X > u)$$

The Pickands-Balkema-De Haan theorem states that, for a large class of underlying distribution functions F and, large u, F_u is well approximated by the generalized Pareto distribution, in the following sense. Suppose that there exist functions $a(u), b(u)$ with $a(u)>0$ such that

$F_u\left(a(u)y + b(u)\right)$ as $u \to \infty$

converges to a non-degenerate distribution, then such limit is equal to the Generalized Pareto Distribution:

$$F_u\left(a(u)y + b(u)\right) \to G_{\xi,\beta}(y)$$

where:

- $G_{\xi,\beta}(x) = 1 - \left(1 - \frac{\xi x}{\beta}\right)^{-\frac{1}{\xi}}$ if $\xi \neq 0$
- $G_{\xi,\beta}(x) = 1 - e^{-\frac{x}{\beta}}$ if $\xi = 0$

In addition, suppose X has GPD with parameters ξ <1 and β, then for a threshold u, we have:

$$e(u) = E(X - u | X > u) = \frac{\beta + \xi u}{1 - \xi} \quad (1)$$

This last equation is linear in u.

Fitting GPD parameters

In addition, suppose X has GPD with parameters ξ <1 and β, then for a threshold u, we have:

$$e(u) = E(X - u|X > u) = \frac{\beta+\xi u}{1-\xi} \quad (1)$$

This last equation is linear in u.

The linearity of equation (1) will be used on the database of Total Damage, adjusted ('000 USD) which is a series of observations $X_1, \ldots, X_n$. These observations are independent and identically distributed ("iid") and the empirical mean excess function is defined by:

$$e_n(u) = \frac{1}{N_u} \sum_{i \in \Delta_n(u)} (X_i - u) \text{ , } u > 0$$

The linearity of equation (1) suggests a graphical approach. On the available database, u is chosen as 200 mUSD and a graphical approach is suggested such that:

$e_n(x)$ is approximately linear for $x \geq u$

Fitting GPD parameters - Concretely

Empirical mean excess function:

$$e_n(u) = \frac{1}{N_u} \sum_{i \in \Delta_n(u)} (X_i - u)\,, u > 0$$

Year	**2023**

Calculation of $e_n(u)$ for different thresholds

									Thresholds u						
		Average $e_n(u)$	2,199,068	2,730,583	3,107,466	3,446,278	3,744,714	3,929,960	4,204,179	4,535,895	4,780,835	5,009,800	5,261,293	5,389,483	5,587,157
Cat number	End Year	Total Damage (adjusted) 000's USD	**200000**	**400000**	**600000**	**800000**	**1000000**	**1200000**	**1400000**	**1600000**	**1800000**	**2000000**	**2200000**	**2400000**	**2600000**
1	2022	2,602,910	2,402,910	2,202,910	2,002,910	1,802,910	1,602,910	1,402,910	1,202,910	1,002,910	802,910	602,910	402,910	202,910	2,910
2	2022	2,602,910	2,402,910	2,202,910	2,002,910	1,802,910	1,602,910	1,402,910	1,202,910	1,002,910	802,910	602,910	402,910	202,910	2,910
3	1986	2,613,119	2,413,119	2,213,119	2,013,119	1,813,119	1,613,119	1,413,119	1,213,119	1,013,119	813,119	613,119	413,119	213,119	13,119
4	2013	2,615,947	2,415,947	2,215,947	2,015,947	1,815,947	1,615,947	1,415,947	1,215,947	1,015,947	815,947	615,947	415,947	215,947	15,947
5	1998	2,616,936	2,416,936	2,216,936	2,016,936	1,816,936	1,616,936	1,416,936	1,216,936	1,016,936	816,936	616,936	416,936	216,936	16,936
6	2014	2,636,306	2,436,306	2,236,306	2,036,306	1,836,306	1,636,306	1,436,306	1,236,306	1,036,306	836,306	636,306	436,306	236,306	36,306
7	2000	2,654,196	2,454,196	2,254,196	2,054,196	1,854,196	1,654,196	1,454,196	1,254,196	1,054,196	854,196	654,196	454,196	254,196	54,196
8	2000	2,654,196	2,454,196	2,254,196	2,054,196	1,854,196	1,654,196	1,454,196	1,254,196	1,054,196	854,196	654,196	454,196	254,196	54,196
9	2012	2,654,266	2,454,266	2,254,266	2,054,266	1,854,266	1,654,266	1,454,266	1,254,266	1,054,266	854,266	654,266	454,266	254,266	54,266
10	1993	2,659,790	2,459,790	2,259,790	2,059,790	1,859,790	1,659,790	1,459,790	1,259,790	1,059,790	859,790	659,790	459,790	259,790	59,790
11	2005	2,661,737	2,461,737	2,261,737	2,061,737	1,861,737	1,661,737	1,461,737	1,261,737	1,061,737	861,737	661,737	461,737	261,737	61,737
12	2009	2,698,524	2,498,524	2,298,524	2,098,524	1,898,524	1,698,524	1,498,524	1,298,524	1,098,524	898,524	698,524	498,524	298,524	98,524
13	2023	2,700,000	2,500,000	2,300,000	2,100,000	1,900,000	1,700,000	1,500,000	1,300,000	1,100,000	900,000	700,000	500,000	300,000	100,000
14	1976	2,703,899	2,503,899	2,303,899	2,103,899	1,903,899	1,703,899	1,503,899	1,303,899	1,103,899	903,899	703,899	503,899	303,899	103,899
15	2011	2,709,192	2,509,192	2,309,192	2,109,192	1,909,192	1,709,192	1,509,192	1,309,192	1,109,192	909,192	709,192	509,192	309,192	109,192
16	2011	2,709,192	2,509,192	2,309,192	2,109,192	1,909,192	1,709,192	1,509,192	1,309,192	1,109,192	909,192	709,192	509,192	309,192	109,192
17	2005	2,730,418	2,530,418	2,330,418	2,130,418	1,930,418	1,730,418	1,530,418	1,330,418	1,130,418	930,418	730,418	530,418	330,418	130,418
18	2003	2,732,998	2,532,998	2,332,998	2,132,998	1,932,998	1,732,998	1,532,998	1,332,998	1,132,998	932,998	732,998	532,998	332,998	132,998
19	2017	2,734,761	2,534,761	2,334,761	2,134,761	1,934,761	1,734,761	1,534,761	1,334,761	1,134,761	934,761	734,761	534,761	334,761	134,761

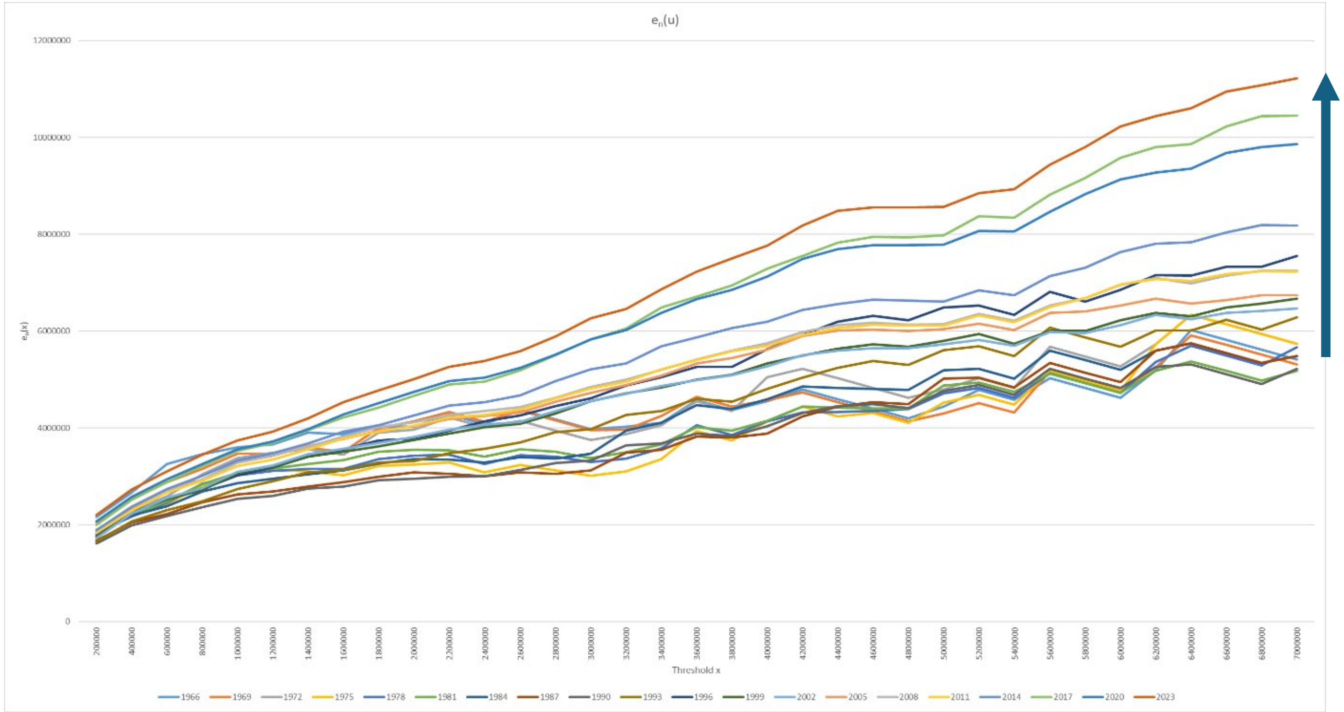
Selection of cat numbers according to the observed year

In theory, $e_n(u)$ is linear in u.

Excess loss over u for different u X_i-u

Fitting GPD parameters

Empirical mean excess function $e_n(x)$ for different years:



The slope seems to increase !

Fitting GPD parameters

Once slope and intercept depending on the year are estimated (e.g. using the function "slope" and "intercept" of excel), it is easy to find the parameters of the GPD distribution:

$$\xi_{Year} = \frac{Slope_{year}}{1 + \ Slope_{year}}$$

Then, in terms of Pareto parameter, we have:

$$\alpha_{year} = \frac{1}{\xi_{Year}}$$

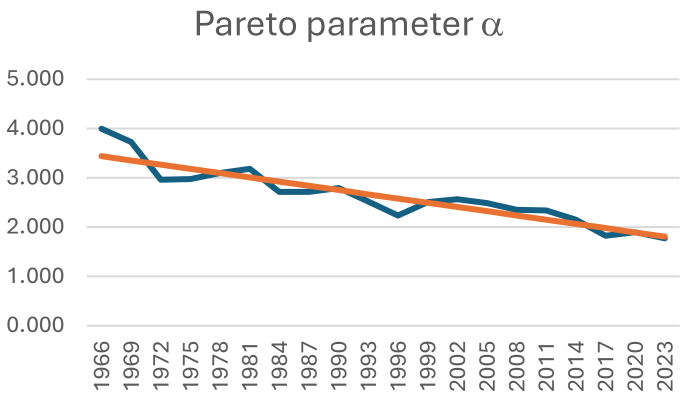
As for a Pareto distribution, there is no expected value as soon as α is below 1.

This is the definition of the "end of times"

Let's look at the shape of the α_{year} !

Projecting GPD parameters

Let's look at the shape of the α_{year} !

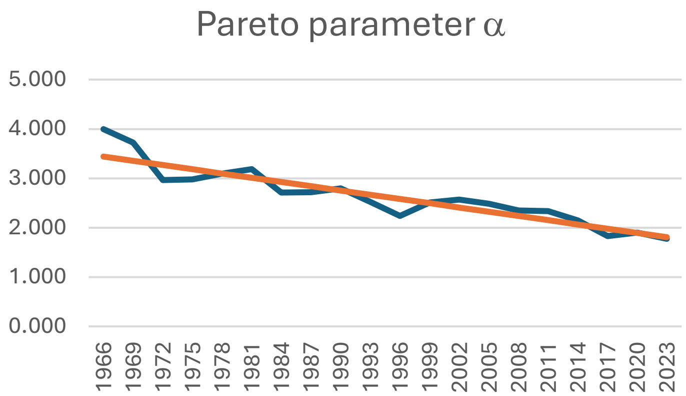


Slope = t = -2.865%
Intercept = b = 59.77

A simple linear projection implies that α is going to be above 1 until the year 2051 !

The "end of times" should be around the year 2051

Potential applications



Pricing NatCat / Property risk

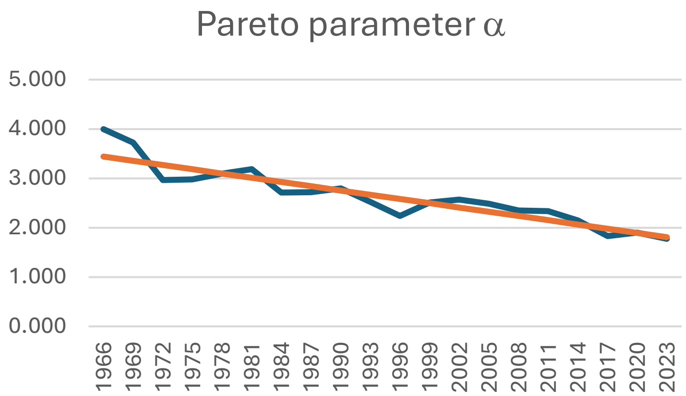
- Year on year, when using a pareto distribution/commercial model (e.g. RMS), an “aggravation” factor should be added to the price.
  - Corresponds to an **indexation of the priorities** year on year (factor independent from the priority level)
  - It is an **inflation** increasing as time goes by!

$$Factor = \frac{\frac{\alpha_{n+1} x_m}{\alpha_{n+1} - 1}}{\frac{\alpha_n x_m}{\alpha_n - 1}} = \frac{-t}{\alpha_n(\alpha_{n+1} - 1)}$$

Year	Inflation	Alpha
2025		1.750099
2026	2.27%	1.721448
2027	2.40%	1.692796
2028	2.55%	1.664144
2029	2.71%	1.635492
2030	2.89%	1.60684
2031	3.08%	1.578188
2032	3.30%	1.549536
2033	3.55%	1.520884
2034	3.83%	1.492232
2035	4.14%	1.46358
2036	4.50%	1.434928
2037	4.91%	1.406276
2038	5.40%	1.377624

Year	Inflation	Alpha
2039	5.96%	1.348972
2040	6.63%	1.32032
2041	7.44%	1.291668
2042	8.43%	1.263016
2043	9.68%	1.234365
2044	11.28%	1.205713
2045	13.42%	1.177061
2046	16.40%	1.148409
2047	20.83%	1.119757
2048	28.09%	1.091105
2049	42.05%	1.062453
2050	79.78%	1.033801
2051	538.26%	1.005149

Potential applications



Capital requirement - Solvency

- Year on year, the capital requirement for natcat products should increase as the alpha decreases.
- For a quantile p, we have:

$$Factor = \frac{(1-p)^{-\frac{1}{\alpha_{n+1}}}}{(1-p)^{-\frac{1}{\alpha_n}}}$$

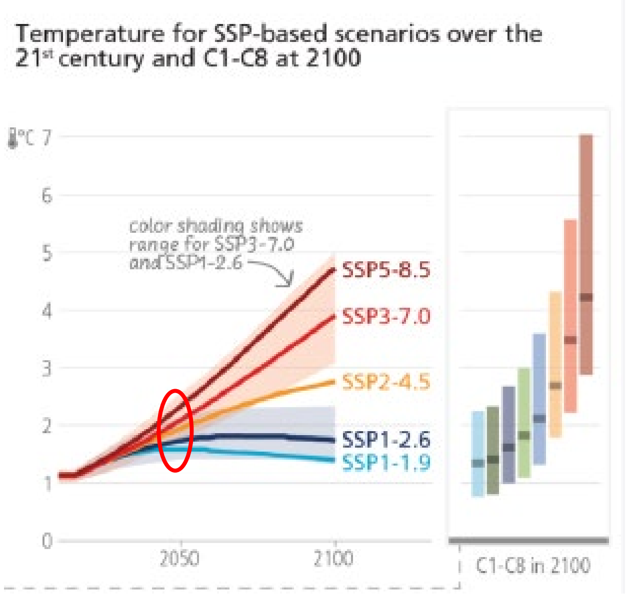
Year	Alpha	Capital increase (VaR 99.5%)
2025	1.750099	
2026	1.721448	5.2%
2027	1.692796	5.3%
2028	1.664144	5.5%
2029	1.635492	5.7%
2030	1.60684	5.9%
2031	1.578188	6.2%
2032	1.549536	6.4%
2033	1.520884	6.7%
2034	1.492232	6.9%
2035	1.46358	7.2%
2036	1.434928	7.5%
2037	1.406276	7.8%
2038	1.377624	8.2%

Year	Alpha	Capital increase (VaR 99.5%)
2039	1.348972	8.5%
2040	1.32032	8.9%
2041	1.291668	9.3%
2042	1.263016	9.8%
2043	1.234365	10.2%
2044	1.205713	10.7%
2045	1.177061	11.3%
2046	1.148409	11.9%
2047	1.119757	12.5%
2048	1.091105	13.2%
2049	1.062453	14.0%
2050	1.033801	14.8%
2051	1.005149	15.7%

p=99.5%

Conclusions

- It is possible to predict a time where climate is going to become extremely harsh.
- This will affect insurance companies (but not only …).
- It is possible to adapt premiums and capital requirements based on this study.
- But … it is going to be very costly !
- And whatever we do now, the future by 2050 is known (see IPCC 6th report (below)):



Q&A Session

Should you have any questions or comments,
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